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Modelling Firm and Market Dynamics - A Flexible Model Reproducing Existing Stylized Facts

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Abstract:

This paper presents a firm and market model that is able to reproduce the empirically observed patterns on firm growth and its statistical characteristics. It goes beyond the existing firm models by reproducing all stylized facts established in the literature. Furthermore, the model is flexible so that it can be adapted to certain industries and life-cycle stages. We analyse and discuss the options that are provided by the various parameters in this sense.

Keywords: Firm model, firm growth, market model, industry dynamics.

JEL Classifications: L11, L13, L22, L25, O12, C15.

I. Introduction

Economic growth is an important issue in science as well as in society. Most research in this field is done on national economic growth and the growth of firms. In recent years regional growth processes have received increasing attention (see Breinlich et al. 2014 for an overview). All kinds of studies can be found in this field, empirical studies as well as conceptual theoretical works and mathematical modelling approaches. On the level of firms the empirical literature is especially rich (see Coad et al. 2009 for an overview). As a consequence, the statistical knowledge about firm growth and firm size distribution is quite comprehensive. Although many firm theories exist (e.g. Penrose 1959), so far no firm model exists that is able to reproduce all well-established stylized facts.

In this paper we develop such a model. Conceptually, we build on the model of monopolistic competition by Dixit and Stiglitz (1977). As a consequence, we argue that firm dynamics can only be adequately modelled in connection with modelling market dynamics. The proposed model represents market dynamics, including the emergence and disappearance of submarkets, as well as firm dynamics, including innovation processes and firm foundation. The approach by Brenner and Werker (2007) is used to validate the model.

We show that the model is able to reproduce all well-known stylized facts on firm growth and their relation to firm size and age. We neither intend to nor obtain one simulation model. Instead, we identify a number of parameter sets that all lead to realistic outcomes. The different parameter sets are able to represent different market situation, such as different industries or different stages in the industrial life-cycle. The meanings of the various parameters in this context are discussed. The paper proceeds as follows. In the next section the simulation approach is described. Section 3 provides an overview on the existing firm models as well as on the empirical knowledge about firm growth. Our simulation model is developed in Section 4. Section 5 contains the calibration of the model and the discussion of the implied characteristics of firm growth. Section 6 concludes.

II. Simulation approach

II.1 Fundamental considerations

Our simulation approach follows the proposal for an abductive approach by Brenner and Werker (2007). The basic idea of this approach is a distinction between two spheres: (A) The sphere of the model and (B) the sphere of the implications. This distinction is an analytical tool, which is helpful because separate literature – theoretical and empirical – exists for each of the spheres. The spheres might refer to different spatial levels, but this is not a necessary condition. For example, the two spheres might be (A) firm growth and (B) regional growth, but they might also be (A) the mechanisms in firms underlying firm growth and (B) the statistical characteristics of firm growth in a firm population.

The simulation approach builds on the fact that (1) knowledge is available on both spheres and (2) there is a logical link (L) between the spheres that can be built into the simulation model. Most simulation approaches in the literature try to find one model (model sphere) that is able to reproduce the known facts on the implication sphere, given the logical link. The intention of the approach by Brenner and Werker is to find all models that are in line with the knowledge about the model sphere and the implication sphere. This is done in a two-step procedure.

The first step is to build various possible models which all together build the set of model specifications (see Figure 1). The aim is to develop a set of models that is quite general, restricting the models only as far as it can be justified by the available knowledge (this is done here in Section 4). Generality of the model is reached by using many parameters that are not or only vaguely fixed. A set of possible model specifications results (see Figure 1). Simulating one model specification leads to a theoretical realisation (logical link). Due to random elements in the models, rerunning a model might lead to different results, so that for each model specification a set of theoretical realisations results (see Figure 1).

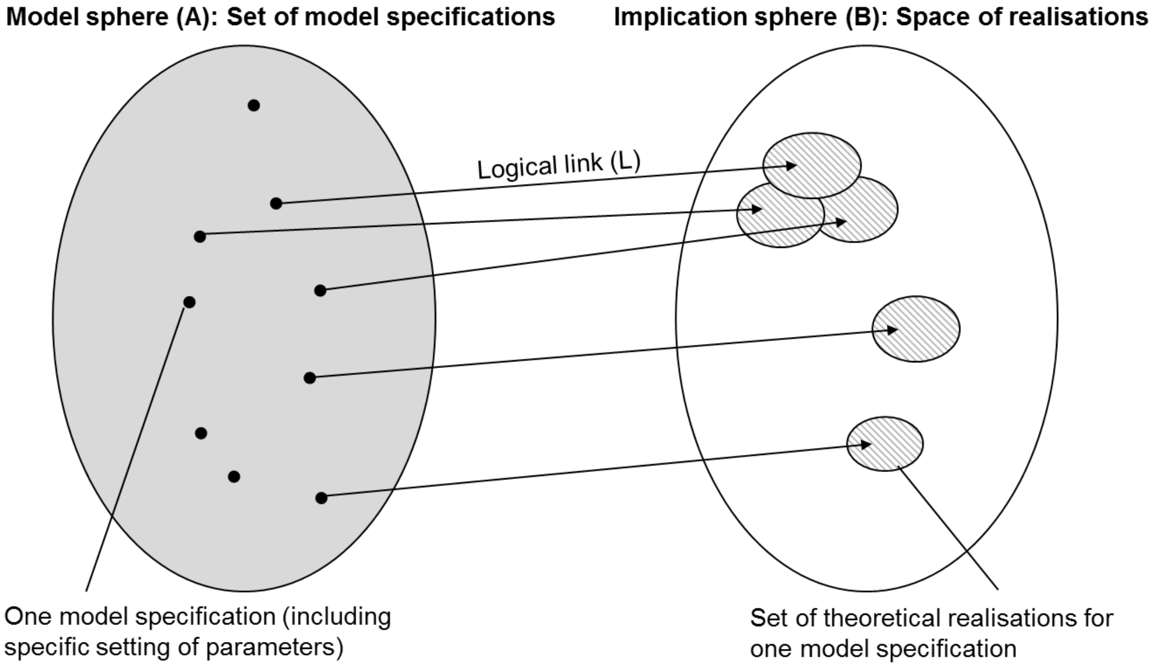

Figure 1: First step: Simulating the logical link from the model sphere to the implication sphere.

In a second step, numerous model specifications (parameter settings) are simulated and the implications are compared with the knowledge on the implication level (this is done here in Section 5). We might represent the empirical knowledge within the implication sphere as a subset representing the empirical realisations (see Figure 2). All model specifications that lead, at least partly, to realisations within this subset are in line with the knowledge on the implication level. Through this, we are able to decide for each simulated model specification whether it is a realistic setting and obtain an area of realistic parameter settings (see Figure 2). This validation step narrows down the set of model specifications to a smaller potentially realistic set.

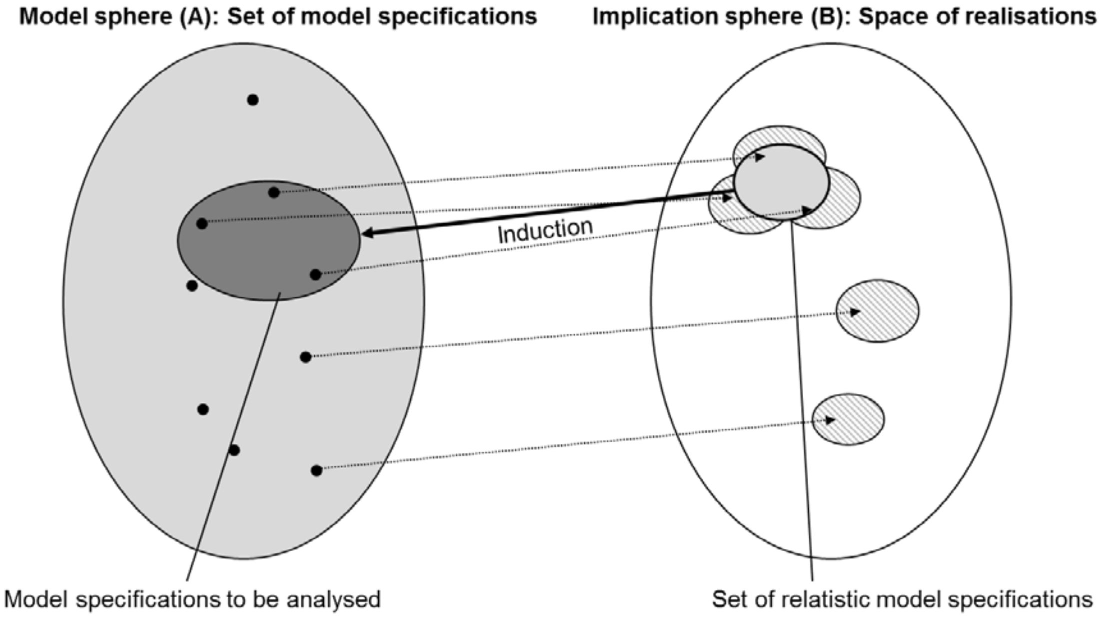

Figure 2: Second step: Inducing from knowledge on the implication level (empirical realisations) on the set of realistic model specifications in the model sphere.

The final result of this simulation approach is still not one simulation model, but a set of model specifications, containing all specifications that are in line with the available knowledge on the model sphere as well as the implication sphere.

II.2 Application to firm growth

In the context of this paper the model sphere (A) is defined as the sphere containing the mechanisms and processes within firms that lead to firm growth. The implication sphere (B) is defined as the level on which the statistical characteristics of firm growth are analysed. Hence, the first step is to set up a firm model that is in line with the mainly theoretical knowledge about firm and market processes (Section 4). The model is kept very general, containing many only vaguely restricted parameters. The second step is to simulate various parameter settings of this model (Section 5, in total 34200 specifications are simulated). The results of these simulations are compared to the empirical knowledge about the statistical characteristics of firm growth (this knowledge is described in Section 3). This allows to narrow down the set of realistic firm models tremendously.

III. Literature on firm growth

III.1 Modelling firms

Many perspectives are used in the literature to model firms and their size. (1) A traditional approach is to model firm output (as a measure of size) with a production function, which relates output to inputs such as labour and capital (e.g., Griliches and Mairesse 1995, Brynjolfsson and Hitt 1995). (2) Another traditional approach focuses on sales and deduces them from competition models. Various market structures and mechanisms are used in this strand of literature. For our approach only the idea of monopolistic competition (Dixit and Stiglitz 1977) is relevant. (3) A quite different perspective is taken in studies that aim to determine the optimal size of firms (e.g., Lucas 1978). (4) Finally, within the literature on firm growth a number of models have appeared in recent years that are built in an attempt to reproduce the known statistical characteristics of firm growth. Interestingly, these models do not build on the other three approaches.

The main reason for this is that the first three approaches do not aim at explaining dynamics. Furthermore, the first approach rather deals with the question of how a firm can use inputs to produce a certain given output. Hence, the output is not the *explanandum* but the *explanans*. In contrast, the second and third approach have the size of firms as *explanandum*. Hence, although they do not aim at explaining the dynamics of firm sizes, they can be of help in modelling them. In particular, we believe that firm size and its dynamics cannot be explained without considering market success and, thus, competition on the market.

However, the starting point for our modelling are the already existing models. These models are foremost concerned to explain the emergence of empirically observed patterns in firm growth, like the puzzling deviation from normality in the growth rate distributions or the scaling of their variance with firms' size. To move beyond Gibrat's (1931) model of proportional growth, which by most researchers is rejected on ground of these empirical findings and the assumption of an independent, random growth process, several models have been developed. Amaral et al. (1998) introduced a hierarchical model with subunits that evolve according to a random multiplicative process, which yields a Laplace distribution of growth rates and a power-law in the variance-scaling relationship. Fu et al. (2005) show that proportional growth is still able to reproduce these two statistical patterns if both the number of subunits and the size of the subunits are allowed to grow. In Schwarzkopf et al. (2010), a different mechanism within the hierarchical structure is assumed. Here, the subunits are replaced from a replication distribution, and originate either as new ones or are taken from another firm. In order to avoid any assumptions on the internal structure of firms, Bottazzi and Secchi (2006a) model the growth rate distribution by drawing on Penrose's (1959) idea of competencies and learning and by explicitly considering the market dimension. Basically, they assume that the firms' ability to take up new business opportunities increases with the number of opportunities already exploited. A self-reinforcing mechanism in the competition for limited market opportunities ultimately leads to fat-tailed growth rate distributions. However, these models focus only on specific stylized facts. Other empirical observations, like the temporal auto-correlation structure, are often neglected or stand even in contrast to the models' implications (Coad 2012). Besides, as these models aim to be very general, they are only loosely grounded on economic mechanisms. For instance, in Schwarzkopf et al. (2010), the Laplace distributed growth rates directly follow from the assumption of power-law replication function.

Hence, more complex Agent-Based models have been developed, which try to include knowledge on economic processes and mechanisms in their assumptions. While Delli Gatti et al. (2005) model explicitly the demand and supply on the credit market and the financial fragility of firms, Metzig and Gordon (2014) match employees to firms, which are required to produce goods and to generate profit. Here, firms compete both for aggregate demand and workforce. While these models are able to reproduce a larger number of stylized facts, they are quite specific, focusing on specific aspects. As discussed in Section 2, our aim is to develop a rather general model based on the knowledge about processes and mechanisms within firms. According to the simulation approach by Brenner and Werker (2007), the modelling on the model sphere should be not influenced by any knowledge on the implication level. In the case of firms, this is not completely possible because firms develop individually so that little general knowledge about their development is available. The existing theoretical models are often developed with some statistical characteristics of firm growth in mind (see above). All that we can do is to orient our own modelling approach more on the firm models that are more general and do not explicitly aim to reproduce all statistical characteristics of firm growth.

Furthermore, we can use theoretical literature on firms that is not connected to any empirical examination of firm growth. Two theoretical ideas are of interest here. First, Dixit and Stiglitz (1977) developed the model of monopolistic competition, which is based on the idea that products differ so that submarkets exist in which certain firms might dominate and are able to decide about prices more freely. Second, Ijiri and Simon (1977) developed the idea that each firm faces a set of growth opportunities. As a consequence, firms grow in dependence of how many of these opportunities they are able to realise. Bottazzi and Secchi (2006a) developed this idea further by assuming that the total number of growth opportunities for all firms is limited.

By combining both ideas we might assume that a market consists of many sub-markets (determined by specific consumer preferences) with firms competing for these sub-markets and each firm dominating some of them. According to Penrose's (1959) theory of the growth of the firm, firms compete on basis of their competencies, and from a dynamic perspective they also compete on how they develop and advance their capabilities (Teece et al. 1997). However, markets are also not static. Here, innovation processes are repeatedly seen in the literature as an important aspect in the competition for new markets. Nelson and Winter (1978) have framed the notion of „Schumpeterian competition“ in this context. Ericson and Pakes (1995) modelled firm behavior as an exploration of evolving market places by investing in research. Hence, we might assume that firms compete for existing and emerging sub-markets on basis of their competencies and the research they perform. Such a perspective is used below.

III.2 Characteristics of firm growth

For the second step of our simulation approach we have to collect the available empirical knowledge on the implication sphere, meaning the knowledge about the statistical characteristics of firm growth. Many empirical studies have been conducted examining various aspects of firm growth.

First of all, the observed patterns of firm growth are more stable and invariant than the corresponding size and age distributions, which are strongly history-, industry- and country-specific (Bottazzi et al. 2007, Dosi et al. 2010). Some of the detected statistical characteristics, like the variance-scaling relationship or the fat-tailed growth rate distributions already mentioned above, are even regarded as universal laws which seem to hold in the growth of all complex organizations. That means they are independent of the particular details of systems (Lee et al. 1998). Hence, stylized facts on firm growth represent adequate selection criteria in the implication sphere for identifying reasonable specifications of the model.

Studies testing Gibrat's law of proportionate growth find that the average growth rate is not independent of but decreases with the size of firms (e.g., Evans 1987, Dunne and Hughes 1994, Bottazzi et al. 2011). Although some deviation from this negative relationship is observed for specific sub-populations of firms (see Coad 2009 for a more extensive discussion), this "statistical regularity" (Sutton 1997) is often considered for the construction and validation of theoretical firm growth models. Furthermore, these studies observe that this negative dependence of average growth is also valid for firm age. In the literature, it is often argued that firm age, which is strongly correlated with firm size, is causally even closer connected with average growth (Evans 1987, Dunne et al. 1989, Geroski and Gugler 2004).

More recently, the literature has departed from merely focusing on average growth rates. In the econophysics literature (e.g., Stanley et al. 1996) it was pointed out that the variance of growth rates scales with the logarithm of firm size as a power law. A variance-scaling parameter coefficient between -0.15 (Stanley et al., 1996) and -0.20 (Amaral et al., 1997) is often reported, although some studies on firm dynamics in France (Bottazzi et al. 2011) and China (Duschl and Peng 2014) find a coefficient as low as -0.07 and -0.06, respectively. As it is generally true for power laws (Gabaix 2009), many possible generating mechanisms exist for the variance-scaling relationship of firm growth rates, with Amaral et al. (2001) and Bottazzi and Secchi (2006b) providing two alternative theoretical models.

Another stylized fact concerns the unconditional distribution of firms' growth rates. The earlier studies in the literature (e.g., Stanley et al. 1996, Bottazzi et al. 2001) show that growth rates significantly deviate from normality due to the presence of fat tails and are much closer to the tent-like shaped Laplace distribution. This finding is confirmed for firms in various countries (e.g., Duschl et al. 2014 for Germany), in different industries (Bottazzi and Secchi 2003) and for alternative measures of firm size (Erlingsson et al. 2013). More recently, tails that are even fatter than expected from the Laplace distribution and an asymmetric shape of the growth rates distribution is observed (Bottazzi et al. 2011, Reichstein and Jensen 2005). Hence, Bottazzi and Secchi (2011) suggest to use the more flexible Asymmetric Exponential Power (AEP) distribution, which has five parameters, to account both for fatness of the tails and asymmetry in the growth rate distributions. By comparing this theoretical distribution to alternative distributions, like the Student-t, Cauchy or Levy-stable distribution, Fagiolo et al. (2008) demonstrate empirically that it is the preferred specification, as it is especially flexible in the tail behaviour.

Finally, the empirical literature on firm dynamics has also strongly focused on the temporal auto-correlation structure of growth rates. The results from decades of research are, however, mixed and often conflicting (Caves 1998, Coad 2009). Depending on the country, industry and number of lags considered, some studies report positive autocorrelations over time (e.g., Chesher 1979), others report negative ones (e.g., Bottazzi et al. 2011) or even fail to detect any temporal correlation structure (Lotti et al. 2003). New light on the study of temporal auto-correlation has been shed by Coad (2007) who additionally takes into account the size of the firm as well as the growth rate itself. First, he demonstrated that the larger the firm, the larger also the auto-correlation coefficient. For very small firms, the coefficient tends to be negative, as they follow a more stochastic growth path, while for large firms it becomes positive and indicates a more stable growth path. Secondly, firms in the tails of the unconditional growth rate distribution are less likely to repeat their extreme growth experience in the subsequent year.

In contrast to the statistical characteristics of growth rates, a more heterogeneous picture emerges for the corresponding distributions of firm size and firm age. Whereas an exponential distribution seems to fit reasonable well the age distribution (Coad 2010), an ongoing debate exists on the appropriate distributional model for firm size. Theoretical and empirical evidence exists both for the log-normal (e.g., Cabral and Mata 2003) as well as the Pareto distribution (e.g. Axtell 2001), which seems to fit better beyond a certain size threshold. Even if common patterns in terms of the functional form might exist, the parameters are strongly influenced by the size of the economy (Gaffeo et al. 2003) or by historical contingencies, like the bumps in the age distributions due to both world wars. Furthermore, these patterns do not survive at the more disaggregated level of industries. Here, often bimodal size distributions are observed (Bottazzi et al. 2011). Therefore, we restrict the analysis of the simulation outcomes to one of the few robust stylized facts about the size and age distribution of firms, which has not been put into question since Gibrat (1931): both distributions are found to be highly skewed to the right. Put differently, the coefficients of the location and scale of the size and age distribution can be only used as additional information in the assessment of the simulation outcomes, but not as criteria to reject the corresponding simulation model specification.

IV. Simulation model

IV. 1 Basic considerations

A common feature of many previous firm models that are able to resemble the known statistical characteristics of firm growth is the definition of sub-units or business opportunities. Shortly after realising that firm growth does not show completely unstructured, random dynamics – as assumed by Gibrat (1931) – the existence of sub-units was used to explain the observed statistical characteristics (Amaral et al. 1998). This leads to discrete steps in the development of the size of firms instead of continuous dynamics, which is more in line with empirical findings. An alternative model (e.g. Bottazzi & Secchi 2006a), which tries to avoid assumptions on the internal structure of the firm, uses business opportunities as units, moving the concept of units to the market side. The original idea was introduced by Ijiri and Simon (1977), who argue that growth results from the number of opportunities a firm is able to take up.

While Bottazzi and Secchi (2006a) focus on new opportunities, their approach can be generalised by interpreting each business opportunity as an existing sub-market. Bottazzi and Secchi (2006a) represent competition in their model by limiting the growth of the complete firm population to a given finite set of new opportunities. Interpreting this in the framework of monopolistic competition (Dixit and Stiglitz 1977), each business opportunity can be seen as a sub-market that is dominated by one firm.

Since we believe that competition is the main underlying mechanism of firm growth we go beyond the existing models and explicitly consider sub-markets and competition for these sub-markets. Sub-markets are not assigned to firms once at the time they appear, but there is permanent competition for sub-markets. However, following the idea of monopolistic competition, at each time each sub-market is dominated by one firm that is a quasi-monopolist in this sub-market. Furthermore, we explicitly consider firm characteristics that might change with time and assume that firms compete for sub-markets on the basis of these characteristics, an idea rooted in the dynamic capabilities approach (Penrose 1959, Teece et al. 1997).

As a consequence, the model contains two kinds of units of observation: markets and firms. In the following two subsections we discuss the modelling of the dynamics of markets and firms one after the other. All parameters of the model are denoted by μ and σ with the respective indices. $N(\mu,\sigma)$ stands for the normal distribution with the respective mean and variance.

IV. 2 Market dynamics

As outlined above, we assume monopolistic competition and model the market as a set of sub-markets that are each dominated by one firm. We call the sub-markets 'market packages' to signify that they have a certain *size* and are *possessed* by certain firms. Competition works on these market packages with always one firm winning the competition and possessing the whole market package.

As a consequence, the respective market is given at any point in time t by the set of market packages $M_{tot,t}$. Each market package $m \in M_{tot,t}$ is characterised at each point in time t by its size $d_{m,t}$ and its age $a_{m,t}$. Hence, the firm that possesses a certain market package m – meaning that it supplies this sub-market – faces a demand of $d_{m,t}$ from this sub-market. To keep the model simple, we do not care about further characteristics of market packages. As a consequence, we have to model three processes: (1) new market packages might appear (the process modelled by Bottazzi and Secchi 2006), (2) market packages might disappear, and (3) market packages might change their size.

Appearance of market packages

The simplest way of modelling the appearance of market packages is an independent random process. Hence, new market packages appear at each point in time (each day) with a certain probability $\mu_{m,new}$.

If a market package appears, its size $d_{m,t}$ is randomly drawn from a uniform distribution between 0 and $\mu_{d,max}$.

Disappearance of market packages

A similarly simple way is used for the disappearance: Each existing market package has at each point in time (each day) a certain probability ($\mu_{m,ex0}+\mu_{m,exa}*a_{m,t}$) *to disappear*. This probability increases with the age of the market package.

However, there is a further process that leads to the disappearance of market packages. The market packages randomly change in size (see below). If the size of a market package falls below *0.005* (million EUR), it also disappears. However, in most settings this happens very rarely.

Changes in the size of market packages

Market packages change their size randomly. This random change is assumed to follow a normal distribution and to be proportional to the actual size. Hence, it can be mathematically written by

$$d_{m,t+1}=d_{m,t}\cdot\left(1+N\left(\mu_{d,mean},\sigma_{d}\right)\right) \quad . \tag{1}$$

Each day, the market packages fluctuate in their size randomly, some packages becoming larger while others become smaller. Hence, Gibrat's (1931) proportional growth model is introduced here at the level of market packages. In addition, there is also a trend in the development of the whole market given by $\mu_{d,mean}$. Besides the appearance probability $\mu_{m,new}$, this allows to model growing and shrinking markets.

In addition, we assume a yearly fluctuation in order to consider fashion trends or other medium-term fluctuations in demand. Hence, the real demand $D_{m,t}$ for each market package is given by

$$D_{m,t}=d_{m,t}+N_{m,y}\left(0,\sigma_{y}\right) \quad , \tag{2}$$

where $N_{m,y}$ is independently drawn for each market package *m* in each year *y*.

IV. 3 Firm dynamics

Firms are modelled on the basis of the market packages that they possess. Hence, competition plays a major role in our model for determining the size of firms. Firm exits are not explicitly modelled. Firms exit if they lose their last market package. Each firm *f* is characterised at each point in time *t* by the set of market packages $M_{f,t}$ that it possesses. Thus, the demand it faces is given by

$$D_{f,t}=\sum_{m\in M_{f,t}} D_{m,t} \quad . \tag{3}$$

Therefore, we have to model mainly the competition process that determines which market package is possessed by which firm. Empirical studies show that only a small part of firm success can be explained by factors that are easily observed from the outside. Decisions and structures within the firm are decisive. Part of the story is whether firms approach the right markets at the right time, part of the story are internal processes and decisions that affect competitiveness. In an abstract approach, both parts have to be modelled as random processes. The former is modelled below. In order to reflect variations in the competitiveness of firms, a competitiveness value $C_{f,t}$ is assigned to each firm when it is founded. The initial competitiveness $\mu_{c,init}$ is the same for all firms. Then every year the competitiveness is updated according to

$$C_{f,t+1\,year}=\mu_{c,r}*C_{f,t}+N(0,\sigma_c) \quad , \qquad (4)$$

meaning that the development of competitiveness follows a random walk process. Two kinds of competitions have to be modelled: (1) the competition for new market packages and (2) the competition for existing market packages. While some firms focus on new markets – we might call them innovators –, other firms focus on existing markets – we might call them imitators. To model this difference, a random value s_f between 0 and 1 is drawn for each firm when it is founded. The higher s_f, the more innovative is a firm.

Competition for existing market packages

We assume that markets show a certain stability in the sense that, in general, sub-markets stay with the same firm. Specific events, such as technological changes or fashion changes, are necessary to lower the grip of firms on sub-markets and allow other firms to enter. For simplicity, we assume that such events occur randomly, so that at each point in time *t* there is a certain probability $p_{mov,m,t}$ that an existing sub-market becomes the object of competition. This probability might increase with the age of a market package as well as with the time that it is owned by the same firm. Therefore, we define the probability $p_{mov,m,t}$ as

$$p_{mov,m,t}=\mu_{mov,0}+\mu_{mov,a}\cdot a_{m,t}+\mu_{mov,o}\cdot o_{m,t} \quad , \qquad (5)$$

where $o_{m,t}$ denotes the time that market package *m* is owned by the same firm.

If a market package becomes the object of competition, the firm currently owning the market package has to compete with one randomly drawn other firm. The competing firm might be a new firm or an already existing firm. We define a probability $\mu_{com,start,exist}$ which stands for the basic probability that the competitor is a new firm. Hence, with this probability a new firm is created that tries to gather the market package. Otherwise, the competing firm is drawn from the existing firms with the likelihood of each firm being proportional to

$$1+\mu_{p,turn}\cdot T_{f,t} \quad , \qquad (6)$$

meaning that larger firms have a higher likelihood. In this draw, a new firm is also considered with a likelihood proportional to $\mu_{T,new}$ in order to reflect the fact that the probability of new firms entering is higher if the actual market activity (total turnover of existing firms) is lower. This can be seen as part of a life cycle development in which entries become less likely the more established the market and the existing firms are (e.g. Klepper 1996).

The competition strength $C_{exist,m,f,t}$ of a firm *f* with respect to an existing market package is calculated according to

$$C_{exist,m,f,t}=\left(1+C_{f,t}+\mu_{exist,cap}\cdot\left(T_{f,t}-D_{f,t}\right)^3\right)\cdot\left(1+\mu_{exist,s}\cdot\left(1-s_f\right)\right) \quad , \tag{7}$$

where $\mu_{exist,cap}$ and $\mu_{exist,s}$ are parameters. $\mu_{exist,cap}$ determines the importance of capacity effects. Production and, hence, turnover gradually adapts to demand (see below). As a consequence, if demand changes quickly, a firm might have problems to adapt, which might have an impact on the chance and willingness of the firm to obtain or loose further market packages. We assume that such an effect becomes relevant only for large difference between turnover and demand and, therefore, use the cubic form of this difference above. $\mu_{exist,s}$ determines the importance of the strategy variable, implying that firms that focus on imitative behaviour have higher chances to obtain existing market packages.

As mentioned above, we assume that the ownership of market packages shows some stability. Therefore, another firm is only able to gather a market package if it has a competence value $C_{exist,m,f,t}$ that exceeds the one of the currently owning firm by a certain amount $\mu_{threshold}$.

Competition for new market packages

The appearance of new market packages is determined as described above (Section IV.2). The competition for new market packages is modelled similar to the competition for existing market packages. Again there is a certain probability $\mu_{com,start,new}$ that a new firm obtains the new market package. For each existing firm *f* a competition strength $C_{new,m,f,t}$ is calculated according to

$$C_{new,m,f,t}=1+\mu_{new,turn}\cdot T_{f,t}+\mu_{new,size}\cdot T_{f,t}\cdot D_{m,t}+\mu_{new,cap}\cdot\left(T_{f,t}-D_{f,t}\right)^3+\mu_{new,inno}\cdot I_{f,t} \quad , \tag{8}$$

where $\mu_{new,turn}$, $\mu_{new,size}$, $\mu_{new,cap}$ and $\mu_{new,inno}$ are parameters. $\mu_{new,turn}$ determines whether larger firm have a higher probability to win new market packages. $\mu_{new,size}$ stands for the fact that larger market packages might be more likely won by larger firms while smaller market packages are more likely won by smaller firms. $\mu_{new,cap}$ has the same meaning as described for $\mu_{exist,cap}$ above. $\mu_{new,inno}$ determines the dependence of the competition strength on the innovation potential $I_{f,t}$ of the firm (see below). Again, a potential start-up is assigned a competition strength of $\mu_{start,new}$, and the competition process is a random draw of a winning firm.

Development of firm characteristics

The potential demand $D_{f,t}$ that a firm *f* faces at time *t* is given by Equation (3). However, the turnover of a firm must not automatically be the same as the potential demand. Especially if new sub-markets appear, it might take some time before a firm is able to satisfy the demand and/or before the customers are aware of the new possibility. Hence, we define a turnover variable $T_{f,t}$ that represents the real sales of a firm. We assume that this variable adapts towards the potential demand at each time step according to

$$T_{f,t+1}=T_{f,t}+\mu_{T,adapt}\cdot\left(D_{f,t}-T_{f,t}\right) \quad , \tag{9}$$

where $\mu_{T,adapt}$ is a parameter that determines the speed of this adaptation.

Furthermore, we explicitly model the innovation process of firms. Following other approaches (such as Nelson and Winter 1978 and Ericson and Pakes 1995), we assume that innovation activities play an important role in firm growth and that firms develop routines and strategies that make them more or less innovative. Therefore, in our model firms are characterised by their innovativeness $I_{f,t}$ at each point in time *t*. A parameter $\mu_{inno,start}$ is defined which represents the innovativeness of a firm when it is founded. During the existence of a firm, its innovativeness develops according to

$$I_{f,t+1} = \mu_{red,inno} \cdot I_{f,t} + D_I(f,t) \cdot s_f \quad , \qquad (10)$$

where $\mu_{red,inno}$ is a parameter and $D_I(f,t)$ is a function that is 1 if firm *f* wins a new market package – which can be interpreted as an innovation – at time *t* and 0 otherwise. Hence, the innovativeness of firms increases with each successful innovation and decreases slowly (the speed is given by $\mu_{red,inno}$) thereafter. Innovation success is assumed to increase the innovative capability of firms. The increase of the innovative capability with each innovation success is modelled proportional to the strategy variable s_f, meaning that firms with a higher focus on innovation stay on average on a higher level of innovativeness.

V. Model calibration and implications

V.1. Simulation procedure

The above model contains in total 28 parameters. The following model calibration has two aims. First, we check whether the model is able to generate realistic firm dynamics. Second, we identify those parameter sets that lead to realistic firm dynamics.

One central finding regarding the first aim is that realistic firm dynamics are only obtained if we simulate the whole development of an industry or economy. Starting the simulation model with a random firm population has caused deviations from the known characteristics of firm growth. It would have been necessary to start with an empirical firm population, which would not be in line with testing whether the simulated firm population matches empirical knowledge. Therefore, we start all simulations with a situation with one market package and one firm. We run each simulation for a randomly drawn time period between 10 and 50 years. Realistic industry developments are represented by changing probabilities for the appearance of new market packages and the disappearance of existing market packages. New market packages become first more and then less likely. The disappearance of market packages becomes more likely with time. Through this, we resemble industry life-cycle dynamics. Then, 10 further years are simulated and the statistical characteristics of firms and firm growth are studied. All simulations are run with time steps of one day.

For each simulation run we randomly draw each parameter from its range (see Table A.1 in the appendix). Then it is checked whether the parameter set leads to realistic firm characteristics (Section V.2). The aim is to identify not only one but many parameter sets. We argue that a firm model with some flexibility is needed so that, for example, different industries can be simulated.

V.2. Model calibration

Knowledge about the statistical characteristics of firm growth, as outlined in Section III.2, is used to identify the realistic parameter space of the simulation model (see the description of the calibration approach in Section II). As common in the literature, growth rate *g* of firm *f* is defined as the log-difference of its size, here measured by its turnover T_f, between time *t+1* and *t*:

$$g_f = \log(T_{f,t+1}) - \log(T_{f,t}) . \quad (11)$$

According to the literature discussed in section III.2, the growth rates g_f should fulfil the following criteria (see also Table 1). First, if g_f is fitted unconditionally to the Asymmetric Exponential Power (AEP) density (for mathematical details and related issues regarding the inference, see Bottazzi and Secchi 2011), the estimated shape parameters b_{left} and b_{right} should indicate the existence of fat tails on both sides of the distribution. The smaller these parameters, the fatter the tails of the density function at the respective side of the mode. In case of $b_{left} = b_{right} = 2$, it converges to the normal distribution, and in case of $b_{left} = b_{right} = 1$ to the Laplace distribution. Hence, we set the cut-off value to 1.3, which is one of the highest values observed in literature for b_{left} or b_{right}, although most studies find values lower than 1.

Secondly, the variance of g_f should scale negatively as a power law with the log size of *f*. We follow Bottazzi et al. (2014) in modelling this variance-scaling relationship directly by introducing a heteroskedasticity term into the stochastic growth process. The resulting scaling parameter *beta* should lie within the highest and lowest values observed in the literature, that is in the interval [-0.25 < *beta* < -0.06].

Next, to assess the stylized facts concerning the average growth rate, we estimated a Gibrat-like growth regression (Audretsch and Lehmann 2005), in which g_f is regressed on the log of size log($T_{f,t}$), age a_f, past growth rate $g_{f,t-1}$ and an interaction term between $g_{f,t-1}$ and log($T_{f,t}$):

$$g_f = b_0 + b_1 \log(T_{f,t}) + b_2 a_f + b_3 g_{f,t-1} + b_4 \log(T_{f,t}) * g_{f,t-1} + u , \quad (12)$$

with *u* denoting an independent, but possibly not normal distributed error term. Therefore, equation (12) is estimated using least absolute deviations (Dasgupta and Mishra 2004). To assess the temporal auto-correlation structure along the entire conditional growth rate distribution, the equation is also estimated at the quantiles $\theta_{0.1}$, $\theta_{0.25}$, $\theta_{0.75}$ and $\theta_{0.9}$ applying quantile regression techniques. To conform to the literature, b_1 and b_2 should be negative, indicating negative dependence of average growth on size and age, respectively. Although the literature is inconclusive about the absolute value of b_3, it shows that the temporal auto-correlation should be smaller at the tails of the conditional growth rate distribution, meaning that extreme growth events are less likely to repeat. Furthermore, temporal auto-correlation is known to be larger for larger firms, hence the coefficient of the interaction term b_4 should be positive.

Finally, less robust knowledge exists about the distribution of firm population characteristics, like size and age. Here, we only exclude simulation outcomes in which one basic fact is not fulfilled: the right-skewedness of the size and age distribution. This can be assessed by comparing the mean and median of the corresponding distributions.

Table 1: Selection criteria for empirical model validation

Statistical characteristic	*Parameters*	*Selection values*
Fat-tailed growth rate distribution	Shape parameter of AEP (b_{left}, b_{right})	[b_{left} < 1.3] [b_{right} < 1.3]
Variance-scaling relationship	Scaling parameter *beta*	[-0.25 < *beta* < -0.06]
Negative dependence of average growth on size	Coefficient of size b_1 growth regression	$b_1<0$
Negative dependence of average growth on age	Coefficient of age b_2 growth regression	$b_2<0$
Smaller auto-correlation coefficient at tails of growth rate distribution	Auto-correlation coefficient b_3 in growth regression at different quantiles θ	$b_{3,\theta 0.1} < b_{3,\theta 0.25} < b_{3,\theta 0.5}$ $b_{3,\theta 0.9} < b_{3,\theta 0.75} < b_{3,\theta 0.5}$
Positive size-dependence of auto-correlation coefficient	Coefficient of interaction term b_4 of lagged growth and size in growth regression	$b_4>0$
Right-skewed size distribution	Mean and median of size T_f	mean(T_f) > median(T_f)
Right-skewed age distribution	Mean and median of age a_f	mean(a_f) > median(a_f)

In total we examine 34200 model specifications and find nine specifications that lead to firm growth processes that satisfy all our selection criteria. Hence, our first aim is reached. The developed model is, indeed, able to reproduce many characteristics on firm growth that are observed in reality. However, we also find that the ranges of the model parameters have been chosen much too large. Only approximately 0.03 % of our model specifications show realistic dynamics. The huge parameter ranges have been chosen to be sure that no realistic specification is missed. For future approaches the ranges can be narrowed and the discussion in the next subsection helps to define adequate restrictions.

V.3. Identified models

As argued above, we identify a number of parameter sets that lead to realistic firm characteristics and dynamics. In total, nine realistic parameter sets result from 34200 model specification that we tested. They are listed in Table 2. In addition, the average size and age is given in Table 2.

Table 2: Realistic parameter sets

Parameter	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7	Set 8	Set 9
$\mu_{d,max}$	8.12	8.15	.21	.26	.21	1.88	1.45	2.66	2.91
$\mu_{d,mean}$	-.00010	-.00015	-.00097	-.00041	-.00024	.0000048	.00014	.00036	.00021
$\mu_{m,ex0}$	.00082	.000022	.00020	.00027	.00031	.000048	.00011	.00064	.00025
$\mu_{m,exa}$	.00017	.00019	.00019	.00012	.00059	.00048	.00082	.00060	.00029
$\mu_{c,init}$	.012	.015	.000040	.0044	.000058	.012	.0012	.0030	.045
$\mu_{c,r}$	.94	.85	.81	.85	.91	.82	.80	.99	.83
$\mu_{T,adapt}$	.0014	.00021	.00065	.00019	.00014	.071	.0050	.047	.024
$\mu_{inno,start}$	.37	.079	.0012	.0021	.037	.059	.21	.0071	.37
$\mu_{red,inno}$	.994	.998	.994	.997	.995	.996	.997	.994	.9998
$\mu_{move,0}$	.035	.036	.058	.069	.063	.032	.018	.050	.0052
$\mu_{move,a}$	.000048	.00087	.000038	.00067	.00092	.000025	.000014	.00075	.00067
$\mu_{move,o}$	.0000057	.000013	.000012	.000026	.000010	.00078	.000029	.000018	.000064
$\mu_{threshold}$	.00011	.00023	.00013	.00023	.0019	.0036	.0016	.0086	.025
$\mu_{com,start,exist}$	.0020	.0021	.0181	.0082	.0097	.0095	.0081	.0087	.064
$\mu_{p,turn}$	.0094	.0014	.0111	.0082	.018	.0046	.0062	.0059	.0076
$\mu_{T,new}$	21.87	1.43	18.05	28.48	15.87	5.05	3.40	1.68	3.35
$\mu_{exist,cap}$	.00014	.000025	.000031	.051	.034	.00086	.00049	.00038	.0043
$\mu_{exist,s}$	.000011	.0040	.128	.17	.033	.000046	.012	.045	.00066
$\mu_{m,new}$	4.80	.918	6.68	5.04	3.32	.67	.16	.171	.20
$\mu_{start,new}$	308	353	2.02	4.18	9.41	333	550	233	4.67
$\mu_{com,start,new}$	.066	.013	.10	.26	.30	.025	.013	.071	.24
$\mu_{new,turn}$	-.0084	.0055	.0092	.0023	.0039	.0027	-.0031	.0093	.0018
$\mu_{new,size}$	4.0 E-07	.0000050	1.1 E-08	4.8 E-07	9.9 E-09	1.7 E-10	1.2 E-09	1.6 E-10	1.4 E-09
$\mu_{new,cap}$	.89	.0016	.0022	.13	.0000042	.32	.000016	.83	.0025
$\mu_{new,inno}$	.015	.00012	.00067	.17	.036	.000021	.072	.00089	.021
σ_d	.0022	.0025	.0079	.0066	.0021	.00055	.0056	.0048	.0070
σ_y	.00065	.0052	.0056	.0070	.069	.053	.0057	.10	.065
σ_c	.00013	.0097	.00090	.00071	.00012	.0039	.00093	.00056	.0032
Mean $T_{f,t}$	10.0	45.8	.25	.32	.32	1.85	1.55	3.30	3.39
Mean $a_{f,t}$	5.00	6.21	10.5	8.75	8.07	16.4	10.8	15.1	5.52

The sets of realistic parameter settings are interesting for two reasons. First, since we tested wide ranges for the parameters, the realistic parameter values provide information about the strength and importance of mechanisms. Second, the various realistic parameter sets represent different situations, so that they enable the modelling of industries with different characteristics.

Quite some of the parameters seem to be not of crucial importance for whether the simulation shows realistic firm behaviour or not. However, some parameters are in all realistic simulation runs quite similar and the originally fixed parameter ranges are found to be too large. Six parameters of this kind can be identified and will be discussed in the following. The increase ($\mu_{m,exa}$) of the disappearance probability with the age of market packages is rather on the upper end of the originally set range, meaning that this is an important, realistic mechanism. The stability ($\mu_{c,r}$) of firm's competition strength never takes values below 0.8 in realistic simulation runs, meaning that competition strength changes rather slowly in reality. This aspect is strengthened by the fact that also the variance (σ_c) of competition strength never takes higher values in realistic simulation runs. Values on the upper end of the original range are not found for the innovative ability of start-ups ($\mu_{inno,start}$), meaning that an extremely high innovativeness of new firms is not a realistic feature. The increase ($\mu_{move,o}$) of the probability to lose a market package with the time of owning it lays in all realistic simulation runs on the lower end of the parameter range. Put differently, this dependence is very weak if existing at all. Maybe we could even withdraw this effect from the model. A similar result is obtained for the dependence ($\mu_{new,size}$) of winning a new market package on the matching between the size of the firm and the size of the market package. This dependence is either weak or not existing.

The second intention is to find parameter variations that can be used to simulate different realities such as different industries or different stages in the industrial life-cycle. The parameter sets identified and listed in Table 2 show that this aim is reached. While three parameter sets (Sets 3 to 5) are quite similar, all other identified parameter sets show clear differences. Hence, they indeed represent different kinds of markets and can be used to simulated different industrial realities. This will be discussed in more detail in the following.

A general observation is that the maximal market package size ($\mu_{d,max}$) influences strongly the average size of firms. However, this often comes along with other characteristics. Some of the parameters are not independent of each other. To produce realistic model behaviour, some parameters have to take corresponding values. The maximal market package size ($\mu_{d,max}$) is clearly connected to two other parameters: High average market package sizes come together with high initial competition strengths ($\mu_{c,init}$), implying a high stability of competitiveness, and low probabilities of losing market packages to start-ups ($\mu_{com,start,exist}$). Hence, large market packages are connected to a high stability of market package ownership and lead to large average firm size.

In general, larger market package size come also together with lower competition strengths of start-ups ($\mu_{T,new}$) and a lower number of new (innovative) market packages ($\mu_{m,new}$). However, Set 1 builds an exception in this context and deserves more detailed discussion. While in general Set 1 is quite similar to Set 2 with relatively large market packages and firm sizes, it represents a market with a high number of new (innovative) market packages ($\mu_{m,new}$) and a high competitiveness of start-ups ($\mu_{T,new}$), two characteristics that are found rather in volatile markets with small average firm sizes (Sets 3 to 5). This higher volatility results also in a lower average firms size compared to Set 2. However, to some extent this volatility is counterbalanced by a high probability of firms with free production capacities to enter new (innovative) markets ($\mu_{new,cap}$). Thus, Set 1 represents a market that is dominated by rather large and stable firms, but shows also a high dynamic in terms of a frequent occurrence of new sub-markets and firms.

While Sets 2 to 5 can be seen as the typical cases for a market dominated by large incumbent firms (Set 2) and a volatile market with many small firms (Sets 3 to 5), the Sets 6 to 8 lie somehow between these two extreme cases. Besides this, Set 6 shows only three specificities: firms are able to adapt their production capacities quickly ($\mu_{T,adapt}$), it is impossible for firms to own market packages for a very long time ($\mu_{move,o}$) and firm strategy ($\mu_{exist,s}$) and prior innovation success ($\mu_{new,inno}$) are of low importance. Hence, Set 6 seems to represent a rather low-tech, competitive market with low fix costs in production and a very stable firm population (high average firm age).

In contrast, Set 7 is characterised by a high dependence on prior innovation success ($\mu_{new,inno}$), a high probability of smaller firms to gather new, innovative market packages ($\mu_{new,turn}$<0) and a high innovativeness of start-ups. This is somewhat counterbalanced by a low frequency of new market packages ($\mu_{m,new}$). Thus, Set 7 represents a market that is, in general, quite stable but contains a part with very innovative small firms and start-ups.

Set 8 is characterised by a very high stability of firm's competitive strength ($\mu_{c,r}$). From year to year 99% of the competitive strength remains constant. In addition, the year to year variation in competitive strength (σ_c) is also rather low. In contrast, the year to year variation in market package size (σ_y) is quite high. New market packages are rather rare ($\mu_{m,new}$). Furthermore, firms are able to adapt their production capacities quickly ($\mu_{T,adapt}$) to the demand for their products. Hence, Set 8 describes a situation in which markets rather change in size, innovations are rare and firms are flexible and quite stable (high average age of firms).

Set 9 has a number of characteristics that makes it quite specific, so that it is discussed here in more detail. First, it shows the highest value for the innovativeness of start-ups ($\mu_{inno,start}$) and an exceptionally high stability of the innovativeness of firms ($\mu_{red,inno}$). In combination with a high success-breads-success element in the innovation activity ($\mu_{new,inno}$), this leads to a strong accumulation of research capacities within firms. In addition, the ownership of market packages is very stable (highest value of $\mu_{threshold}$ and lowest value of $\mu_{move,0}$). If the ownership of a market package changes, start-ups have good chances ($\mu_{com,start,exist}$). New market packages are rather rare ($\mu_{m,new}$). Thus, Set 9 reflects a market in which existing firms build innovation capacities and dominate the rather rare events of sub-market emergence. Sub-market ownership changes rarely, but if it happens, start-ups are most likely to enter.

VI. Conclusions

A model of firm and market dynamics is developed in this paper. The model builds on general assumptions about competition and innovation processes: firms compete for monopolistic positions in narrow sub-markets, which in turn develop independently. Depending on their competitive strength, existing as well as new firms gain or lose these so-called market packages. New market packages are conquered as an outcome of successful innovation.

Following the approach by Brenner and Werker (2007), all processes and mechanisms, originating from knowledge about firms and markets, are modelled as general as necessary by using many parameters that are not or only vaguely fixed. Established stylized facts about the statistical characteristics of firm growth provide the knowledge to identify those parameter sets that lead to realistic behaviours of the model. Running 34200 parameter sets, nine parameter sets are identified as realistic.

A detailed view on these parameter sets has shown that they, indeed, can be seen as representing different markets or industries with specific characteristics. Hence, the aim to develop a flexible model that is in line with well-established knowledge about firm growth and that is able to represent different industrial situations is reached. Testing further parameter sets would provide further realistic parameter sets. However, the range of potential characteristics that the model is able to represent is already well outlined by the nine parameter sets that are identified and discussed here.

Various extensions of this simulation model are possible. First, employment dynamics can be modelled by assuming that changes in the number of employees are adjustments to the turnover a firm has realized (or is expected to realize). This requires the introduction of an adjustment function, like in Schlump and Brenner (2010). Besides, the firm model developed here might be used to study growth processes at higher levels of aggregations. This is straightforward because regional or national growth is the sum of the growth of all firms and production sites, plus firm founding, site opening and site/firm closure, within these spatial units. Hence, to study the implications of the above firm model for regional growth, the location of the firms has to be considered. Additional parameters might take into account different kinds of benefits from location, namely technological spillovers, agglomeration externalities for market competition and agglomeration externalities for innovation activities. In the literature on New Economic Geography and agglomeration studies, it is argued that these mechanisms are crucial for spatial concentration of economic activities to emerge and stabilize. An alternative explanation of industrial clusters can be found in Klepper (2006) showing that new firms are more often established in locations in which similar firms are already located. Our firm simulation model would allow to discriminate which mechanisms matter most under which circumstances, such as the life cycle phase or the innovation propensity of an industry.

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Appendix

Table A.1: Parameter ranges

Parameter	Minimum	Maximum		Parameter	Minimum	Maximum
$\mu_{d,max}$	.1	10		$\mu_{p,turn}$	0	.01
$\mu_{d,mean}$	-.002	.001		$\mu_{T,new}$	1	100
$\mu_{m,ex0}$	.00001	.002		$\mu_{exist,cap}$	.00001	.1
$\mu_{m,exa}$	.000001	.001		$\mu_{exist,s}$	.00001	1
$\mu_{c,init}$	.00001	1		$\mu_{m,new}$	.01	100
$\mu_{c,r}$	.2	1		$\mu_{start,new}$	1	1000
$\mu_{T,adapt}$	.0001	.1		$\mu_{com,start,new}$	.01	.3
$\mu_{inno,start}$	.001	10		$\mu_{new,turn}$	-.01	.01
$\mu_{red,inno}$	.99	1		$\mu_{new,size}$	.0000000001	.0001
$\mu_{move,0}$	.0001	.1		$\mu_{new,cap}$	.000001	1
$\mu_{move,a}$	.00001	.001		$\mu_{new,inno}$	.00001	1
$\mu_{move,o}$	.000005	.001		σ_d	0	.01
$\mu_{threshold}$	.0001	.1		σ_y	0	.1
$\mu_{com,start,exist}$	.001	.1		σ_c	.0001	.1