## Representation Theory of Lie Groups

Seminar in the Summer Semester 2024 Friday 8-10, SR X (Lahnberge) Oliver Goertsches Preliminary meeting: Tuesday, Feb 6, 9am, SR X For registration please write an email to: goertsch@mathematik.uni-marburg.de

In this seminar, we will read some of the later chapters of the book *Lie Groups, Lie Algebras, and Representations* by Brian C. Hall. We will assume knowledge of Lie groups, as provided for instance in the course *Lie groups and Lie algebras*.

## Lecture plan:

Lecture 1: Weyl group and Weyl chambers. Sections 8.4 and 8.5 (large parts of 8.4 already known; maybe start with Definition 8.20.)

Lecture 2: Integral and dominant integral elements. Sections 8.7 and 8.8.

Lectures 3 and 4: Theorem of the highest weight, part I. Section 9.1. (Note that for Theorem 9.4 one needs parts of Chapter 6) Give Proposition 6.17 as an example.

Lecture 5: Verma modules and universal enveloping algebras. Sections 9.2 and 9.3.

Lecture 6 (optional): Proof of the Poincaré–Birkhoff–Witt Theorem. Section 9.4.

Lecture 7: Construction of Verma modules. Section 9.5

Lecture 8: Theorem of the highest weight, part II. Sections 9.6 and 9.7.

Lecture 9: The weights of a finite-dimensional highest-weight representation. Section 10.1.

Lecture 10: The Casimir element and complete reducibility. Sections 10.2 and 10.3.

Lecture 11: The Weyl character formula. Section 10.4

Lecture 12: The Weyl dimension formula. Section 10.5.

Lecture 13: Kostant's multiplicity formula. Section 10.6

Lecture 14: Proof of the character formula. Sections 10.7 and 10.8.