

Multi-Granular Conflict and Dependency Analysis in Software Engineering based on Graph Transformation

Leen Lambers
Hasso-Plattner-Institut Potsdam,
Germany
leen.lambers@hpi.de

Daniel Strüber
Universität Koblenz-Landau,
Germany
strueber@uni-koblenz.de

Gabriele Taentzer, Kristopher
Born, Jevgenij Huebert
Universität Marburg, Germany
{taentzer,born,huebert}@mathematik.
uni-marburg.de

ABSTRACT

Conflict and dependency analysis (CDA) of graph transformation has been shown to be a versatile foundation for understanding interactions in many software engineering domains, including software analysis and design, model-driven engineering, and testing. In this paper, we propose a novel static CDA technique that is multi-granular in the sense that it can detect all conflicts and dependencies on multiple granularity levels. Specifically, we provide an efficient algorithm suite for computing *binary*, *coarse-grained*, and *fine-grained* conflicts and dependencies: Binary granularity indicates the presence or absence of conflicts and dependencies, coarse granularity focuses on root causes for conflicts and dependencies, and fine granularity shows each conflict and dependency in full detail. Doing so, we can address specific performance and usability requirements that we identified in a literature survey of CDA usage scenarios. In an experimental evaluation, our algorithm suite computes conflicts and dependencies rapidly. Finally, we present a user study, in which the participants found our coarse-grained results more understandable than the fine-grained ones reported in a state-of-the-art tool. Our overall contribution is twofold: (i) we significantly speed up the computation of fine-grained and binary CDA results and, (ii) complement them with coarse-grained ones, which offer usability benefits for numerous use cases.

CCS CONCEPTS

•Software and its engineering → Automated static analysis;

1 INTRODUCTION

Conflicts and dependencies are fundamental phenomena in software engineering. For example, when a software system is developed collaboratively [61], a change operation can facilitate or prohibit other change operations. In concurrent programming, conflicts may arise from data races [28, 55] when a thread writes to a memory location accessed by another thread. From unrecognized conflicts and dependencies, severe consequences may arise, ranging from productivity obstacles to fatal safety hazards. Therefore, there is a need for techniques to detect conflicts and dependencies automatically.

Graph transformation [17, 18, 52] has been shown to be a versatile foundation for supporting conflict and dependency detection in software engineering, based on the following three principles: First, graphs are used for representing structures of interest, such as states of computation [1, 7] or versions of the system structure [8]. Second, certain changes, such as state or structure modifications, are described using graph transformation rules. Third, the provided

transformation specification is fed to the static *conflict and dependency analysis* (CDA) of graph transformations [27, 47]: Given a set of transformation rules, all conflicts and dependencies arising from a given pair of rules are identified. A conflict arises, for example, if the first rule application deletes an element required by the second rule application. A key benefit of graph transformation is its mature formal foundation, which supports CDA techniques that are correct by design: all conflicts and dependencies can be detected.

Based on these principles, the CDA of graph transformations has enabled a large number of *use-cases in software engineering*, including analysis and design, model-driven engineering, and testing. For example, graph transformations can be used to model the execution behavior of Java programs in terms of preconditions and effects on the object structure; identified conflicts and dependencies are then used as oracles for test generation [1, 53]. In software product line engineering, feature interactions can be detected by specifying features as graph transformations and identifying conflicts and dependencies with CDA [33]. In model-based refactoring [45], graph transformations and CDA are used to find a suitable order of refactoring steps. One contribution of this paper is a literature survey that overviews 25 papers describing such use-cases.

Although generally helpful for the task at hand, however, several authors report that the used CDA technique showed severe limitations. In our survey, we identify three key requirements for an improved CDA technique for software engineering: it shall be (i) *domain-independent* to be applicable to a large variety of software engineering domains, (ii) *usable* in the sense that it should display a reasonable amount of information to support understandability, and (iii) *efficient* when applied to software projects of realistic size.

To address these requirements, we present a novel static CDA technique for software engineering based on graph transformation. The technique is based on the notion of *granularity* of conflicts and dependencies introduced in [10]: Often, the user merely requires to know if a given rule pair can induce conflicts or dependencies at all, while details are irrelevant (*binary granularity*). At the next level, the user wants to pinpoint certain elements that present the root causes of conflicts or dependencies (*coarse granularity*). At the final level, a complete description of each potential conflict and dependency is required (*fine granularity*). To enable an efficient computation, we present an algorithm suite which can compute binary, coarse-grained, and fine-grained results rapidly. The computation of fine-grained ones harnesses coarse-grained ones. Moreover, we conducted a user study, in which the participants found coarse-grained analysis results more understandable and easier to work with than fine-grained ones reported by a state-of-the-art tool.

In summary, this paper presents a **multi-granular CDA technique based on graph transformation** achieving the same level of (i) domain-independence as the state of the art, while providing major (ii) understandability and (iii) performance improvements. Specifically, we make the following contributions:

- A **literature survey** of existing CDA use-cases (Sec. 3), focusing on granularity requirements.
- A **formalization** of different granularity levels of conflict and dependencies (Sec. 4), for ensuring the well-foundedness of our technique.
- An **algorithm suite** supporting the computation of CDA results at multiple granularity levels (Sec. 5).
- An **implementation evaluation**, in which we study the performance of our algorithm suite (Sec. 6).
- A **user study** to determine the usefulness of coarse-grained conflict results in comparison to fine-grained ones (Sec. 7).

With these contributions, we aim to improve on the state-of-the-art CDA technique, critical pair analysis [42, 58] (CPA). CPA does not distinguish between granularity levels: Since its goal is to provide all conflicts and dependencies, its output—a list of *critical pairs* depicting each conflict situation in a minimal context—always exhibits fine granularity. From the lack of support for different granularity levels, two main drawbacks arise. First, computing all critical pairs can be computationally vastly expensive. Second, comprehending the critical pairs of a set of rules can be a daunting task, since the list of critical pairs generally reflects numerous options to combine the involved root causes.

Our work is the first to provide a general CDA technique for graph transformations supporting multiple granularity levels. Earlier usage scenarios either used CPA, or task-specific CDA techniques relying on the structure of the involved rules [31, 35]. Our earlier work [10, 40] serves as a formal foundation for the current one. We now amend the existing declarative definitions with constructive characterizations that support efficient computations. To the best of our knowledge, we provide the first, albeit preliminary, empirical evidence regarding the usefulness of CDA techniques.

2 RUNNING EXAMPLE

Our running example deals with requirement elicitation for a web shop, a service-oriented software system that enables a retailer to sell goods on a website. Customers can perform orders and inspect the information on goods and orders. The retailer uses the available information to manage its business processes. We focus on two business processes, one for *order management* and one for *data mining*, which we do not want to interfere with each other. This means that activities of one process should not render activities of the other process impossible; hence, conflicts between activities of different processes should not occur. To be able to analyze conflicts, we specify activity requirements with graph transformation. A transformation rule specifies pre- and post-conditions of activities.

In our example, we consider two activities of the selected business processes to be specified by rules: Rule *returnUnpaidGood* on the left of Fig. 1 returns a *Good* into the *Stock* by deleting the corresponding *OrderItem* and *BillItem*. In addition, the returned good is removed from the customer. Rule *offerGift* specifies a pattern for data mining which looks for an *Order* with at least two *OrderItems*.

The *Customer* ordering and owning these items is marked for a *GiftOffer*. Note that a customer may own a good without having it ordered and paid since it may be a gift. Both rules are depicted in an integrated form where annotations specify which graph elements are deleted, preserved, and created. While the preserved and deleted elements form the left-hand side (LHS) of a rule, the preserved and created elements form its right-hand side (RHS).

Conflict and granularity considerations. To run the selected business processes for *order management* and for *data mining* concurrently, they shall not interfere with each other. This is the case if their activities do not interfere pairwise (neglecting any potential control flow on activities). We focus our investigations on activity *returnUnpaidGood* being part of the order management process and *offerGift* being some data mining activity. To investigate interferences between these two activities, we analyze two rules specifying them. If an application of the first rule renders an application of the second rule impossible or, if the second rule is still applicable, but not at the original match anymore, a conflict occurs. To reduce the amount of conflicts, we need to identify all their potential sources.

If the developer is just interested in knowing whether a given pair of rules can induce a conflict, this information can be easily given in a table such as Table 1 where a ‘+’ marks that a conflict for the given rule pair arises while ‘-’ marks that there is no conflict.

Rule 1 / Rule 2	returnUnpaidGood	offerGift
returnUnpaidGood	+	+
offerGift	-	-

Table 1: Binary information about conflicts

To get a *coarse understanding* of conflicts, the developer may be interested in knowing which rule elements can cause conflicts. Since rules *returnUnpaidGood* and *offerGift* specify activities of two processes that shall not interfere, we are especially interested in understanding all conflicts related to this rule pair. Two reasons for conflicts are shown in the middle of Figure 1. Conflict-causing elements are included in minimal graphs that describe the needed overlap of participating rules (depicted on the left and on the right) to cause an actual conflict on their applications. The upper graph specifies the deletion of an order item needed to offer a gift. The lower one shows the deletion of an *owns*-edge needed to offer a gift. Each of these graphs with their embeddings into rules are called *minimal conflict reasons*. Overlapping the rules along such a conflict reason yields two conflicting rule applications called *critical pair*.

For a *fine-grained representation* of all conflicts, we also have to consider all possible combinations of root causes. This means that all possible compositions of minimal conflict reasons describe further conflict situations which we just call *conflict reasons*. One example of such a conflict reason is shown in Figure 2, where the order item as well as the targeting *owns*-edge for one and the same good are deleted, both needed to offer a gift. This graph induces one of altogether 6 conflict reasons caused by rule *returnUnpaidGood* on rule *offerGift*. The conflict reasons not shown are analogous to the three ones depicted in Figures 1 and 2. The remaining three ones overlap 5: *OrderItem* and 6: *Good* of rule *returnUnpaidGood* with 3: *OrderItem* and 4: *Good* of rule *offerGift*.

The state-of-the-art CDA which computes *essential critical pairs* [42], however, yields more results for this rule pair, namely 10 pairs

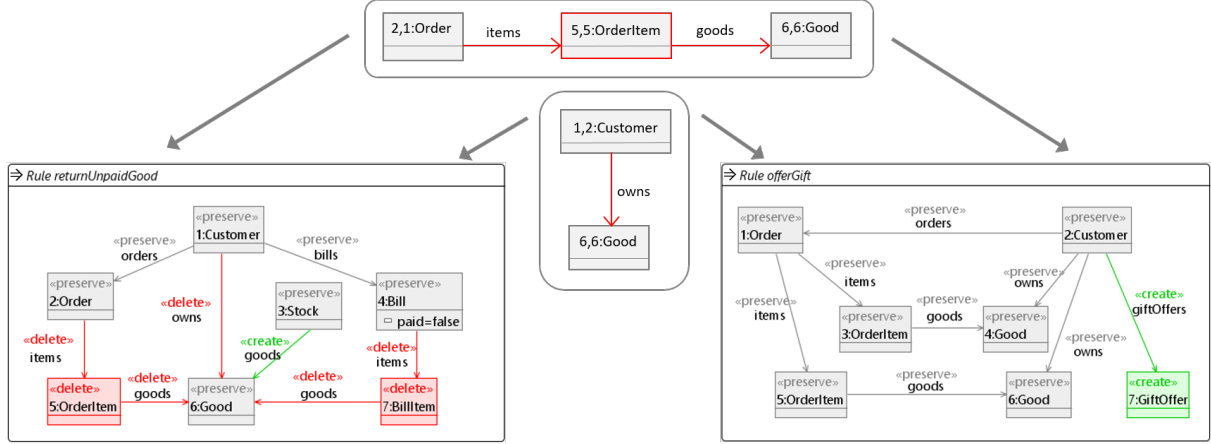


Figure 1: Rules *returnUnpaidGood* and *offerGift* and two minimal conflict reasons – Coarse-grained information



Figure 2: One of 6 conflict reasons of rule *returnUnpaidGood* on rule *offerGift* – Excerpt of fine-grained information

of conflicting transformations. While 6 of them directly correspond to the ones discussed above, there are 4 further results which do not show basically new conflict reasons but just nuances of the considered ones. Hence, the existing CDA provides an even more fine-grained information about conflicts. Table 2 compares the numbers of coarse and fine-grained results for our example, distinguishing our intended fine-grained analysis (indicated by 'New Fine') from the existing CDA (indicated by 'Ex. Fine'). While the entry for rule pair (*returnUnpaidGood*, *offerGift*) shows a moderate increase of numbers, this is already more striking for rule pair (*returnUnpaidGood*, *returnUnpaidGood*) demonstrating a bigger difference between numbers of coarse and existing fine-grained conflict information (3 versus 19). In general, a lot of different critical pairs can exist and it may be tedious to go through all of them. In Section 6, we found an example rule pair with 1588 ess. critical pairs vs. 24 min. conflict reasons.

Rule 1 / Rule 2	<i>returnUnpaidGood</i>	<i>offerGift</i>
<i>returnUnpaidGood</i>	Coarse: 3 New Fine: 7 Ex. Fine: 19	Coarse: 4 New Fine: 6 Ex. Fine: 10

Table 2: Number of conflict reasons and ess. critical pairs, resp., in coarse and fine-grained representations

Conclusion. Instead of overwhelming the user with a large number of conflicts as the state-of-the-art fine-grained CDA does, a multi-granular analysis supports a continuously deeper understanding of conflicts where needed. In this example, rule pair (*returnUnpaidGood*, *offerGift*) is of special interest since it may be conflicting and specifies activities of two processes that should run independently of each other. From the analysis, we can deduce that data mining on unpaid goods may lead to inconsistencies. To avoid such conflicts, the activities may be adapted by, e.g., offering a gift only for goods that have been already paid.

3 LITERATURE SURVEY

We conducted a literature survey to explore the variety of software engineering domains in which critical pair analysis (CPA) has been applied, the designated state-of-the-art technique for conflict and dependency analysis (CDA) based on graph transformation. First, we present collected statements to performance and usability of the CPA. Then we elicit requirements regarding the actual granularity level needed when performing CDA in specific use-cases.

To identify use-cases in the literature, we applied the search clause "critical pair*" AND ("graph transformation" OR "model transformation") to the five major CS online libraries of ACM, IEEE, Elsevier, Wiley, and Springer, restricting the search to mentions in title and abstract. This search yielded an initial body of 37 papers, to which we added 11 based on our knowledge about CPA uses. We discarded those that focused on theoretical results, and those represented by other papers in the same line of work on the same overall use-case. We grouped the remaining 25 papers in four main software engineering application domains (see Table 3).

Performance and usability statements. In several papers such as [13, 23, 43, 62], the authors recognized severe *performance problems* when using the state-of-the-art implementation of the CPA in AGG [58]. They conclude that CPA does not scale for industrial use. Often a too large number of critical pairs is computed which makes the *manual inspection* of the CPA results *nearly impossible*. As one solution to increase the performance and to drop the number of results, authors tried to constrain the allowed graphs. Another way out of this dilemma was to replace the static CDA by a runtime check. However, these actions either change the kind of graphs considered or switch from static to dynamic analysis. From this review of *performance and usability* statements, we conclude that a highly performant static analysis is needed which produces a concise set of results that is easy to inspect manually. The essential CPA [42], also available in AGG, was introduced as a first solution to these requirements yielding a considerably smaller set of results in a smaller amount of time. We observed, however, that even the essential CPA often returns *too fine-grained results* representing an obstacle for performance as well as for usability when doing CDA. Therefore, we analyze now the requirements w.r.t. the actual level of granularity needed when performing CDA.

Granularity SE domain	Analysis and design of software systems	MDE techniques	Testing	Optimization of rule-based computations
Binary				Graph parsing [12], Activity diagram validation [20], Edit operation recognition [35], Nondeterminism detection [23, 29]
Coarse-grained	Consistency validation of use-cases [26], service-based systems [38, 41], context-aware and adaptive systems [13, 15, 43], activity diagrams [20]; Feature interaction detection [2, 33, 44, 62]	Model versioning [39, 59], Refactoring recommendation [45, 49], Detecting & resolving model inconsistencies [46]	Test case generation [31] & validation [53]	
Fine-grained		Verification of model transformations [6, 30]		

Table 3: Granularity requirements and software engineering domains of the CPA in literature survey

SE domains and granularity requirements. We describe our findings w.r.t. granularity requirements of the CPA use cases along their application domains in SE as illustrated in Table 3.

In *software system design and analysis*, the conformance of behavior models such as activity models and live sequence charts, with the rule-based specification of activities (methods, operations, or services) is investigated. The CPA is mostly used to find and understand conflicts and dependencies in the data flow and to reason about their plausibility w.r.t. the considered system. In this context, a *coarse-grained CDA* seems to be sufficient to start with.

Model-driven engineering (MDE) techniques are often specified on the basis of model transformations. The CPA can be used to detect and to reason about the plausibility of conflicts and dependencies between specified transformations. In model version management, the CPA is moreover used to resolve conflicts between model changes. The *coarse-grained analysis* seems to be sufficient to find conflicts and dependencies between transformation specifications here. Confluence proofs, however, have to be performed based on a *fine-grained analysis* since these proofs are based on completeness of the CPA results which is only given in the fine-grained case.

In *testing*, the CPA is used for reasoning about and generating interesting test cases. A *coarse-grained analysis* of rule interdependencies seems adequate to understand the specified activities.

In *optimization of rule-based computations*, the CPA has been used to find out which rule pairs are conflicting or dependent at all and to exploit this information for improving the computation. The non-existence of conflicts or dependencies may allow computations without backtracking. Hence, *binary information* about the existence of conflicts or dependencies is usually sufficient to either avoid or deliberately postpone backtracking. For further optimization, the elimination of existing conflicts or dependencies may be a choice. Further information, i.e., a *coarse-grained analysis*, is then needed to understand their causes and to modify rules accordingly.

Threats to Validity Regarding *construct validity*, we omit investigating expressiveness of the analyzed graph transformations w.r.t. advanced transformation features such as negative application conditions, which is orthogonal to the required granularity level. The extension of our technique to these features is ongoing work. While not being trivial, we are confident that this is possible, since the underlying theory is given in a category-theoretical setting.

Conclusion. We deduce the following granularity requirements for CDA: (1) *Binary granularity* refers to the situation where the relevant information is whether a conflict/dependency between two graph transformations rules exists or not. Such information

is sufficient, e.g., to trim a solution space of possible alternatives. (2) *Coarse granularity* refers to the situation where users need to inspect individual conflicts and dependencies, but do not need to know the precise details of each possible conflict or dependency situation. (3) *Fine granularity* is needed to inspect each conflict and dependency situation in-depth. This is necessary, for example, to reason about confluence of a state transition relation. Our observations (in Table 3) demonstrate that for most considered applications of conflict and dependency analyses, a binary or coarse-grained analysis would have been sufficient or would have represented a good starting point for analysis that can – only if necessary – still be refined to a more fine-grained one. These granularity requirements support the *need for a multi-granular approach* to CDA.

4 CONFLICT AND DEPENDENCY CONCEPTS

We revisit conflict and dependency concepts for graph transformation with varying granularity level in Sec. 4.1. As a new contribution, we instantiate the binary, coarse and fine granularity level (as identified in Sec. 3) of our multi-granular CDA technique with adequate formal conflict and dependency concepts in Sec. 4.2.

4.1 Background

We recall main concepts from graph transformation with the conflict notion underlying our work [17, 42]. Then, conflict and dependency concepts with varying granularity level are recalled [10, 40].

Graph transformation. Representing complex structures as graphs, *graph transformation* is one of the main paradigms to describe their rule-based modification. A *rule* mainly consists of two graphs: L is the left-hand side (LHS) of the rule representing a pattern that has to be found to apply the rule. After the rule application, a pattern equal to R , the right-hand side (RHS), has been created. The intersection $K = L \cap R$ is the part that is not changed, the part that is to be deleted is defined by $L \setminus K$, while $R \setminus K$ defines the part to be created. To make the deletion part of a rule a graph, we add all *boundary nodes* $B \subseteq K$, hence obtain *deletion graph* $C = L \setminus (K \setminus B)$. If C is empty, the rule is called *non-deleting*.

A *graph transformation* $G \xRightarrow{r,m} H$ between two graphs G and H is defined by first finding a *match* m , that is a mapping of the LHS L of rule r into G such that m is injective and fulfills the *dangling condition* [17]: all adjacent graph edges of a graph node to be deleted must be deleted as well. Second, we construct H in two passes: (1) build $D := G \setminus m(L \setminus K)$, i.e., erase all graph elements that are to

be deleted; (2) construct $H := D \cup m'(R \setminus K)$ such that a new copy of all graph elements that are to be created is added.

Example [Graph rule and transformation] Figure 3 shows a graph to which rules *returnUnpaidGood* and *offerGift* in Figure 1 are applicable. Considering, e.g., rule *returnUnpaidGood* the red and the grey parts form the LHS and the RHS consists of the green and the grey parts. Hence, K is represented by the grey part. Boundary nodes are $1:Customer$, $2:Order$, $4:Bill$, and $6:Good$. Deletion graph C consists of the red rule part together with all boundary nodes.

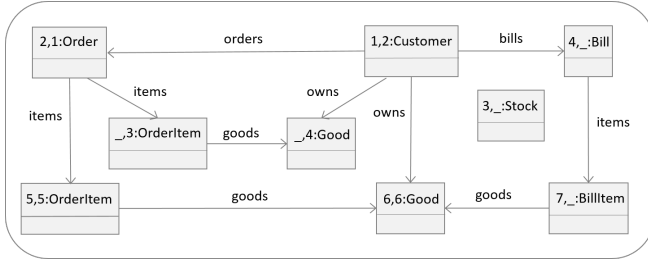


Figure 3: Example graph to which rules *returnUnpaidGood* and *offerGift* are applicable, with indicated matches

The rule's matches are indicated by numbers. For example, $1:Customer$ of rule *returnUnpaidGood* and $2:Customer$ of rule *offerGift* are both mapped to $1,2:Customer$. An ' ' indicates that this node is not in the corresponding match. Note that rule *returnUnpaidGood* can be applied in two different ways to this graph. Since rule *offerGift* is non-deleting, the dangling condition is always fulfilled. Rule *returnUnpaidGood*, however, deletes two nodes: $5:OrderItem$ and $7:BillItem$. Their images in the graph are not allowed to have dangling edges, i.e., edges without origins in the LHS. This is the case here, hence both mappings fulfill the dangling condition.

The effect of applying rule *returnUnpaidGood* at the given match is the deletion of edge *owns* from $1,2:Customer$ to $6,6:Good$ as well as of nodes $5,5:OrderItem$ and $7, -:BillItem$ with adjacent edges. In addition, a new edge from $3, -:Stock$ to $6,6:Good$ is added.

Conflict. Given a graph G there are, in general, several rules applicable at different matches. A pair of transformations $(G \xRightarrow{r_1, m_1} H_1, G \xRightarrow{r_2, m_2} H_2)$ is *in conflict* if the first rule application deletes graph elements used by the second one, i.e., if $m_1(C_1 \setminus B_1) \cap m_2(L_2)$ is not empty. Since matches are injective, we can build a *conflict part* $m_1^{-1}(m_1(C_1 \setminus B_1) \cap m_2(L_2)) \subseteq C_1$ that can be completed by adding incident boundary nodes to a *conflict graph* S_1 , being a subgraph of the deletion graph C_1 of rule r_1 . Moreover, we have a mapping $e_2 = m_2^{-1} \circ m_1$ of S_1 into L_2 . Together with its embedding into C_1 , a *span* $s_1 = (C_1 \xleftarrow{e_1} S_1 \xrightarrow{e_2} L_2)$ for rule pair (r_1, r_2) can be defined which distills the cause of a conflict and therefore, is called *conflict reason* for $G \xRightarrow{r_1, m_1} H_1$ and $G \xRightarrow{r_2, m_2} H_2$. Given a span $s_1 = (C_1 \xleftarrow{e_1} S_1 \xrightarrow{e_2} L_2)$ for rule pair (r_1, r_2) and mappings $m_1 : L_1 \rightarrow G$ and $m_2 : L_2 \rightarrow G$ we say that these mappings *overlap* in s_1 if $m_1(S_1) \cap m_2(e_2(S_1)) \subseteq m_1(L_1) \cap m_2(L_2)$.

Example [Conflict] In the graph in Figure 3 with the given matches of rules *returnUnpaidGood* and *offerGift*, the resulting pair of transformations is in conflict since $m_1(C_1 \setminus B_1) \cap m_2(L_2)$ contains node $5,5:OrderItem$ with adjacent edges and edge *owns* from $1,2:Customer$

to $6,6:Good$, i.e. these elements are deleted by the first transformation and used by the second one. The corresponding conflict reason is given by the conflict graph in Figure 2, its embedding into the deletion graph of rule *returnUnpaidGood*, and its mapping into the LHS of rule *offerGift*. Both embeddings are given by numbers (compare Figures 1 and 2). We focus on the conflict graph of a conflict reason while embeddings are just given by corresponding numbers as explained above.

Conflict concepts. Heading towards a static conflict analysis, we do not investigate each pair of graph transformations but analyze *rule pairs* instead. Rule pair (r_1, r_2) is *in conflict* if there is any pair of conflicting transformations applying rule r_1 and then r_2 .

We further concentrate on rule parts that may cause conflicts. The minimal building bricks of conflict causes are called *conflict atoms*. An atom is derived from an *atom candidate* being a span $a_1 = (C_1 \xleftarrow{e_1} A_1 \xrightarrow{e_2} L_2)$ for rule pair (r_1, r_2) , where A_1 is a single deleted node or edge incident with preserved nodes. It is deleted by the first rule and used by the second one. A deleted edge with at least one incident deleted node is not considered as atom candidate, since the edge is deleted together with the deleted node anyway. If a pair of transformations exists so that their match mappings overlap on the atom candidate, it is called *conflict atom*. Note that, in general, the matches of such a pair of transformations may overlap also in graph elements other than the conflict atom.

A *conflict reason* $s_1 = (C_1 \xleftarrow{e_1} S_1 \xrightarrow{e_2} L_2)$ for rule pair (r_1, r_2) subsumes all atoms being involved in a conflict. Among the conflict reasons for two rules, there are *minimal reasons* describing minimal compositions of atoms leading to conflict reasons.

In contrast to conflict reasons showing conflict-causing *rule overlaps*, a *critical pair* consists of two conflicting *transformations* applying two rules in a minimal context, where all elements stem from L_1 or L_2 or both. Critical Pair Analysis (CPA) [27, 47] is the state-of-the-art static CDA for graph transformation. The set of critical pairs has the important property that it is *complete*: each possible conflicting pair of transformations is represented by some critical pair. Two important subsets of critical pairs being still complete have been identified in the literature: An *essential critical pair* [42] is one where the rules' LHSs overlap merely deletion graph elements of one rule with LHS elements of the other rule, i.e. only the conflict reason is overlapped. The rationale is that overlapping additional preserved elements (as done in regular CPA) does not contribute to new conflicts. (Essential) Critical pairs can be computed with the state-of-the-art implementation of the CPA in AGG [58] and VeriGraph [5]. An *initial conflict* [40] is an essential critical pair without isolated boundary nodes. The latter arise when a preserved node with incident deletion edges of the first rule is overlapped with a node of the second rule without overlapping any incident deletion edge of the first rule. Such overlaps do not contribute to new conflicts. Initial conflicts represent currently the most optimal subset of critical pairs fulfilling the completeness property, but their detection has not been implemented yet.

Example [Conflict concepts]. Considering the conflict reason in Figure 2, it is covered by two conflict atom graphs consisting of node $5,5:OrderItem$ and edge *owns* from $1,2:Customer$ to $6,6:Good$. We can complete conflict atom $5,5:OrderItem$ to a minimal conflict reason by adding both adjacent edges and their incident source or

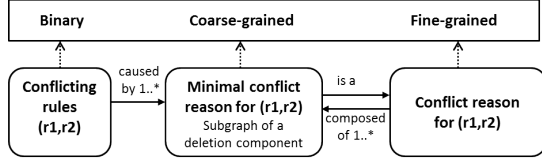


Figure 4: Conflict concepts & mapping to granularity

target nodes. It is shown as top graph in Figure 1. The conflict atom including the *owns* edge already constitutes a minimal conflict reason shown underneath the top graph in Figure 1. Both together form the conflict reason shown in Figure 2. Overlapping the LHSs of both example rules at this conflict reason yields the graph in Figure 3. The resulting LHS embeddings into this graph are actually rule matches since they fulfill the dangling condition (as shown above). The corresponding transformations form a critical pair, which is actually an initial conflict here.

Dependency and dependency concepts. For reasoning about dependencies, where graph elements being produced by the first transformation are used by the second one, we simply consider the dual concepts by inverting the first transformation of a conflicting pair. Based on this analogy the concepts *dependency between rules* and (*minimal*) *dependency reason* are defined accordingly.

4.2 Formalizing granularity levels

As suggested by the granularity requirements derived from our literature survey in Sec. 3, various use-cases may require either fine-grained, coarse-grained, and binary analysis results. To support all of these granularity levels in our technique, we formalized each granularity level with appropriate conflict and dependency concepts as outlined in this section. A full account of formal definitions, characterizations and proofs is given in the appendix.

Overapproximation. Our survey (in Sec. 3) indicates serious performance problems in practice when computing conflict information on a fine-grained level in the form of (essential) critical pairs, as done in the state-of-the-art implementation in AGG [58]. Therefore we propose as first improvement a well-chosen *overapproximation* of results that is easier to compute. It is based on the idea that, if there is a conflict for a pair of rules (r_1, r'_2) , then an equivalent conflict exists for the rule pair (r_1, r_2) with r_2 being the *non-deleting variant* of rule r'_2 . The other direction does not hold, since the dangling condition of rule r'_2 could be violated then.

Mapping conflict concepts. Fig. 4 gives an overview of how we further map conflict concepts to granularity levels.

Since initial conflicts represent currently the most optimal subset of critical pairs being still complete [40], we have chosen to return all *conflict reasons* for (r_1, r_2) corresponding to initial conflicts as new *fine-grained results*. An initial conflict for (r_1, r_2) with r_2 being non-deleting simply corresponds to the overlap of such a conflict reason. As reported in Sec. 6, this choice represents a very good trade-off between precision and performance.

Moving to the *coarse-grained level*, we selected *minimal conflict reasons*, since our considered conflict reasons at the fine-grained level are composed of minimal ones. The conflict graph of minimal conflict reasons for a rule pair (r_1, r_2) with r_2 non-deleting can be characterized as subgraph of a so-called deletion component. The

deletion part $L_1 \setminus K_1$ of a given rule r_1 may consist of several disjoint fragments, called *deletion fragments*. Completing a deletion fragment to a graph by adding all incident boundary nodes from B_1 yields a *deletion component*. Each two deletion components overlap in boundary nodes only; the union of all deletion components coincides with the deletion graph C_1 of the rule. Deletion components thus specify maximal parts of C_1 that need to be overlapped in order to find corresponding conflicting transformations. Overlapping more elements than present in a deletion component can never lead to additional minimal conflict reasons since, if the dangling condition of r_1 was not fulfilled before, it will never be fulfilled.

Example [Deletion components] Rule *returnUnpaidGood* in Figure 1 has three deletion fragments: Node 5:OrderItem with adjacent edges, node 7:BillItem with adjacent edges, and edge *owns* from node 1 to node 6. For completing them to deletion components, boundary nodes 1:Customer, 2:Order, 4:Bill, and 6:Good are needed.

On the *binary level*, we report if a *rule pair is conflicting*, since this means by definition that there is at least one pair of conflicting transformations via these rules. In particular, we can check if there is a minimal conflict reason for the given rule pair (r_1, r_2) with r_2 non-deleting, equivalent with the rule pair being conflicting.

Example [Granularity levels] Two minimal conflict reasons of rule pair *(returnUnpaidGood, offerGift)* were discussed above. Hence, these two rules are in conflict. Two further minimal conflict reasons arise if node 5 of the first rule overlaps with node 3 of the second rule, and if node 6 of the first rule is overlapped with node 4 of the second one. These four minimal reasons form the coarse analysis result of the considered rule pair. Together with all their possible combinations, we get 6 conflict reasons being reported as “New Fine” analysis result in Table 2.

Mapping dependency concepts. As mentioned before, a *dependency* can be understood as dual concept to conflicts. Analogous to the conflict case, we overapproximate produce-use dependencies by produce-read dependencies. Then *dependencies between rules*, *minimal dependency* and *dependency reasons* are mapped to the binary, coarse and fine granularity level accordingly.

5 ALGORITHM SUITE

We present an algorithm suite for computing binary, coarse and fine-grained conflicts. It is implemented in Henshin [3, 56], a model transformation framework based on graph transformation concepts.

Given a pair of rules, we consider a non-deleting variant of the second rule, supporting the overapproximation presented in Section 4.2. The algorithm in Figure 5 computes minimal conflict reasons (*coarse granularity*). The algorithm in Figure 6 uses these results to compute all conflict reasons (*fine granularity*). To support use-cases where *binary granularity* is sufficient, we can stop the computation as soon as one minimal conflict reason is discovered.

Computing minimal conflict reasons. For efficiency, we use the characterization of minimal conflict reasons as introduced in Sect. 4.2. The key idea is to compute first the set of all *conflict atom candidates* (line 3). To this end, all conflict-inducing elements of r_1 are identified in line 9. Each match of such an element to r_2 (line 10) is extended to a span (lines 11–14) yielding an atom candidate. Next, we try to extend each atom candidate to minimal conflict reasons (line 4) recursively and return the results (line 5).


```

1: funct COMPUTE_MIN_REASONS( $r_1 : \text{Rule}, r_2 : \text{Rule}$ )
2:   var reasons  $\leftarrow \emptyset$ 
3:   for each  $c \leftarrow \text{COMPUTE\_ATOM\_CANDIDATES}(r_1, r_2)$  do
4:     COMPUTE_MIN_REASONS_RECURSIVELY( $r_1, r_2, c, \text{reasons}$ )
5:   return reasons
6:
7: funct COMPUTE_ATOM_CANDIDATES( $r_1 : \text{Rule}, r_2 : \text{Rule}$ )
8:   var candidates  $\leftarrow \emptyset$ 
9:   for each  $el_1 \leftarrow r_1.\text{conflictInducingElements}$  do
10:    for each  $el_2 \leftarrow r_2.\text{lhs.occurrenceOf}(el_1)$  do
11:      var  $S_1 \leftarrow \text{new Graph}(\{el_1\})$ 
12:      var  $embed_{r_1} \leftarrow \text{new Mapping}(\{el_1 \mapsto el_1\})$ 
13:      var  $embed_{r_2} \leftarrow \text{new Mapping}(\{el_1 \mapsto el_2\})$ 
14:      candidates += new Span( $S_1, embed_{r_1}, embed_{r_2}$ )
15:   return candidates
16:
17: funct COMPUTE_MIN_REASONS_RECURSIVELY( $r_1 : \text{Rule}, r_2 : \text{Rule}, s_1 : \text{Span}, \text{reasons} : \text{Set}(\text{Span})$ )
18:   var ( $G, m_1, m_2$ )  $\leftarrow \text{constructOverlap}(r_1, r_2, s_1)$ 
19:   var  $isCR_{s_1} \leftarrow \text{findDanglingEdges}(r_1, m_1).isEmpty()$ 
20:   if  $isCR_{s_1}$  then reasons +=  $s_1$ ; return; else
21:     for each  $s_2 \leftarrow \text{EXTEND\_SPAN}(r_1, r_2, s_1, \text{reasons})$  do
22:       COMPUTE_MIN_REASONS_RECURSIVELY( $r_1, r_2, s_2, \text{reasons}$ )
23:
24: funct EXTEND_SPAN( $r_1 : \text{Rule}, r_2 : \text{Rule}, s_1 : \text{Span}, \text{reasons} : \text{Set}(\text{Span})$ )
25:   var ( $G, m_1, m_2$ )  $\leftarrow \text{constructOverlap}(r_1, r_2, s_1)$ 
26:   var  $dangling \leftarrow \text{findDanglingEdges}(r_1, m_1)$ 
27:   var  $fixing \leftarrow dangling.\text{foreach}(\text{findFixingEdges}(r_1, r_2, s_1))$ 
28:   return aggregate( $fixing.\text{foreach}(\text{enumerateExtensions}(s_1))$ )

```

Figure 5: Computing minimal conflict reasons.

To determine efficiently if a particular candidate can be extended to a minimal conflict reason recursively, we check if it gives rise to a critical pair by computing the overlap graph G of the rules' LHSs along the candidate (line 18) and checking if the corresponding embeddings $m_1 : L_1 \rightarrow G$ and $m_2 : L_2 \rightarrow G$ are rule matches (line 19). Since we assume r_2 to be non-deleting, m_2 is automatically a match. We are finished if m_1 is a match as well (i.e. if r_1 can be applied at embedding m_1 without producing dangling edges) meaning that we found a reason (line 20). Otherwise, we extend candidate spans in potentially several ways (line 21), discarding further extension opportunities in case we identified a conflict reason. We computed all extensions by function `extendSpan()` (line 24). A span s_1 has to be extended if the rule' LHS embeddings into a given overlap graph do not fulfill the dangling condition (line 25). An extension is performed stepwise by first identifying all dangling edges (line 26). For each of them a set of fixing edges in r_2 is searched (line 27). Suitable candidates for this purpose are all adjacent edges e in $C_1 \setminus S_1$ of e 's adjacent node in S_1 being identified by calling `findFixingEdges()`. For each fixing edge, function `enumerateExtensions()` (line 28) yields a set of spans, each extending s_1 by an edge and its adjacent node if not previously included in s_1 . All these sets have to be aggregated to obtain the result (line 28). Since we know that a minimal conflict reason is always part of a deletion component, we have a stopping criterion for the extension process, as we can focus on adjacent edges when computing extensions.

Computing conflict reasons. Starting from a set of minimal conflict reasons, all conflict reasons can be composed of them. Function `ComputeConflictReason` in Figure 6 picks any minimal reason mr out of a given set (line 4), adds it as reason (line 5), and composes it with the set $subReasons$ of all conflict reasons computed from the remaining set of minimal reasons (lines 7–12). All conflict reasons in $subReasons$ are first of all conflict reasons themselves (line 9) and second to be composed with mr if composable (lines 11–12).

```

1: funct COMPUTE_CONFLICT_REASONS( $minReasons : \text{Set}(\text{Span})$ )
2:   var reasons  $\leftarrow \emptyset$ 
3:   if  $minReasons.nonEmpty()$ 
4:     then var  $mr \leftarrow minReasons.pickAny()$ 
5:       reasons.add( $mr$ )
6:       if  $minReasons.size \geq 2$ 
7:         then var  $subReasons = \text{COMPUTE\_CONFLICT\_REASONS}(minReasons.remove(mr))$ 
8:           reasons.add( $subReasons$ )
9:       for each  $s : subReasons$  do
10:         if  $isComposable(mr, s)$ 
11:           then reasons.add( $\text{compose}(mr, s)$ )
12:   return reasons

```

Figure 6: Computing conflict reasons.

Computing dependency reasons. We invert rule r_1 , compute all conflict reasons, and interpret them as dependency reasons.

Implementation. We implemented the algorithm suite for the Henshin model transformation language, which is based on graph transformation concepts and the Eclipse Modeling Framework (EMF). Our implementation includes a comprehensive test suite, in which the test oracle is provided by computing fine-grained conflicts in the state-of-the-art CDA framework AGG. Section 6 studies the performance of our implementation.

6 IMPLEMENTATION EVALUATION

We evaluated our analysis via comparison to the existing analysis in AGG, focusing on two research questions: **RQ1**: *How fast are our coarse and fine-grained analyses in relation to the existing analysis in AGG?* **RQ2**: *What is the degree of the overapproximation of the multi-granular CDA technique?*

We performed our evaluation on three subject rule sets, all of them representing use-cases identified in Sect. 3: REFAC is a set of refactoring rules as used in refactoring recommenders [45]. FMEDIT is a set of editing rules that was proposed in [57] as a benchmark for edit operation detection [35]. NANOXML is a set of rules that was reverse-engineered from the Java code for a small XML parser [1] to provide an oracle for test case generation [53]. The implementation and evaluation artifacts are available at <https://github.com/KristopherBorn/multiCDA>.

RQ1. We considered the *essential* CPA in AGG and compared it with our computations of minimal conflict/dependency reasons (MCR/MDR) as coarse-grained analysis and conflict/dependency reasons (CR/DR) as fine-grained one. We analyzed delete-use-conflicts between each rule pair considering a non-deleting version of the “use” rule in each case. Moreover, we preprocessed the rule set and meta-model such that they fit to the features supported by AGG and by our implementation. In the results (Table 4), we observed that our approach achieved a major speed-up from e.g. over 8 minutes to 5 seconds for 1681 rule pairs. In terms of quantity, we see that the mean number of results dropped considerably for the larger cases, e.g. from 2.6 to 0.5 for FMEDIT rule pairs. There were even more striking results for individual rule pairs. In the most extreme case, we had 1588 ess. critical pairs, 644 conflict reasons, and 24 min. conflict reasons. A trend towards excessive individual cases is reflected in a higher standard deviation of results, which in the FMEDIT case amounted to 1.4 for coarse-grained, 18.2 for fine-grained, and 49.1 for the essential CPA results.

Rule set	#Rule pairs	Runtime (m:ss.x)			#Results (mean)		
		MCR	CR	ess.	MCR	CR	ess.
		MDR	DR	CPA	MDR	DR	CPA
REFAC	64	0:00.3	0:00.3	0:08.7	1.0	1.4	2.6
FMEDIT	1681	0:05.3	0:16.3	8:41.1	0.5	1.3	2.6
NANOXML	1296	0:04.6	0:04.7	1:30.4	0.2	0.2	0.2

Table 4: RQ1 results from our coarse (MCR/MDR) and fine-grained (CR/DR) vs. the existing analysis in AGG (ess. CPA).

Rule set	prec.	recall
REFAC	0.88	1.0
FMEDIT	0.98	1.0
NANOXML	1.0	1.0

Table 5: Overapproximation results

RQ2. We computed the sets of essential critical pairs (ess. CPs) for an original rule pair and the one for a pair where the second rule is the non-deleting variant of the original one. Both sets of ess. CPs have been filtered w.r.t. initial conflicts (dependencies). The precision is the percentage of rule pairs with equal numbers of initial conflicts (dependencies). Since we do an overapproximation, the recall is always 1.0, i.e., we do not miss a critical pair when switching to the non-deleting rule variant. The precision, however, happens to be smaller than 1.0, i.e., false positives can occur. As the results in Table 5 show, the resulting precision is still acceptable.

Threads to validity. External validity can be questioned since we focus on a limited number of rule sets being preprocessed according to unsupported features such as application conditions [19] and amalgamation [11]. We intend to support more transformation features in the future, thus enabling a more comprehensive study with more expressive subject rule sets.

7 USER STUDY

The goal of our user study is to test the usefulness of our technique’s output compared to that of its predecessor, critical pair analysis (CPA). As discussed in Section 3, in many use-cases, coarse-grained results may provide a more suitable level of detail than the fine-grained ones produced by CPA. Focusing on such use-cases, we investigated the following research question: *How useful are our coarse-grained results compared to fine-grained ones?*

We conducted a user study in which comprehension tasks had to be solved based on conflict analysis results. The analysis results were embedded as screenshots into an online questionnaire. With this web-based setup, we aimed to recruit a sufficient number of participants with appropriate expertise in graph transformations. A replication package with all tasks and data from our user study is available at <http://uni-marburg.de/y35ak>.

Focusing on the usefulness concerns of user performance and perception, our null hypotheses were as follows: ($H_{0_{perf}}$) *Users can solve comprehension tasks equally well using the given coarse- and fine-grained results;* ($H_{0_{perc}}$) *Users perceive the usefulness of the given coarse- and fine-grained results as equal.* We used a standard experimental setup involving independent, dependent, and controlled variables. Granularity was the independent variable. User performance and perception were the dependent variables. We controlled the used examples and the chosen visualization by keeping them constant. All analysis results were shown in the result visualization of AGG [58].

Methods, participants, materials. Our study design is a *cross-over study*, a variant of *within-subject design* [34] in which all participants are sequentially exposed to both treatments (here: coarse- and fine-grained analysis results). We selected this design because it minimizes the number of participants necessary to identify statistically significant differences between the result types. The main threat to the validity for this design are learning effects. We later discuss threats and adopted mitigation measures.

The experiment took place in Summer 2017. To recruit subjects with appropriate expertise, we invited participants of recent software engineering conferences focusing on model and graph transformations, and authors of the papers surveyed in Sect. 3. We sent invitations to 131 persons, 33 of which participated in the survey. Our overall sample consisted of 24 academic/scientists, 5 PhD students, 3 practitioners, and 1 MSc student. To survey the shared software-engineering and graph-transformation background of our participants, we collected demographic data based on five-point Likert scales. 30 respondents rated their experience with UML-based modeling 3 or higher (20 of them ≥ 4), like 32 did for their graph-transformation experience (in 27 cases ≥ 4), 25 participants rated their experience as CDA users as 3 or higher (14 cases ≥ 4).

The questionnaire was made up of three parts: an instruction part, an experimental part, and a survey part. The *instruction* part used an example to revisit basics about conflicts of graph transformations and familiarize the participants with the used visualization. The *comprehension* part included question-based comprehension tasks. In the *survey* part, we aggregated the demographic information and asked the participants for their subjective experiences.

The instruction and comprehension parts used a common example domain, namely the detection of conflicting requirements during the development of an online shop. The rationale for choosing this domain was twofold: First, it did not require expertise in a specialized technical environment. Second, it is based on a use-case in which we hypothesize that coarse-grained results are beneficial, namely consistency validation of use-cases [26]. The example in Sect. 2 is representative for this example domain and use-case.

In the comprehension part, each participant solved two tasks, one for each granularity level. Each task included an example rule pair, a number of conflict analysis results, and two questions: First, whether any conflicts existed for the rule pair. Second, if the first question was answered “yes”, we asked to name the specific elements causing the conflicts. The rule pairs and their order were the same for each participant. The order of the granularity levels used to present the conflicts was assigned randomly. Based on the questions, the task metrics were measured as follows: A correct answer for the first question was rewarded with 1 point. The second question was more complicated, which was reflected in 2 points for a fully and 1 point for a partially correct answer. The time needed to complete both tasks was measured using the questionnaire.

We tested the tasks in a prestudy with 10 participants, in which we experimented with various difficulty levels. We aimed to balance complexity—the drawbacks of fine-grained results are more obvious for complex examples and tasks—and simplicity, to avoid participant exhaustion and to benefit completion rate. To this end, we decided to drop additional tasks concerning dependency analysis and conflict repair. The actual tasks used in our experiment were based on the conflict example shown in Sect. 2 (Task 1) and a

Table 6: User study results.

Conflict type	Correctness	Completion time	Understandability 5=hard to understand	Simplicity 5=hard to solve tasks	Effort 5=much effort
Fine-grained	93.9 % (± 13.0 %)	7.7 min (± 6.6 min)	2.6 / 5 (± 0.9)	2.5 / 5 (± 1.0)	2.9 / 5 (± 1.1)
Coarse-grained	91.9 % (± 14.5 %)	5.7 min (± 2.9 min)	2.0 / 5 (± 0.9)	2.0 / 5 (± 1.1)	2.0 / 5 (± 0.9)
	$p = 0.42$	$p = 0.10$	$p = 0.004$	$p = 0.008$	$p \leq 0.001$

comparable one (Task 2). Fine-grained results represent essential critical pairs, as provided by the state-of-the-art CDA tool AGG. Coarse-grained results are those of our multi-granular CDA.

In the survey part, to measure user perception, the participants were asked for their subjective assessment of both granularity levels. Three metrics were collected using five-point Likert scales: ratings of understandability ("How easy was it to understand the results of type X?"), difficulty of solving tasks ("How difficult was it to answer the questions using the results of type X?"), and perceived effort ("How much effort was required to answer the questions using the results of type X?"). Finally, we asked them for their overall preference between both granularity levels on a five-point Likert scale. Additional qualitative information regarding the subjective assessment was collected using free-form text fields.

For statistical hypothesis testing, we used the Wilcoxon signed-ranked test [22] and, where applicable, the paired-samples t-test [63]. Wilcoxon is a standard nonparametrized test that supports high-confidence inferences for paired data. The t-test provides greater statistical power than Wilcoxon for normally distributed data. We checked for normality using the Shapiro-Wilk test [54].

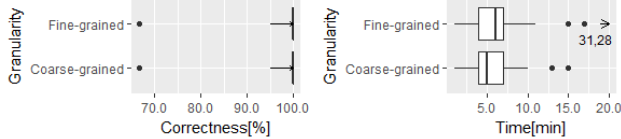


Figure 7: Correctness and completion time.

Results. Table 6 summarizes the results in terms of descriptive and inferential statistics. The task metrics are visualized with boxplots in Fig. 7. Irrespective of whether coarse- or fine-grained detection results were used, the vast majority of all participants answered the tasks with full correctness scores. The mean time required to complete the tasks was shorter by 33 % (2 minutes) when coarse-grained results were used. Yet, the completion times for the fine-grained case included two excessive data points of 28 and 31 minutes, which can be considered as outliers. After removing them, the mean completion time for coarse-grained is still lower by 0.6 minutes. The completion times did not differ to a statistically significant extent ($p=0.10$). In summary, since we cannot find a significant effect on correctness and completion time, our first null hypothesis $H_{0_{perf}}$ cannot be rejected.

The perception metrics of understandability, difficulty, and effort are visualized in Fig. 8. In all cases, the participants reported more positive (70–79 % vs. 36–55 %), fewer negative (9–12 % vs. 18–36 %), and fewer neutral scores (12–18 % vs. 24–33 %) when working with the coarse-grained detection results: they experienced better understandability, ease of solving tasks, and less effort. The differences were strongly statistically significant ($p \leq 0.01$). In summary, our second null hypothesis $H_{0_{perc}}$ is rejected.

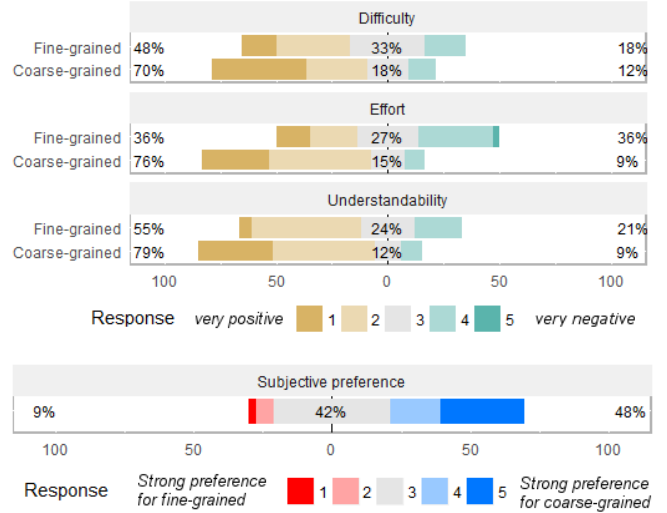


Figure 8: User perception and preference.

Finally, we asked for an overall subjective preference. Fig. 8 gives a distribution overview: coarse-grained results attracted more subjective preferences than fine-grained ones. In fact, the overall sample is mostly divided into two groups: those participants with a preference for the coarse-grained results (48%, 30 % of them with a strong preference), and those without a preference (42 %).

In qualitative information collected using the questionnaire, we found that the reasons for preferring were largely in line with our motivation: Participants found that “less results provided a quicker overview”, and that coarse-grained results “boil down the problem to actual distinguish conflicts”. Some subjects extrapolated from our tasks to more complex cases: “Assuming that we have a larger set of rules that conflict with each other, I’d assume that the [fine-grained] results will become very complex and hard to understand”.

We also found reasons why this preference was not shared unanimously. Some participants found the redundancy of fine-grained beneficial as it “presented conflicts more consistently” and felt that coarse-grained results did not mirror their understanding of conflicts: “I believe that [fine-grained] displays all conflicts, whereas [coarse-grained] only displays selected conflicts.” This group of users may still benefit from our faster computation of fine-grained results.

Threats to Validity. *Construct validity.* For our effort and understandability measurements, we rely on subjective ratings by the participants. However, such subjective measures are highly correlated with objective measures of cognitive load [24]. Moreover, we aimed to avoid participant bias in favor of the experimenters by replacing the names of the used tools and concepts with pseudonyms.

Internal validity. The main threat in our within-subject design are learning effects, in particular, since the same questions were

asked for each analysis result type (albeit for different tasks). We mitigated this threat by using counterbalancing, i.e., randomizing the order in which the results were shown to the participants.

External validity. We addressed external validity by recruiting a sufficiently large number of participants with relevant software engineering and CDA expertise. However, our web-based setup required some trade-offs: the sample analysis results were relatively small (3–10 entries) and were shown using screenshots instead of in the actual tool. Generalizations to a greater variety of use-cases are threatened since we considered a single domain and use-case. While our discussion in Sect. 3 highlights key similarities to other domains, a definitive verdict on the practical usefulness of coarse-grained results is outside the scope of this paper.

Summary. Having identified use-cases in which the level of detail offered by our coarse-grained results seems sufficient (Sect. 3), we set out to empirically study their usefulness in one such use-case. While we did not detect an effect on participants’ performance, the participants perceived coarse-grained results as easier to understand and work with than fine-grained ones. The relative majority of participants preferred coarse-grained over fine-grained results. Our user study complements significant performance benefit achieved by our multi-granular computations and, to the best of our knowledge, provides the first empirical evidence on the usefulness of CDA for graph transformation.

8 RELATED WORK

In this paper, we have presented a generic, multi-granular CDA technique based on graph transformation (GT) which is statically analyzing transformation rules; it is fully automatic and state-independent. We compare with other analysis techniques along these aspects.

Further GT analysis techniques. Another *static analysis* technique for GT checks for invariants [7, 9]. In contrast, *model checking techniques* for GT [21, 50] need an initial start graph and then generate for this graph and a given rule set a corresponding state space in order to analyze more complex temporal logic properties. Whereas both kinds of analysis techniques are fully automatic, there exist also *interactive theorem proving techniques* dedicated to GT that are able to prove partial correctness of graph programs [25, 48].

With respect to designing and performing *analysis techniques with different granularity levels*, we can relate our work in a broader sense to the work on *counter example guided abstraction refinement* (CEGAR) initially described in [14] and applied to the analysis of GT in [37]. Our approach contains a similar idea in the sense that we perform a stepwise analysis technique with different levels of accuracy and terminate with the level of accuracy that is needed. In the CEGAR approach the desired level of accuracy is obtained if either the desired property for the analyzed system is successfully verified or a real counterexample has been detected. In our approach there can be many different use-cases to be satisfied with a certain level of accuracy of the analysis as described in detail in the literature review in Section 3.

Other generic CDA techniques. Besides GT there exist other formal approaches which allow for static and dynamic CDA of specified systems. A Church-Rosser-Checker for equational specifications in Maude [16] looks for critical pairs between conditional

term rewriting rules and tries to join them. There are also logic-based approaches such as *conditional transformations* [36] providing static CDA techniques. These techniques are generic and therefore language-independent but the analysis techniques provided are not multi-granular. Further logic-based approaches such as *constraint networks* and *model checking* approaches such as Alloy [32] are not state-independent.

Language-specific CDA techniques. There are many sophisticated CDA techniques in the context of sequential and concurrent programming. In the context of concurrent threads, conflicts are called *data races*; they occur if two different threads access the same memory location and at least one of them is a write (see e.g. [28, 55]). Common challenges of these techniques are dynamic object creation, dynamic thread creation, and references to objects [51]. Similarly, there has been a long line of works developing *data flow analyses* to find dependencies between object changes and accesses; for example, static taint analyses have been recently developed for Android applications as e.g. FlowDroid [4]. These CDA techniques differ considerably to ours in several aspects: they are not generic but have been specifically developed to find bugs in implementations. To the best of our knowledge, there are no language-specific CDA techniques for reasoning about software engineering activities. Language-specific CDA techniques are mostly dynamic and therefore state-dependent. This is often not suitable for engineering activities such as refactorings which are specified state-independently. Moreover these CDAs are usually not multi-granular so that gradual analysis time reduction and easy understanding are not especially supported.

9 CONCLUSION

In this paper, we presented a novel static CDA technique for graph transformation which can detect conflicts and dependencies in software engineering on multiple granularity levels. Compared to the state-of-the-art CDA, we were able to significantly speed-up the computation of fine-grained CDA results and to complement them with coarse-grained ones offering usability benefits for numerous use-cases. Our technique is especially advantageous for analyzing interactions on complex and dynamic object structures as, e.g., for feature interaction in software product line engineering.

Our multi-granular CDA technique currently uses transformation rules without *advanced features* such as application conditions [19] and amalgamation [11], for which the state-of-the-art CDA technique has been investigated already. In the future, we aim to support these more complex concepts in our technique as well. Due to the major speed-up we could achieve with our multi-granular approach, this technique has now the potential to be applied in fields where performance plays a larger role.

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A FORMALIZATION

In Section 4.2, we explain how *conflict reasons* and *minimal conflict reasons* serve as a basis for computing conflicts on the fine and coarse granularity level in our algorithm suite. We provided informal descriptions and set-theoretical characterizations of these notions. We now formalize these informal descriptions and prove the correctness of the characterizations. Our formalization assumes certain preliminaries on rules and conflicts, which are recapped in Section A.1. The new definitions and proofs are introduced in A.2. We assume that the reader of this appendix is familiar with basic categorical concepts such as (initial) pushouts and pullbacks. We refer to [17, 60] for a gentle introduction to these notions in the context of graph transformation systems.

A.1 Recap: Preliminaries

This section shares materials with [10, 40], which introduce the consideration of conflict concepts on different granularity levels as well as the notion of initial conflicts.

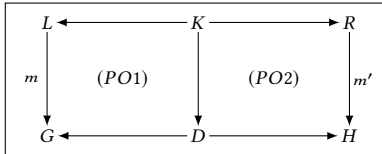
The underlying paradigm of our considerations is the double-pushout approach to graph transformation as presented in [17]. Furthermore, we formally introduce the notion of conflicting transformations [42] used in this paper.

Graph Transformation

In Section 4.1, we informally present the notion of graph transformation in a set-theoretical way. Here we recap the alternative definition of graph transformation based on category theory, since it is needed for presenting formally all further concepts. Moreover, this categorical setting paves the way for extending our technique to more expressive kinds of graphs, e.g. typed, attributed ones [17], in the future.

In the following categorical definition of graph transformation, the deletion and creation of elements prescribed by the rule is given by two pushouts, respectively. In the diagram below, the existence of (PO1) for a given injective morphism m from the LHS graph L to a graph G , and the LHS morphism of the rule is equivalent with the satisfaction of the dangling condition of m .

Definition A.1 (Rule and transformation). A rule r is defined by $r = (L \hookrightarrow K \hookrightarrow R)$ with L, K , and R being graphs connected by two graph inclusions. A (direct) transformation $G \xrightarrow{m,r} H$ which applies rule r to a graph G consists of two pushouts as depicted below. Morphism $m : L \rightarrow G$ is injective and is called *match*. Rule r is *non-deleting* if $L = K$. Rule r is *applicable* at match m if there exists a graph D such that (PO1) is a pushout.



Conflicts

The classical definition of delete-use conflicts [17] (based on category theory) states that the match for the second transformation

cannot be found anymore after applying the first transformation. In the following, we consider an equivalent alternative definition [42] for a pair of transformations to be in delete-use conflict (as presented set-theoretically in Sec. 4.1). It states that at least one deleted element of the first transformation is overlapped with some used element of the second transformation. This overlap is formally expressed by a span of graph morphisms between the deletion graph C_1 of the first rule, and the LHS of the second rule (Fig. 9). Categorically speaking, an initial pushout construction [17] over the left-hand side morphism of the rule is used to compute the *boundary graph* B_1 consisting of all nodes needed to make $L_1 \setminus K_1$ a graph and the *deletion graph* $C_1 := L_1 \setminus (K_1 \setminus B_1)$. We say that the nodes in B_1 are *boundary nodes*. Graph fragment $C_1 \setminus B_1$ is the *deletion part* of rule r_1 . It may consist of several disjoint fragments, called *deletion fragments*. Completing a deletion fragment to a graph by adding all incident nodes (i.e. boundary nodes) it becomes a *deletion component* in C_1 . Each two deletion components overlap in boundary nodes only; the union of all deletion components is C_1 . If two transformations overlap such that there is at least one element of a deletion fragment included, they are also conflicting.

Definition A.2 (Delete-use conflict). Consider a pair of transformations $(t_1, t_2) = (G \xrightarrow{m_1, r_1} H_1, G \xrightarrow{m_2, r_2} H_2)$ via rules $r_1 : L_1 \xrightarrow{le_1} K_1 \xrightarrow{ri_1} R_1$ and $r_2 : L_2 \xrightarrow{le_2} K_2 \xrightarrow{ri_2} R_2$ as depicted in Fig. 9 with the initial pushout (1) for $K_1 \xrightarrow{le_1} L_1$ and the pullback (2) of $(m_1 \circ c_1, m_2)$ yielding the span $s_1 : C_1 \xleftarrow{o_1} S_1 \xrightarrow{e_2} L_2$. The pair of transformations (t_1, t_2) is in *delete-use conflict* if $s_1 : C_1 \xleftarrow{o_1} S_1 \xrightarrow{e_2} L_2$ satisfies the *conflict condition* i.e. there does not exist any morphism $x : S_1 \rightarrow B_1$ such that $b_1 \circ x = o_1$. We also say that the span s_1 is the *conflict reason* for (t_1, t_2) .

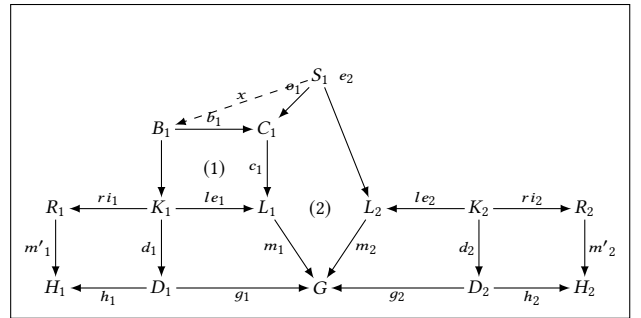


Figure 9: Transformation pair in delete-use conflict

In the rest of this section we abbreviate *delete-use conflict* with *conflict*.

Conflict Concepts

We recall here the formal notions of the conflict concepts on different granularity levels (conflict part, conflict atom, and (minimal) conflict reason) as reintroduced informally in Section 4.1. We thus lift our conflict considerations from transformations to the rule level, i.e., we consider conflicting rules. Two rules are in conflict

if there is a pair of conflicting transformations arising from applications of these rules. According to Definition A.2 there is a span between these rules specifying the conflict reason. In the following definitions, we concentrate on these spans and distinguish several forms of spans showing conflict reasons or parts of it.

We start focusing on minimal building bricks, called conflict atoms. In particular, we consider a conflict atom to be a minimal sub-graph of C_1 which can be embedded into L_2 but not into B_1 (*conflict and minimality conditions*). Moreover, a pair of direct transformations needs to exist for which the match morphisms overlap on the conflict atom (*transformation condition*). Note that, in general, the matches of this pair of transformations may overlap also in graph elements not contained in the conflict atom. Hence, such a pair of transformations may be chosen flexibly, it does not need to show a conflict in a minimal context as critical pairs do. While conflict atoms describe the smallest conflict parts, a conflict reason is a complete conflict part in the sense that all in the reported conflict involved atoms are subsumed by it (*completeness condition*).

Definition A.3 (Basic conflict conditions). Given rules $r_1 : L_1 \xrightarrow{le_1} K_1 \xrightarrow{ri_1} R_1$ and $r_2 : L_2 \xleftarrow{le_2} K_2 \xleftarrow{ri_2} R_2$ with the initial pushout (1) for $K_1 \xrightarrow{le_1} L_1$ as well as a span $s_1 : C_1 \xleftarrow{o_1} S_1 \xrightarrow{e_2} L_2$ as depicted in Fig. 9, *basic conflict conditions* for the span s_1 of (r_1, r_2) are defined as follows:

- (1) *Conflict condition*: Span s_1 satisfies the *conflict condition* if there does not exist any injective morphism $x : S_1 \rightarrow B_1$ such that $b_1 \circ x = o_1$.
- (2) *Transformation condition*: Span s_1 satisfies the *transformation condition* if there is a pair of transformations $(t_1, t_2) = (G \xRightarrow{m_1, r_1} H_1, G \xRightarrow{m_2, r_2} H_2)$ with $m_1(c_1(o_1(S_1))) = m_2(e_2(S_1))$ (i.e. (2) is commuting in Fig. 9).
- (3) *Completeness condition*: Span s_1 satisfies the *completeness condition* if there is a pair of transformations $(t_1, t_2) = (G \xRightarrow{m_1, r_1} H_1, G \xRightarrow{m_2, r_2} H_2)$ such that (2) is the pullback of $(m_1 \circ c_1, m_2)$ in Fig. 9).
- (4) *Minimality condition*: A span $s'_1 : C_1 \xleftarrow{o'_1} S'_1 \xrightarrow{e'_2} L_2$ can be embedded into span s_1 if there is an injective morphism $e : S'_1 \rightarrow S_1$, called *embedding morphism*, such that $o_1 \circ e = o'_1$ and $e_2 \circ e = e'_2$. If e is an isomorphism, then we say that the spans s_1 and s'_1 are *isomorphic*. (See (3) and (4) in Fig. 10.) Span s_1 satisfies the *minimality condition* w.r.t. a set SP of spans if any $s'_1 \in SP$ that can be embedded into s_1 is isomorphic to s_1 .

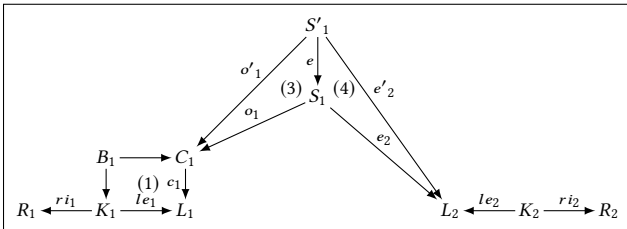


Figure 10: Illustrating span embeddings

Now we are ready to define the building bricks of conflicts on different granularity levels.

Definition A.4 (Conflict concepts). Let the rules $r_1 : L_1 \xleftarrow{le_1} K_1 \xrightarrow{ri_1} R_1$ and $r_2 : L_2 \xleftarrow{le_2} K_2 \xrightarrow{ri_2} R_2$ with initial pushout (1) for $K_1 \xrightarrow{le_1} L_1$ and a span $s_1 : C_1 \xleftarrow{o_1} S_1 \xrightarrow{e_2} L_2$ as depicted in Fig. 9, be given.

- (1) Span s_1 is called *conflict part candidate* for the pair of rules (r_1, r_2) if it satisfies the conflict condition. Graph S_1 is called the *conflict graph* of s_1 .
- (2) A conflict part candidate s_1 for (r_1, r_2) is a *conflict part* for (r_1, r_2) if s_1 fulfills the transformation condition.
- (3) A conflict part candidate s_1 for (r_1, r_2) is a *conflict atom candidate* for (r_1, r_2) if it fulfills the minimality condition w.r.t. the set of all conflict part candidates for (r_1, r_2) .
- (4) A conflict atom candidate s_1 for (r_1, r_2) is a *conflict atom* for (r_1, r_2) if s_1 fulfills the transformation condition.
- (5) A conflict part s_1 for (r_1, r_2) is a *conflict reason* for (r_1, r_2) if s_1 fulfills the completeness condition.
- (6) A conflict reason s_1 for (r_1, r_2) is *minimal* if it fulfills the minimality condition w.r.t. the set of all conflict reasons for (r_1, r_2) .

Table 7 provides a conflict concept overview and basic conditions.

basic condition / conflict concept	conflict condition	transf. condition	compl. condition	min. condition
conflict part cand.	x			
conflict part	x	x		
conflict atom cand.	x			x
conflict atom	x	x		x
conflict reason	x	x	x	
min. conflict reason	x	x	x	x

Table 7: Overview of conflict concepts

The following lemma gives a more constructive characterization of conflict atom candidates compared to their introduction in Def. A.4. Candidates are either nodes deleted by rule r_1 and used by rule r_2 or edges deleted by r_1 and used by r_2 if their incident nodes are preserved by r_1 . Edges with at least one incident deleted node are not considered as atom candidates since their deletion is caused by node deletions.

LEMMA A.5 (CONFLICT ATOM CANDIDATE CHARACTERIZATION).

A conflict atom candidate $s_1 : C_1 \xleftarrow{o_1} S_1 \xrightarrow{e_2} L_2$ for rules $(r_1 : L_1 \xleftarrow{le_1} K_1 \xrightarrow{ri_1} R_1, r_2 : L_2 \xleftarrow{le_2} K_2 \xrightarrow{ri_2} R_2)$ has a conflict graph S_1 either consisting of a node v s.t. $o_1(v) \in C_1 \setminus B_1$ or consisting of an edge e with its incident nodes v_1 and v_2 s.t. $o_1(e) \in C_1 \setminus B_1$ and $o_1(v_1), o_1(v_2) \in B_1$.

The subsequent theorem recaps the interrelation between conflict reasons and conflict atoms [10]. The characterization for conflict reasons presented in the next section relies on this interrelation. The theorem states that each conflict reason is covered by a unique set of atoms, i.e. all atoms that can be embedded into that conflict reason. With atoms we mean conflict atoms as well as boundary atoms, where the latter ones consists merely of a single boundary node.

This means that by investigating the set of conflict atoms one gets a complete overview of graph elements that can cause conflicts in a given conflict reason. Moreover, the set of boundary atoms indicates how this conflict reason might be still enlarged with other conflict-inducing edges. Of course, this result also holds for the special case that the conflict reason is minimal.

Definition A.6 ((Isolated) Boundary atom). A span $s_1^b : C_1 \xleftarrow{o_1^b} S_1^b \xrightarrow{e_1^b} L_2$ is a *boundary part* for rules (r_1, r_2) with initial pushout (1) as in Fig. 9 if there is a morphism $s_B : S_1^b \rightarrow B_1$ such that $b_1 \circ s_B = o_1^b$ and s_1^b fulfills the transformation condition. A non-empty boundary part s_1^b is a *boundary atom* if it fulfills the minimality condition w.r.t. the set of boundary parts for (r_1, r_2) .

Given a conflict reason $s_1 : C_1 \xleftarrow{o_1} S_1 \xrightarrow{e_1} L_2$ and a boundary atom s_1^b with an embedding $e^b : S_1^b \rightarrow S_1$ into this conflict reason, then s_1^b is called *isolated* w.r.t. its embedding e^b if there is no conflict atom $a_1 : C_1 \xleftarrow{o_1^a} A_1 \xrightarrow{e_1^a} L_2$ being embedded into s_1 via embedding morphism $e^a : A_1 \rightarrow S_1$ such that s_1^b can be embedded into that conflict atom via $x : S^b \rightarrow A_1$ with $e^a \circ x = e^b$.

It is straightforward to show that graph S_1^b of a boundary atom consists of exactly one boundary node being the source or target node of an edge that is potentially conflict-inducing.

THEOREM A.7 (COVERING OF CONFLICT REASONS BY ATOMS). Given a conflict reason $s_1 : C_1 \xleftarrow{o_1} S_1 \xrightarrow{e_1} L_2$ for rules (r_1, r_2) , then the set A of all conflict atoms together with the set A^B of all boundary atoms that can be embedded into s_1 covers s_1 , i.e. for each conflict reason $s'_1 : C_1 \xleftarrow{o'_1} S'_1 \xrightarrow{e'_1} L_2$ for (r_1, r_2) that can be embedded into s_1 it holds that, if each atom in $A \cup A^B$ can be embedded into s'_1 , then s'_1 is isomorphic to s_1 .

Initial conflicts

Compared to the classical definition of *critical pairs*, initial conflicts [40] offer a more *declarative* view on pairs of minimal transformations in conflict. In categorical terms, one can use actually the notion of *initial transformation pairs* to obtain this new view on critical pairs. Interestingly, it turns out that each initial conflict is a critical pair but not vice versa. By definition, initial conflicts have the important new characteristic that there exists a *unique* initial conflict for each given pair of conflicting transformations. Additionally, all initial conflicts still satisfy the Completeness Theorem as well as the Local Confluence Theorem.¹ For the category of (typed) graphs, all these requirements hold [17, 40].

In the following, we do not recall the categorical definition of initial conflicts [40], but simply reintroduce its set-theoretical characterization for the category of (typed) graphs. The idea of initial conflicts is that no different pair of transformations can be found that can be embedded into it. Otherwise speaking, it describes the “smallest” conflict that can be embedded into a given conflict. In particular, it has been shown [40] that an initial conflict is a

¹For Completeness of initial conflicts the existence of initial transformation pairs for conflicts is assumed. For Local Confluence, initial POs are required in addition.

transformation pair in conflict with *minimal context* and *maximal unfolding of preserved elements* leading to the following characterization. The first two items characterize critical pairs for a given rule pair (r_1, r_2) in conflict. The first three items characterize essential critical pairs as item 3 follows directly from their construction [42]. In the category of (typed) graphs a critical pair is essential if two injective matches overlap in deleted elements and boundary nodes only [42]. Item 4 is an additional condition that needs to hold for initial conflicts ensuring that no further unfolding of elements is possible. Note that we adapted slightly the characterization of initial conflicts as introduced in [40], since we are interested in the asymmetric case of a delete-use-conflict (according to Def. A.2) in this paper only.

Definition A.8 (Initial conflict). A pair of transformations $ic : (G \xRightarrow{r_1, m_1} H_1, G \xRightarrow{r_2, m_2} H_2)$ as depicted in Fig. 9 is an initial conflict if it has the following properties:

- (1) Minimal context: m_1 and m_2 are jointly surjective.
- (2) At least one element in conflict, i.e. deleted by r_1 and used by r_2 :
 $m_1(L_1) \cap m_2(L_2) \not\subseteq m_1(l_1(K_1))$.
- (3) Overlap in deletion graphs only:
 $m_1(L_1) \cap m_2(L_2) \subseteq (m_1(c_1(C_1)) \cap m_2(L_2)) \cup (m_1(L_1) \cap m_2(c_2(C_2)))$.
- (4) No isolated boundary atom in conflict reason:
 $\forall x \in m_1(c_1(b_1(B_1))) \cap m_2(L_2) :$
 $\exists e \in m_1(c_1(C_1)) \cap m_2(L_2) : x = \text{src}(e) \vee x = \text{tgt}(e)$ and
 $\forall x \in m_2(c_2(b_2(B_2))) \cap m_1(L_1) :$
 $\exists e \in m_2(c_2(C_2)) \cap m_1(L_1) : x = \text{src}(e) \vee x = \text{tgt}(e)$.

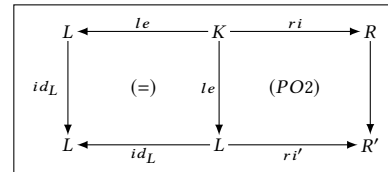
A.2 Formalizing Granularity Levels

In this section, we present more formally our idea for overapproximating conflicts as well as the characterization of minimal conflict reasons (coarse granularity) and conflict reasons (new fine granularity) as described in Sec. 4.2.

Overapproximation

As described in Sec. 4.2, to support an efficient computation, we do not compute an exact overview of all possible delete-use conflicts, but an over-approximation. To this end, we focus on delete-read conflicts: If the second rule of a pair of rules is non-deleting, we say that delete-use conflicts arising from applications of these rules are merely delete-read conflicts. For our computation, we replace the second rule of the considered rule pair with its non-deleting variant and identify all delete-read conflicts.

Definition A.9 (Non-deleting rule variant). Given a graph transformation rule $r : L \xleftarrow{le} K \xrightarrow{ri} R$, then $r' : L \xleftarrow{id_L} L \xrightarrow{ri'} R'$ is the *non-deleting variant* of r with $ri' : L \hookrightarrow R'$ being defined by the pushout of (le, ri) .



THEOREM A.10 (OVER-APPROXIMATING DELETE-USE BY DELETE-READ CONFLICTS). *Given a pair of rules (r_1, r_2) together with the non-deleting variant r'_2 of r_2 , then the following holds: If transformation pair $(t_1, t_2) = (G \xrightarrow{r_1, m_1} H_1, G \xrightarrow{r_2, m_2} H_2)$ is in delete-use conflict, then transformation pair $(t_1, t'_2) = (G \xrightarrow{r_1, m_1} H_1, G \xrightarrow{r'_2, m_2} H'_2)$ is in delete-read conflict.*

PROOF. This follows directly from Def. A.2 since r'_2 , the non-deleting variant of r_2 , has the same LHS as r_2 and match m_2 of t_2 is identical to the match for t'_2 . \square

This theorem shows that, instead of investigating further all delete-use conflicts that may arise for rules r_1 and r_2 , we can also investigate all the delete-read conflicts arising from rules r_1 and r'_2 , the non-deleting variant of r_2 , without missing any conflicts. However, it may happen that some false positives are computed as the following example illustrates. This is because the opposite direction of the theorem does not hold, i.e. if transformation pair (t_1, t'_2) is in delete-read conflict with transformation t'_2 via r'_2 , then there does not necessarily exist a transformation t_2 via r_2 and match m_2 because the dangling condition might be violated.

Example A.11 (False positive conflict). In the following figure, we see the left-hand sides of rule r_1 that deletes an edge and of rule r_2 that deletes a complete handle, i.e., an edge with its source and target node. We choose matches such that the handles of both rules overlap completely. While r_1 is applicable, r_2 is not since its match does not fulfill the dangling condition. If we take the non-deleting variant r'_2 of r_2 , the dangling condition is always trivially fulfilled. Using the same matches as before, we get two transformations. They have a delete-read-conflict since the application of r_1 deletes an edge that is read by the application of r_2 .

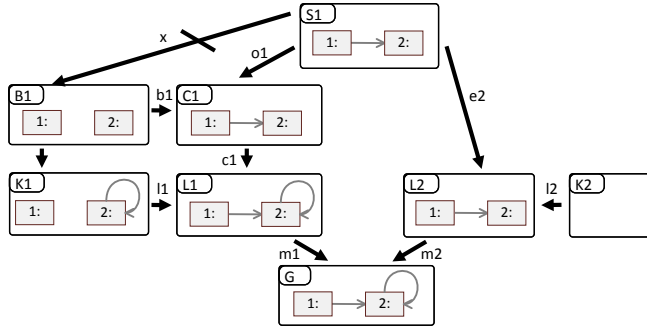


Figure 11: Example for a false positive conflict

In general, false positives occur if rule r_1 contains some context edge at an overlap node that is deleted by r_2 . In addition, r_1 deletes some other item in its overlap with r_2 . The application of r_2 would lead to a dangling edge due to the context edge. Hence, there cannot be a transformation t_2 applying r_2 in the described way. The non-deleting variant of r_2 , however, would not delete overlap nodes and hence, would fulfill the dangling condition. Therefore, there would be a pair (t_1, t'_2) applying r_1 and r'_2 in the corresponding way and this pair would be conflicting.

Mapping of Conflict Concepts

In this section, we argue why conflict reasons corresponding to initial delete-read conflicts (new fine granularity, see Sec. 4.2) are composed of minimal conflict reasons. We also show as stated in Sec. 4.2 that indeed an initial conflict for (r_1, r_2) with r_2 non-deleting simply corresponds to the overlap of such a conflict reason. Moreover, we start with proving that the conflict graph of a minimal conflict reason (coarse granularity, see Sec. 4.2) for some delete-read conflict is always a subgraph of one of the deletion components of the deletion rule. In the following, we thus assume that rule r_2 is non-deleting.

LEMMA A.12 (MINIMAL CONFLICT REASON AND DELETION COMPONENTS). *If a conflict reason $s_1 : C_1 \leftrightarrow S_1 \rightarrow L_2$ for rule $r_1 : L_1 \leftrightarrow K_1 \hookrightarrow R_1$ and non-deleting rule $r_2 : L_2 \leftrightarrow K_2 \hookrightarrow R_2$ is minimal then the conflict graph S_1 is a subgraph of a deletion component of C_1 .*

PROOF. Assuming that the conflict graph S_1 is not a subgraph of one deletion component of C_1 , we argue by contradiction and will show that in this case s_1 is not minimal. Since s_1 satisfies the conflict condition, there exists at least one node or edge x in S_1 that is deleted. Now consider a span s'_1 embedded into s_1 such that its conflict graph S'_1 consists of all elements belonging to S_1 that are mapped to the deletion component in C_1 surrounding x . It is obvious that s'_1 is a conflict part candidate. In particular, the conflict condition is trivially fulfilled. We can, moreover, show that s'_1 fulfills the transformation and completeness conditions. This is the case since for s_1 there exists a pair of transformations with valid matches m_1 and m_2 overlapping the deleted elements of r_1 and read elements of r_2 exactly as specified by s_1 .

Now there exist also two transformations by restricting the matches m_1 and m_2 to overlap merely in the one deletion component specified by s'_1 . We show this by contradiction and by assuming that the accordingly restricted match m'_1 of m_1 would violate the dangling edge condition. This is the case if a node is deleted by r_1 that is matched by m'_1 to a node incident with an edge that is not deleted by r_1 . If this node does not belong to the deletion component C_1 , then m'_1 matches it to a node that can merely be incident with edges from the LHS of rule r_1 , since no overlap on this deletion component takes place. If rule r_1 would not delete one of these incident edges, then it would not be a valid graph rule and this is a contradiction. If, on the other hand, this node belongs to the deletion component in C_1 , then, if m'_1 maps it to a node with dangling edge, m_1 would violate the dangling edge condition since m'_1 and m'_2 are equal to m_1 and m_2 on this deletion component and this is a contradiction. Since r_2 is non-deleting, the dangling edge condition for the restricted match of m_2 is obviously fulfilled. Hence, s'_1 is a conflict reason that can be embedded into s_1 , and is definitely non-isomorphic to s_1 . This is a contradiction to our assumption that s_1 is minimal. \square

Due to Lemma A.12 and Theorem A.7 each minimal conflict reason is covered by conflict atoms only (i.e., no isolated boundary atoms). This is because, if isolated boundary atoms would exist, then the minimal conflict reason would be larger than a deletion component. The minimal conflict reason cannot consist of just one single isolated boundary atom since the conflict condition would

not be fulfilled then. Therefore, there has to be another deletion component and this is a contradiction.

The next result states that each conflict atom can be embedded into a minimal conflict reason. This result is closely related with the subsequent one, which shows that each conflict reason can be covered by minimal conflict reasons and isolated boundary atoms. Hence, we have a hierarchical structure of conflict concepts: Atoms cover minimal conflict reasons (special case of Theorem A.7), which cover conflict reasons, together with isolated boundary atoms (Theorem A.14).

THEOREM A.13 (CONFLICT ATOM \hookrightarrow MINIMAL CONFLICT REASON).

Each conflict atom $a_1 : C_1 \xrightarrow{o_1} A_1 \xrightarrow{e_2} L_2$ for rule $r_1 : L_1 \hookrightarrow K_1 \hookrightarrow R_1$ and non-deleting rule $r_2 : L_2 \hookrightarrow K_2 \hookrightarrow R_2$ can be embedded into a minimal conflict reason for (r_1, r_2) .

PROOF. Since a_1 is a conflict atom, there is a pair of conflicting transformations (t_1, t_2) with matches $m_1 : L_1 \rightarrow G$ for r_1 and $m_2 : L_2 \rightarrow G$ for r_2 such that $m_1(c_1(a_{11}(S_1))) = m_2(a_{12}(S_1))$, i.e., the transformation condition in Def. A.4 holds. Hence, by building the pullback of $(m_1 \circ c_1, m_2)$ we get a conflict reason $s_1 : C_1 \xrightarrow{o_1} S_1 \xrightarrow{e_2} L_2$ for (r_1, r_2) . The conflict condition is fulfilled due to atom a_1 ; the completeness and transformation condition are fulfilled by construction. Moreover, because of the pullback property a_1 can be embedded into s_1 .

Assume that a_1 is isomorphic to s_1 , then a_1 is trivially embedded into s_1 . Now assume that a_1 itself is not isomorphic to s_1 and therefore not a minimal conflict reason yet. Then we can enlarge A_1 by successively adding elements of S_1 belonging to the same deletion component of C_1 . We stop this addition of elements as soon as the matches arising from m_1 and m_2 by restricting them to overlap merely in the elements in A_1 and added to A_1 fulfill the dangling edge condition. This addition terminates at latest when we have reached all elements present in S_1 surrounding A_1 belonging to the same deletion component of C_1 . As argued in the proof of Lemma A.12, we can follow that the dangling condition is fulfilled if a complete deletion component is reached as overlap.

The span that arises by this construction fulfills the conflict and completeness condition by construction and it is minimal because we stop adding elements as soon as the transformation condition is fulfilled in addition. \square

THEOREM A.14 (COVERING OF CONFLICT REASONS BY MINIMAL CONFLICT REASONS AND ISOLATED BOUNDARY ATOMS). *Given a conflict reason $s_1 : C_1 \xrightarrow{o_1} S_1 \xrightarrow{e_2} L_2$ for rule r_1 and non-deleting rule r_2 , then set $M \cup B = \{s_i^m \mid i \in I\} \cup \{s_j^b \mid j \in J\}$ of all minimal conflict reasons and all isolated boundary atoms for (r_1, r_2) that can be embedded into s_1 via a corresponding set of embedding morphisms $E_M = \{e_i \mid i \in I\}$ and $E_B = \{e_j \mid j \in J\}$ covers s_1 , i.e. set $E = E_M \cup E_B$ is jointly surjective.*

PROOF. Assume that E is not jointly surjective. Then there exists at least one graph element x in S_1 that is not in the codomain of one of the morphisms in E .

First assume that x is mapped via o_1 to an edge or a node that is deleted by rule r_1 , then x leads to a conflict atom candidate (either with its incident nodes or the node itself) that can be embedded into s_1 (see Lemma A.5).

Assume otherwise that x is mapped via o_1 to a node that is not deleted by r_1 . Then, either x does not have incident edges to be deleted, i.e., x would lead to an isolated boundary atom, which would be a contradiction since then x would belong to the codomain of one of the morphisms in E_B , or there is an incident edge y in S_1 that is deleted. Now either both incident nodes of y are preserved such that y with its incident nodes represents a conflict atom candidate, or one of the incident nodes is deleted such that this deleted node represents a conflict atom candidate. So, in all cases, we have found a conflict atom candidate that is embedded into s_1 . Therefore, it is a conflict atom in particular.

Because of Theorem A.13 we can complete this conflict atom to a minimal conflict reason for (r_1, r_2) . In particular, we can add a minimal number of elements of the surrounding deletion component of the atom that are also present in the conflict reason s_1 such that the transformation condition is fulfilled and it can be embedded into s_1 . This is a contradiction since x is in the codomain of one of the morphisms in E_M . Therefore, E is indeed jointly surjective. \square

Now we are ready to conclude that conflict reasons corresponding to delete-read initial conflicts (new fine granularity, see Sec. 4.2) are indeed composed of minimal conflict reasons only. In particular, isolated boundary atoms do not occur in this type of conflict reason.

COROLLARY A.15 (COVERING OF INITIAL CONFLICT REASONS BY MINIMAL CONFLICT REASONS). *Given a conflict reason $s_1 : C_1 \xrightarrow{o_1} S_1 \xrightarrow{e_2} L_2$ for (r_1, r_2) with r_2 non-deleting, then set $M = \{s_i^m \mid i \in I\}$ of all minimal conflict reasons for (r_1, r_2) that can be embedded into s_1 via a corresponding set of embedding morphisms $E_M = \{e_i \mid i \in I\}$ covers s_1 , i.e. set E_M is jointly surjective.*

PROOF. This follows from Thm. A.14 and Def. A.8 stating in particular that initial conflicts cannot have isolated boundary atoms in their conflict reason s_1 such that the set B is indeed empty. \square

Finally, we show in the last corollary as stated in Sec. 4.2 that indeed an initial conflict for (r_1, r_2) with r_2 non deleting simply corresponds to the overlap of its conflict reason.

COROLLARY A.16 (INITIAL CONFLICT AND CONFLICT REASON). *Given an initial conflict $(t_1, t_2) = (K \xrightarrow{m_1, r_1} P_1, K \xrightarrow{m_2, r_2} P_2)$ for rule r_1 and non-deleting rule r_2 with conflict reason $s_1 : C_1 \xrightarrow{o_1} S_1 \xrightarrow{q_{12}} L_2$ then $(K, m_1 : L_1 \rightarrow K, m_2 : L_2 \rightarrow K)$ is the pushout of $(S_1, c_1 \circ o_1, e_2)$.*

PROOF. This follows directly from Def. A.2, Def. A.8 and the fact that rule r_2 is non-deleting. \square

Summarizing, Fig. 12 illustrates the new fine granularity for conflicts as introduced in this paper: As for the other granularities we overapproximate delete-use conflicts with delete-read conflicts (see right part of Fig. 12 and Th. A.10) considering conflict reasons of initial conflicts only. When building such conflict reasons, we exploit the fact that each delete-read initial conflict indeed corresponds to the overlap of its conflict reason (see left part of Fig. 12).

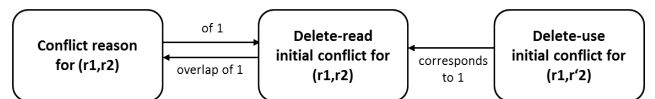


Figure 12: Fine granularity: r_2 non-deleting variant of r'_2