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Product Quality in a Simple OLG Model of Scientific Competition

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Abstract

Using a simple OLG model where the research output of one generation provides inputs for the next, the paper explains how quality standards can become established in scientific competition. Researchers seek status, which they get if their results are used by the next generation. Quality is hereditary in the sense that input quality affects output quality. Hereditary quality allows for simple coordination on quality standards.

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1 The Problem

In this paper, I am concerned with a simple question: Why is the quality of scientific research so high on average? For those who accept the premise, the answer is probably obvious. Science is highly competitive. By science as an institution, I mean academic or open science, that is, the whole system of research-oriented universities, scientific journals, the peer review system, learned societies, and so forth. In this system, the production of high quality research is the way to rise to the top. Researchers compete for positions, research grants, journal space, and status. Thus, there is an incentive to aim at high quality but, of course, no sure-fire method to produce it, which explains why we also observe low quality.

However, pointing out that science is competitive is only half of the answer. High quality in science is what researchers in the respective field consider to be high quality. Why and how do competing researchers coordinate on common quality standards?

One possible answer to this question is that quality standards are imported into science by scientific novices. After all, researchers believe in ideals and accept standards even before they enter science. Curiosity is often considered an important motivation for choosing a career in science, and it is hard to see how somebody could be curious without being interested in the truth about matters. Hence, a love of truth might be imported into science from the outside.

However, neither curiosity nor a love of truth are sufficient for judging the quality of a research paper. According to modern philosophy of science, no statements (or, at least, no statements of scientific interest) can ever be verified or falsified conclusively. Hence, even for those who just seek truth, it is not obvious how to judge the products of science, as everybody knows who has experience with the refereeing process (see, e.g., Seidl et al. 2008, Albert and Meckl 2008). It requires training to make these judgments. Scientific quality standards are not easily explained to outsiders; and they are a matter for debate among insiders. Both facts suggest that quality standards are endogenous to science and not just imported by scientific novices.

Another possible answer is that quality standards are imposed from the outside, by the final consumers of science. This is the traditional explanation in the context of markets. Consider the case of perfect competition where all parties are equally informed about the quality of all goods. Different qualities of the same good can then just be treated as different goods. In equilibrium, consumers select the qualities they want at given prices, and producers select inputs of a certain quality because these inputs allow them to satisfy consumers' wishes at minimal costs. This explanation extends to a production process with intermediate stages where the input quality selected upstream is determined by the output quality demanded downstream.

Open science, however, is not a market; it uses the so-called voluntary contribution mechanism (VCM), not the price mechanism. Researchers publish their ideas and results, which can then be used free of charge by anybody who wishes to do so. Research outputs are non-rival goods anyway; publication turns them into public goods. Since researchers are typically not paid for their publications but receive a fixed income, this is a case of voluntary provision of public goods.

The VCM can be combined with different kinds of incentives. In science, status among one's peers is an important reward in itself and the key to most other rewards, like attractive positions or Nobel prizes. The status of a researcher is determined by his impact on the field, that is, by the extent to which his ideas and results are used by by other researcher.[1]

In this analysis of scientific competition, there are no final consumers. This is not to deny that there are users of research outside science: innovators, administrators, politicians, doctors, lawyers, writers of popular science books and textbooks. However, their preferences do not influence competition within science. Indeed, this is the whole point of scientific competition. It is difficult, if not impossible, for outsiders to evaluate the quality of research, especially basic research. Scientific competition delegates the

[1]See Stephan (1996) and Diamond (forthcoming) on the economic analysis of science and, specifically, on the characteristics of open science in contrast to proprietary science (as research under the protection of intellectual property rights like patents is often called). On incentives for voluntary contributions, see Hackl et al. (2005, 2007). On status as a reward in itself, see Marmot (2004). See Merton (1973) on status as an incentive in science and Hull (1988) on use as the basis of status. See Congleton (1989) on a model of status seeking and competing VCMs.

evaluation to those who have the necessary competence.[2] This works quite well even for final consumers, but only because researchers coordinate on reasonable quality standards and maintain them collectively. This brings us back to the central question: Why and how do they do it?[3]

A third possible answer is based on the analysis of scientific competition sketched above and invokes the folk theorem for infinitely repeated games (see, e.g., Fudenberg and Tirole 1991: 150-165). Researchers might use punishment strategies in order to enforce a certain selection of inputs: for instance, never use the work of a researcher who used the wrong kind of input. However, this kind of punishment strategy requires additional punishment for those who do not punish, which makes correct behavior very difficult to monitor. A simpler way to achieve the same is a trigger strategy: as soon as one researcher deviates from some norm, nobody uses the work of others anymore.

[2]See Dasgupta and David (1994) and David (1998, 2004) for a theoretical and historical explanation of the self-regulating character of open science. This explanation is, however, incomplete; it must be supplemented by a model of scientific competition showing that self-regulation can actually deliver high quality. The explanation problem becomes even more severe once it is recognized that scientific quality standards are endogenous to science; see below.

[3]It is sometimes doubted that the mode of scientific competition under discussion is still relevant today; see, e.g., Ziman (2000). It is certainly true that some science policies, starting with the 1980 U.S. Bayh-Dole act, have aimed at replacing scientific competition by market competition. Still, open science is so far the only institutional alternative to proprietary science and, therefore, worth understanding.

However, these kinds of strategies are just not relevant in science. Interesting research results are not discarded just because the author misbehaved in some way. For instance, plagiarism certainly violates the norms of proper scientific behavior. Nevertheless, it is usually not severely punished by the scientific community because, in contrast to counterfeit data, it does not affect the quality of the research building upon the offending publication.[4]

Subsequently, I explore a fourth mechanism based on a production function for research with a simple but intuitively plausible property: the quality of the input affects, at least stochastically, the quality of the output, that is, quality is hereditary in scientific production.

Hereditary quality in a production process is common sense in most production processes: from food over clothing to housing, high-quality inputs are almost always necessary (if not sufficient) for producing high-quality outputs. For many quality characteristics in science, hereditariness is at least plausible: Using simple models as part of a new model increases the chances to produce a simple model. Using consistent assumptions as a basis increases the chances to come up with a consistent model. Incorporating relations that have survived statistical tests increases the chances of finding a statistical model that will also stand up to empirical scrutiny.[5]

A link between input and output quality provides a rationale for re-

[4]See the critical discussion of Broad and Wade (1982) in Hull (1988).

[5]Albert (2007) shows that hereditariness of quality is an important feature of a falsificationist methodology.

searchers to be selective in the choice of inputs. Researchers use the work of others if they think that it is good enough to build upon it (Hull 1988). Hence, if a paper gets a negative evaluation, it will not be used by other researchers, which means that the author fails in the quest for status. Thus, given hereditary quality, high-quality papers are used and low-quality papers are discarded in equilibrium just because everybody expects this kind of behavior.

The paper proceeds as follows. Section 2 presents a model of the research process along the lines sketched above. Section 3 computes the most relevant equilibria. Section 4 analyzes the dynamics of the research process in equilibrium. Section 4 concludes. Some mathematical derivations are relegated to an appendix.

2 An OLG Model of Scientific Competition

We consider an overlapping-generations (OLG) model. At each point in time $t = 1, \ldots, \infty$, there are n active young researchers and n inactive old researchers who were active at $t-1$. Each active researcher writes one paper, which is published and can be used as an input by young researchers at $t+1$. Production at $t = 0$ uses no papers as inputs. Young researchers at $t > 0$ find, among the papers written in $t-1$, exactly one paper relevant to their own problem, which they might use as an input or not.[6]

[6]This is the simplest assumption. The model becomes more complicated if there are always several relevant papers not all of which can be used. Nevertheless, the three kinds

There are two different qualities for papers, X or Y. Quality $Z = X, Y$ is high quality in equilibrium iff Z-papers are used as inputs with a higher probability than papers falling into the other category. Researchers can determine the quality of a paper; however, they have no intrinsic preference for either X-papers or Y-papers.[7]

Quality is hereditary. Using a Z-paper as an input increases the probability of producing a Z-paper as an output. Thus, let p be the probability of writing an X-paper on the basis of an X-paper. Let q be the probability of writing an X-paper on the basis of an Y-paper. Let r be the probability of writing an X-paper without any paper as an input. We then assume $1 > p > r > q > 0$, implying that a researcher cannot ensure that his paper falls into a certain category.[8]

At time $t > 0$, each active researcher randomly draws one of the n papers written at $t-1$, which is relevant to his problem and may be used as an input. The n drawings are independent and with replacement. The probability of

of strict equilibria on which we focus below remain possible. It seems reasonable, therefore, to skip over the quite interesting questions raised by this extension, e.g.: Are there plausible equilibria where the scientific community splits into subgroups with different quality standards? I am grateful to Harald Uhlig for alerting me to this possibility.

[7]Albert (2006) considers a model with a finite but arbitrary large number of qualities combining into an endogenous quality scale.

[8]This assumption identifies two probabilities that may be different. The probability of writing an X-paper may depend not only on the input but also on one's aim: it may make a difference whether one *tries* to write an X-paper or a Y-paper. A generalization along these lines, however, seems to generate no additional insights.

each paper to be drawn by researcher $i = 1, \dots, n$ is $\frac{1}{n}$. Hence, it is pure chance whether a specific paper becomes relevant to a researcher of the next generation. Each researcher of the next generation decides independently whether to use a relevant paper or not. The expected number of uses of a paper written at time $t-1$ can therefore be computed as follows. Let π_j, $j = 1, \dots, n$ the probability that researcher j will use the paper given that he has drawn the paper. This probability may, among others, depend on the quality of the paper. Given this probability, the probability that researcher j draws and uses the paper is π_j/n. Hence, each researcher adds π_j/n to the expected number of uses, which is therefore equal to the average probability of using the paper, $\frac{1}{n}\sum_{j=1}^{n}\pi_j$.

The state of the research process at time t is the number X_t of X-papers available as inputs. This state is a random variable with possible values in $\{0, 1, \dots, n\}$. The transition probabilities from X_t to X_{t+1} depend on the strategies of the researchers.

Researchers have two actions, using the paper they have drawn as an input (U) or discarding it (D). They maximize their expected utility on the basis of identical utility functions. Utility depends on status and costs of research. Status is equal to the number k of researchers of the next generation who use one's paper as an input for their research.[9] A researcher's costs c are

[9]This is a simplification. Status is rank among one's peers and should therefore be relative to the success of others; see Albert (2006) for such a model. However, this point is only relevant for the computation of the threshold values S_Z on p. 11 below.

low ($c = c_L \geq 0$) if the researcher uses a paper as an input; otherwise, the costs are high ($c = c_H > c_L$). Thus, the quality, X or Y, of the paper used as input enters neither utility functions nor cost functions. We assume a linear utility function $u(k, c) = \alpha k - c$, $\alpha > 0$, which implies risk neutrality. With risk neutrality, the expected utility of a researcher is equal to the $u(a, c)$, where a is the expected number of uses, or expected status.

The features of the research process described so far are assumed to be common knowledge among all researchers. Generation $t > 0$ of researchers has common knowledge of past and current states X_s, $s = 1, \ldots, t$. Past behavior of previous researchers and the papers they have drawn are known, but he quality of the paper drawn by a current researcher is private information in the researcher's generation. The stage game in which researchers decide whether to use the papers they have drawn is simultaneous.

Knowledge about the past is relevant for a researcher only insofar as this knowledge can be used to compute his expected status a_Z depending on whether he writes a Z-paper, $Z = X, Y$. The difference in expected status, then, is the motive for using or not using the paper a researcher has drawn as an input since this decision influences the probability of producing an X-paper. If $|a_X - a_Y|$ is small, a researcher will use any paper because the decrease in costs by $\Delta_c = c_H - c_L$ will dominate his decision. If $|a_X - a_Y|$ is large, it may pay to discard the wrong kind of paper, that is, discard a Y-paper if $a_X > a_Y$, because higher costs are offset by a higher probability of producing an X-paper.

3 Equilibrium Analysis

This model, like many other OLG models, allows for a large number of equilibria. The focus in this paper, however, is on a specific question, namely, whether there are Nash equilibria in which researchers consider only quality when selecting inputs. For this reason, we are looking for equilibria where researchers use only four pure strategies or mixtures of them. The pure strategies are denoted by AB, $A, B \in U, D$, where A is the action upon drawing an X-paper and B is the action upon drawing a Y-paper. We denote the set of these four pure strategies and their mixtures by $\mathcal{Q}$.

For any researcher, only the strategy choices of the next generation of researchers are relevant. Given a mixed-strategy profile drawn from $\mathcal{Q}^n$ of the researchers at time $t+1$, a researcher at time $t = 1, 2, \ldots$ cannot improve expected payoffs by using a strategy drawn from a set larger than $\mathcal{Q}$.

As already shown in the last subsection, expected status is equal to the average probability that the paper is used when drawn. This average probability is determined by the paper's quality and the average taken over the mixed-strategy profile of the next generation, which is itself a mixed strategy from $\mathcal{Q}$ called the average strategy. Such a strategy is denoted by $\pi = (\pi_{UU}, \pi_{UD}, \pi_{DU}, \pi_{DD}) \in \Delta^4$, where Δ^4 is the four-dimensional unit simplex and π_{AB} is the probability of strategy AB.

We consider the best reply of a researcher at time $t = 1, 2, \ldots$ to the average strategy $\pi = (\pi_{UU}, \pi_{UD}, \pi_{DU}, \pi_{DD}) \in \Delta^4$ at time $t+1$. The expected status from writing an X-paper at time t is $\pi_{UU} + \pi_{UD}$, while the expected status from writing an Y-paper at time t is $\pi_{UU} + \pi_{DU}$. Assume that a researcher at time t draws an X-paper. If he uses this paper, he produces an X-paper with probability p. Thus, the expected status from using an X-paper is $a_X = p\pi_{UD} + (1-p)\pi_{DU} + \pi_{UU}$. Analogously, the expected status from using a Y-paper is $a_Y = q\pi_{UD} + (1-q)\pi_{DU} + \pi_{UU}$, and the expected status from discarding the paper, whatever its quality, is $a_D = r\pi_{UD} + (1-r)\pi_{DU} + \pi_{UU}$. Since $p > r > q$, $a_X > a_D > a_Y$ iff $\pi_{UD} > \pi_{DU}$ and $a_Y > a_D > a_X$ iff $\pi_{UD} < \pi_{DU}$.

Expected utility is $u(a, c) = \alpha a - c$. Expected utility from using a Z-paper is $u(a_Z, c_L)$, $Z = X, Y$. Expected utility from discarding the paper drawn is $u(a_D, c_H)$.

Consider first the case $\pi_{UD} > \pi_{DU}$. The optimal strategy is UD or UU: an X-paper should be used because this increases expected status and decreases costs; an Y-paper can be used iff the decrease in expected status is offset by the increase in costs, that is, iff $u(a_Y, c_L) \geq u(a_D, c_H)$ or $\pi_{UD} \leq \pi_{DU} + S_X$, $S_X := \frac{c_H - c_L}{\alpha(r-q)} > 0$. In the limiting case $\pi_{UD} = \pi_{DU} + S_X$, mixing UD and UU is possible.

If, on the other hand, $\pi_{UD} < \pi_{DU}$, the optimal strategy is DU or UU: Y-papers should always be used; X-papers can be used iff the decrease in expected status is offset by the increase in costs, that is, iff $u(a_X, c_L) \geq u(a_D, c_H)$ or $\pi_{DU} \leq \pi_{UD} + S_Y$, $S_Y := \frac{c_H - c_L}{\alpha(r-p)} > 0$. In the limiting case $\pi_{DU} = \pi_{UD} + S_Y$, mixing DU and UU is possible.

Table 1: Best replies at time $t = 1, 2, \ldots$ to the average strategy $\pi = (\pi_{UU}, \pi_{UD}, \pi_{DU}, \pi_{DD}) \in \Delta^4$ at time $t + 1$.

	Average strategy in $t+1$								
	π_{UD}				$\pi_{UU} = 1$	π_{DU}			
	$= 1$	$\in (S_X, 1)$	$= S_X$	$\in (0, S_X)$		$\in (0, S_Y)$	$= S_Y$	$\in (S_Y, 1)$	$= 1$
	1	2	3	4	5	6	7	8	9
1	$\oplus$	$\oplus$	$\oplus$	0	0	0	0	0	0
2	0	0	$\oplus$	0	0	0	0	0	0
3	0	0	$\oplus$	0	0	0	0	0	0
4	0	0	$\oplus$	0	0	0	0	0	0
5	0	0	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	0	0
6	0	0	0	0	0	0	$\oplus$	0	0
7	0	0	0	0	0	0	$\oplus$	0	0
8	0	0	0	0	0	0	$\oplus$	0	0
9	0	0	0	0	0	0	$\oplus$	$\oplus$	$\oplus$
	X-selection	weak X-selection			no selection	weak Y-selection			Y-selection

As explained in the text, we have $\pi UD + \pi_{UU} = 1$ or $\pi DU + \pi_{UU} = 1$ and assume $S_X < 1$ and $S_Y < 1$. The row strategies correspond to the column strategies with the same number. The symbol $\oplus$ in a cell indicates that the row is a best reply to the column; otherwise, the cell contains a zero.

Hence, for individual as well as average strategies at all times, we have $\pi_{UD} + \pi_{UU} = 1$ or $\pi_{DU} + \pi_{UU} = 1$, while $\pi DD = 0$. If $S_X > 1$, $\pi_{UD} = 0$; if $S_Y > 1$, $\pi_{DU} = 0$. In order to exclude no possibilities, we assume $S_Z < 1$, $Z = X, Y$. Table 1 shows the best-reply correspondence.

Since strategy choices at time $t + 1$ restrict strategy choices at time $t = 1, 2 \ldots$, the set of possible sequences of strategy choices in equilibrium is restricted. Appendix A derives nine types of equilibria that are possible under our assumptions from table 1 by symbolic forward induction. Table 2 lists these types of equilibria. All of them are subgame perfect.

While there are infinitely many equilibria, only three of them are strict: the X-selection equilibrium, the Y-selection equilibrium, and the no-selection equilibrium. The non-strict equilibria are rather implausible.[10]

In the Z-selection equilibria, researchers use only Z-papers as inputs. In this sense, Z denotes high quality. However, high quality in this sense derives not from individual preferences (which are neutral between X and Y). The belief that researchers use only Z-papers is a self-fulfilling prophecy.

[10]Under an evolutionary, or social-learning, dynamics, the non-strict equilibria would presumably play no role. However, evolutionary considerations are beyond the scope of the present paper. Moreover, it seems to me that research strategies are often forward-looking, which generates herding behavior, with immediate coordination (as assumed in this paper) as an extreme case.

Table 2: Nine types of subgame perfect Nash equilibria

	Label	Equilibrium strategies
1	X-selection	$\pi_{UD} = 1$ throughout
2	Transition from X-selection to weak X-selection in $t > 1$	$\pi_{UD}^{s<t} = 1$, $\pi_{UD}^{t} \geq S_X$, $\pi_{UD}^{s>t} = S_X$
3	Weak X-selection	$\pi_{UD} = S_X$ throughout
4	Transition from no selection to weak X-selection in $t > 1$	$\pi_{UU}^{s<t} = 1$, $\pi_{UD}^{t} \leq S_X$, $\pi_{UD}^{s>t} = S_X$
5	No selection	$\pi_{UU} = 1$ throughout
6	Transition from no selection to weak Y-selection in $t > 1$	$\pi_{UU}^{s<t} = 1$, $\pi_{DU}^{t} \leq S_Y$, $\pi_{DU}^{s>t} = S_Y$
7	Weak Y-selection	$\pi_{DU} = S_Y$ throughout
8	Transition from Y-selection to weak Y-selection in $t > 1$	$\pi_{DU}^{s<t} = 1$, $\pi_{DU}^{t} \geq S_Y$, $\pi_{DU}^{s>t} = S_Y$
9	Y-selection	$\pi_{DU} = 1$ throughout

Assumption: $S_X < 1$ and $S_Y < 1$.

4 The Dynamics of Equilibrium Production

Subsequently, we focus on the three strict equilibria, where all researchers use the same strategy $\pi = (\pi_{UU}, \pi_{UD}, \pi_{DU}, \pi_{DD})$. This strategy determines the probability π_Z that a researcher drawing a Z-paper will write a Z-paper:

$$\begin{pmatrix} \pi_X \\ \pi_Y \end{pmatrix} = \begin{pmatrix} p & p & r & r \\ q & r & q & r \end{pmatrix} \begin{pmatrix} \pi_{UU} \\ \pi_{UD} \\ \pi_{DU} \\ \pi_{DD} \end{pmatrix}$$

Obviously, $\pi_X \geq \pi_Y$.

The probability that any given researcher at time $t > 0$ produces an X-paper depends on the strategy π and the state X_t, that is, the number of X-papers produced at $t-1$. The probability of drawing an X-paper in state $X_t = j$ is $\frac{j}{n}$; the probability of producing an X-paper in state $X_t = j$, then, is

$$w_j = \pi_Y + \frac{j}{n}(\pi_X - \pi_Y)\,. \tag{1}$$

This implies that the state X_t develops according to a regular Markov chain with $(n+1) \times (n+1)$ transition matrix $\mathbf{T}$.[11] Let $b(n,k,p) := \binom{n}{k} p^k (1-p)^{n-k}$ the probability mass function of the binomial distribution. The transition probability from $X_t = j$ to $X_{t+1} = k$ is $t_{jk} = b(n, k, w_j)$; the rows of

[11] On Markov chains, see, e.g., Grimmett and Stirzaker (1992). Regularity means that the transition matrix T has no zero entries. A regular Markov chain is irreducible, or ergodic.

the transition matrix, then, are given by the values of the probability mass function of $b(n,k,p)$ with expectations

$$E(X_{t+1}|X_t = j) = \sum_{k=0}^{n} t_{jk}k = nw_j\,. \tag{2}$$

Regularity implies that a unique stationary probability distribution of the states exists, which is determined by the equation $\mathbf{eT} = \mathbf{e}$, where $e_j > 0$, the jth component of the row vector $\mathbf{e}$, is the stationary probability of state j and $\sum_j e_j = 1$.

We introduce some notation. Let $Q = f(X)$ be a random variable depending on the state of the process, with $f(j) = q_j,\ j = 0, 1, \ldots, n$ and, therefore, $Q \in \{q_0, q_1, \ldots, q_n\}$. Let $\mathbf{q} = (q_0, q_1, \ldots, q_n)$ be the row vector of the possible values of Q. The expected value of Q in the long run, then, is $E(Q) = \mathbf{eq}'$ (with $'$ for transposition). Specifically, $X \in \{0, \ldots, n\}$ is the state and $\mathbf{x} = (0, 1, \ldots, n)$ is the row vector of possible states. Moreover, $W \in \{w_0, \ldots, w_n\}$ denotes the probability of producing an X-paper depending on the state, with the row vector $\mathbf{w} = (w_0, w_1, \ldots, w_n)$ of possible probabilities.

We now compute some expected values. First, we compute the long-run expected value of the state, $E(X)$. From (2), we get $\mathbf{Tx}' = n\mathbf{w}'$, since $E(X_{t+1}|X_t = j) = nw_j$ is the expected value of the binomial distribution in row $j - 1$ of $\mathbf{T}$. Hence,

$$E(X) = \mathbf{ex}' = \mathbf{e}T\mathbf{x}' = n\mathbf{ew}' = nE(W)\,. \tag{3}$$

From (1), we get $E(W) = \pi_Y + \frac{1}{n}(\pi_X - \pi_Y)E(X)$. With $w := \frac{\pi_Y}{1+\pi_Y-\pi_X}$, this yields $E(X) = nw$ and $E(W) = w$.

Let $A \in \{a_0, a_1, \ldots, a_N\}$ be the expected status as a function of the state. Let $\mathbf{a} = (a_0, a_1, \ldots, a_n)$ be the row vector of the possible values of A. The long-run expected value of A, then, is $E(A) = \mathbf{e}\mathbf{a}'$. We have

$$\begin{aligned} a_j \quad := \quad & \tfrac{j}{n}\left[a_X(\pi_{UU} + \pi_{UD}) + a_D(\pi_{DD} + \pi_{DU})\right] + \\ & \tfrac{n-j}{n}\left[a_Y(\pi_{UU} + \pi_{DU}) + a_D(\pi_{DD} + \pi_{UD})\right] . \end{aligned}$$

From this, we find $E(A)$ for the three strict equilibria (see table 3):

- In the X-selection equilibrium, $\pi_{UD} = 1$ and, thus, $a_X = p$ and $a_N = r$. Hence, $a_j = \frac{j}{n}p + \frac{n-j}{n}r = r + \frac{j}{n}(p - r) = w_j$ and, consequently, $E(A) = w < 1$, where $w = \frac{r}{1+r-p}$.

- In the no-selection equilibrium, $\pi_{UU} = 1$ and, thus, $a_X = a_Y = 1$. Hence, $a_j = 1$ and, consequently, $E(A) = 1$.

- In the X-selection equilibrium, $\pi_{DU} = 1$ and, thus, $a_Y = 1 - q$ and $a_N = 1 - r$. Hence, $a_j = \frac{n-j}{n}(1 - q) + \frac{j}{n}(1 - r) = 1 - q + \frac{j}{n}(q - r)$ and, consequently, $E(A) = 1 - q + (q - r)w < 1$, where $w = \frac{q}{1+q-r}$ and, therefore, $E(A) = \frac{1-r}{1+q-r}$.

Expected status in the no-selection equilibrium is higher than in the Z-selection equilibria. The same goes for expected utility because researchers in Z-selection equilibria have higher expected costs because the discard one category of papers.

Table 3: The Three Strict Nash Equilibria: Overview

General value	X-selection	No selection	Y-selection
$(\pi_{UU}, \pi_{UD}, \pi_{DU}, \pi_{DD})$	$\pi_{UD} = 1$	$\pi_{UU} = 1$	$\pi_{DU} = 1$
$\pi_X = p(\pi_{UU} + \pi_{UD}) + r(\pi_{DU} + \pi_{DD})$	$\pi_X = p$	$\pi_X = p$	$\pi_X = r$
$\pi_Y = q(\pi_{UU} + \pi_{DU}) + r(\pi_{UD} + \pi_{DD})$	$\pi_Y = r$	$\pi_Y = q$	$\pi_Y = q$
$w_j = \pi_Y + \frac{j}{n}(\pi_X - \pi_Y)$	$w_j = r + \frac{j}{n}(p - r)$	$w_j = q + \frac{j}{n}(p - q)$	$w_j = q + \frac{j}{n}(r - q)$
$w = \frac{\pi_Y}{1+\pi_Y-\pi_X}$	$w = \frac{r}{1+r-p}$	$w = \frac{q}{1+q-p}$	$w = \frac{q}{1+q-r}$
$a_X = p\pi_{UD} + \pi_{UU} + (1-p)\pi_{DU}$	$a_X = p$	$a_X = 1$	$a_X = 1 - p$
$a_D = r\pi_{UD} + \pi_{UU} + (1-r)\pi_{DU}$	$a_D = r$	$a_D = 1$	$a_D = 1 - r$
$a_Y = q\pi_{UD} + \pi_{UU} + (1-q)\pi_{DU}$	$a_Y = q$	$a_Y = 1$	$a_Y = 1 - q$
$a_j = \frac{j}{n}a_X(\pi_{UU} + \pi_{UD}) + \frac{j}{n}a_N(\pi_{DD} + \pi_{DU}) + \frac{n-j}{n}a_Y(\pi_{UU} + \pi_{DU}) + \frac{n-j}{n}a_N(\pi_{DD} + \pi_{UD})$	$a_j = r + \frac{j}{n}(p - r)$	$a_j = 1$	$a_j = 1 - q + \frac{j}{n}(q - r)$
$E(A)$	$E(A) = \frac{r}{1+r-p} < 1$	$E(A) = 1$	$E(A) = \frac{1-r}{1+q-r} < 1$

The X-selection equilibrium has a higher expected status than the Y-selection equilibrium iff $p > 1 - \frac{r}{1-r}q$. In the case $r = 0.5$, this condition implies $p > 1 - q$: higher expected status requires a higher probability of reproduction. A higher expected status means also lower costs because higher status is only possible if more potential inputs are used and less are discarded. Thus, with $r = 0.5$, a higher probability of reproduction means a higher expected utility. Thus, from the point of view of the researchers, the no-selection equilibrium is best *ex ante* and in the long run, while the two other equilibria are ranked according to the values of the reproduction probabilities.

5 Conclusion

The model of scientific competition proposed in this paper shows how endogenous quality standards can emerge in scientific competition without invoking the folk theorem. However, not any kind of quality standard can be established in this way. Participants must believe that quality is hereditary in the production process, that is, they must believe that the quality of the input they use, which consists of ideas and results from other researchers, has an influence on the quality of their research output.

In scientific competition, production decisions are not governed by the evaluations of final consumers. From the perspective of researchers, their output serves only as an input for further research; there is no final output. Users outside science itself may profit from research, or they may be hurt by it, but these effects are external to scientific competition. The value of research outputs in the eyes of the producers is determined by the demand of an infinite sequence of downstream producers who are all in the same position. Hence, the results of research have no fundamental value within science. Because quality is hereditary, every researcher tries to guess what the next researcher would like to use, and selects his own inputs accordingly. Since there are no contracts fixing a price for papers satisfying some quality norm, production proceeds on the basis of these guesses. This game has the structure of a sequential beauty contest.[12]

There is a certain similarity to stock markets, where a similar beauty contest also leads to the coordination on evaluation standards (Pratten 1993). However, the need to coordinate on evaluation standards is more obvious on stock markets, which use the price mechanism. Science, in contrast, uses the voluntary contribution mechanism. Every researcher produces a unique public good. Hence, there is no need to find a common price. Without the assumption of hereditary quality, which forges a link between input quality and output quality, there would be no beauty contest.

The equilibrium in scientific competition is not necessarily *ex ante* efficient from the researchers' point of view. Comparing an equilibrium where a quality standard becomes established with an equilibrium where this is not the case, we find that the former is more competitive than the latter:

[12] On simultaneous beauty contests, together with experimental results, see, e.g., Camerer (2003: 209-218).

researchers discard some inputs, which lowers expected utility. A similar effect exists in other beauty contests, where coordination on an equilibrium reduces average payoffs. Thus, as in market competition, the competitive mechanism diminishes producers' rents. Whether this is good or bad for society as a whole depends, of course, on the the quality standard itself. There is no guarantee that the quality standard established in science reflects the preferences of outsiders.

The model of this paper provides a basis for a critical discussion of different methodologies. Logical positivists tried to reduce methodology to logic. However, Popper (1959) argued convincingly that methodological standards are social conventions (see also H. Albert 1978, Jarvie 2001). The present paper shows how certain methodological standards can become established in science. The equilibria considered in this paper can be changed through unexpected events that change expectations. Thus, methodological discussions are potentially relevant: new arguments can change expectations and shift the equilibrium. However, the equilibria have an objective basis in hereditary quality; not just any quality standard can become established. Hence, the present model provides a test for methodologies: if the qualities required by a methodology are not hereditary in the research process, the methodology is moot.

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A Finding Nash Equilibria by Forward Induction

Consider the grouping of strategies in table 1: group 1 is X-selection ($\pi_{UD} = 1$); groups 2 to 4 are different cases of weak X-selection ($1 > \pi_{UD} > 0$, $(\pi_{UD} + \pi_{UU} = 1)$); group 5 is no selection ($\pi_{UU} = 1$); groups 6 to 8 are different cases of weak Y-selection ($1 > \pi_{DU} > 0$, $(\pi_{DU} + \pi_{UU} = 1)$); and group 9 is Y-selection ($\pi_{DU} = 1$).

Table 1 gives a best-reply correspondence on the basis of this grouping. We denote the 9×9 matrix of table 1 by $\mathbf{\Theta}$, that is,

$$\mathbf{\Theta} = \begin{pmatrix} \oplus & \oplus & \oplus & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \oplus & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \oplus & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \oplus & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \oplus & \oplus & \oplus & \oplus & \oplus & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \oplus & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \oplus & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \oplus & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \oplus & \oplus & \oplus \end{pmatrix}$$

Each row is a profile $(s_1, \ldots, s_9)$ with $s_i \in \{0, \oplus\}$, $i = 1, \ldots, 9$. Let $\mathcal{O}$ be the set of all such profiles. The 9×9 matrix of table 1 says for each group of strategies whether the strategies of this group are best replies to the strategies of the other groups. If the ith element of the jth row is $\oplus$, then the strategies of group j are best replies to the strategies of group i. Or in other words: in equilibrium, the strategies from group j can be played at time $t+1$ if a strategy from group i was played at time t.

We can describe a process of "forward induction" by multiplying the matrix $\mathbf{\Theta}$ with itself and with row vectors from $\mathcal{O}$. We use the standard rules of matrix multiplication together with the following rules for multiplying symbols: $\oplus \times \oplus = \oplus$, $x \times 0 = 0 \times x = 0$, $\oplus + \oplus = \oplus$, $x + 0 = 0 + x = x$ where $x = 0, \oplus$.

Let $\mathbf{o} \in \mathcal{O}$ denote the best replies at time t. Then $\mathbf{o\Theta}$ gives the best replies at time $t+1$ that are consistent with $\mathbf{o}$. Specifically, let $\mathbf{o}^j \in \mathcal{O}$, $j = 1, \ldots, 9$ be the jth base profile, that is, a profil with $\oplus$ in place j and zeroes everywhere else. Then $\mathbf{o}^j\mathbf{\Theta}$ describes the strategy groups that are possible at time $t = 2$ if strategy group j contains the best reply at time $t = 1$. Similarly, $\mathbf{o}^j\mathbf{\Theta}^2$ describes the strategy groups that are possible at time $t = 3$, and so forth. These are the forward-induction steps. Obviously, $\mathbf{\Theta}^2 = \mathbf{\Theta}$ and, therefore, $\mathbf{\Theta}^s = \mathbf{\Theta}$ for all $s = 1, 2, \ldots$, which simplifies the considerations.

A Nash equilibrium can be found by forward induction in the following way: Start with a base profile. Multiply with $\mathbf{\Theta}$ as above. Select one of the possible resulting strategy groups, that is, a base profile. Repeat *ad infinitum*.

A single forward-induction step leads to the following results:

$$\begin{aligned}
&\mathbf{o}^1\mathbf{\Theta}{=}\mathbf{o}^1+\mathbf{o}^2+\mathbf{o}^3\\
&\mathbf{o}^j\mathbf{\Theta}{=}\mathbf{o}^3 \text{ if } j=2,3,4\\
&\mathbf{o}^5\mathbf{\Theta}{=}\mathbf{o}^3+\mathbf{o}^4+\mathbf{o}^5+\mathbf{o}^6+\mathbf{o}^7\\
&\mathbf{o}^j\mathbf{\Theta}{=}\mathbf{o}^7 \text{ if } j=6,7,8\\
&\mathbf{o}^9\mathbf{\Theta}{=}\mathbf{o}^9+\mathbf{o}^8+\mathbf{o}^7
\end{aligned} \tag{4}$$

Thus, starting from $\mathbf{o}^1$, a transition to $\mathbf{o}^3$ is possible at any time, with a single intermediate appearance of $\mathbf{o}^2$; a deviation from $\mathbf{o}^3$ is impossible. From considerations of this kind follow the Nash equilibria of table 2 in the text.