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FDI and FPI - Strategic Complements?

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Abstract

We show in a dynamic investment setting whether firms choose FDI or international portfolio investment (FPI) in the presence of stochastic productivity taking into account differences in flexibility of both investments. Isolated FPI and FDI investments are compared to combined FPI and FDI investments. FDI requires higher investment specific costs than FPI. Thus, it is not possible to adjust FDI to environmental changes every period. In contrast, FPI bears lower fixed costs and can be adjusted immediately to short-term changes in the environment. Additionally, as a result of the investors' control position FDI yields a higher return than FPI. Hence, there is a trade-off between flexibility and higher return for firms deciding between FDI and FPI. We explore whether as a consequence of higher investment specific fixed costs and lower flexibility in the case of FDI, small firms prefer FPI and larger firms invest in FDI. We show that a combined strategy dominates the isolated strategy always in times. Further, combined international investment comprises a higher incentive for firms to invest in r&d-investment and consequently firm productivity increase faster than with isolated international investment. Depending on the success-probability and the correlation between the various investment possibilities, even small firms (low productivity) invest in FDI.

JEL: F21; F23; G11

Keywords: FDI, Portfolio Investment, Risk Diversification, endogeneous Productivity

FDI and FPI - Strategic Complements? Barbara Pfeffer¹

1 Introduction

The recent World Investment Report 2006 highlights that Foreign Direct Investment (FDI) flows and growing FDI stocks are now at an unparalleled level with most going to industrial countries. At the same time flows of international portfolio investments (FPI) exceeded FDI flows twice at the beginning of the nineties while more recently FPI growth slowed down and both capital flows converged (WTO 1996). What are the motives for firms to invest in one or the other and how are they to be explained?

Previous studies on FDI explained the motives for FDI with differential rates of return, differences in interest rates and risk diversification (Dunning (1973)). Following Andersen and Hainaut (1998) these determinants lost explanatory power and recent theoretical and empirical studies document that FDI is undertaken to exploit cost advantages (vertical FDI) or to serve different markets locally to avoid trade costs (horizontal FDI).², ³ If FDI no longer serves risk diversification does FPI fill the gap and are these capital flows strategic complements rather than substitutes?⁴

We analyse whether firms choose FDI or FPI in the presence of stochastic productivity taking into account differences in flexibility of both investments. As FDI requires higher investment specific costs it is not possible to adjust FDI to environmental changes every period.⁵ In contrast, FPI bears lower fixed costs and can be adjusted immediately to short-term changes in the environment. In particular the assumption is that FDI is less flexible than FPI and this reduced flexibility entails a higher rigidity of FDI. A further distinction between FDI and FPI is the possibility to exert control. FDI encloses control rights for the investor. Thus, the investor is manager and owner in one person. He has easy access to all information and has the possibility to navigate the investment according to his own interests. FPI on the other hand does not comprise control rights. In this case, the investor and the manager are different persons with

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 $^{^{2}}$ Grossman, Helpman, Szeidl (2005) discuss in which states firms decide to outsource or offshore some of their production stages. Acemoglu, Aghion, Griffith and Zilibotti show that vertical integration is more common if the technology intensity differs significantly.

³See Helpman, Melitz, Yeaple (2003) for a detailed survey whether firms decide to serve a foreign market through export or FDI - horizontal FDI. Studies of complex FDI strategies can be found for example in Helpman (2006) or Grossman, Helpmann, Szeidl (2003).

⁴In contrast to the consumer theory, the cross-price elasticity of demand is not the decisive factor for the present distinction between complements and substitutes. In the present analysis "strategic-complements" represents the conscious choice of the investor to combine both investment instruments. The decision whether to combine both investments depends not only on the price / costs of the investment but the risk correlation and flexibility of the respective investment instrument.

⁵See Goldstein and Razin (2005) for a discussion of the different costs for FDI and FPI.

differing interests. Information asymmetries and agency problems can arise. Consequently, the investment project is not necessarily completely managed in line with investors' interest. Following Goldstein and Razin (2004), as a result of the investors' control position FDI yields a higher return than FPI. Hence, there is a trade-off between flexibility and higher return for firms deciding between FDI and FPI. We explore whether as a consequence of higher investment specific fixed costs and lower flexibility in the case of FDI, small firms prefer FPI and larger firms invest in FDI.

We show that the combined investment strategy (FDI and FPI at the same time) always starts the international investment activity earlier in time than the isolated strategy (FDI or FPI). Additionally, with combined international investment, there is a higher incentive for firms to invest in research and development (R&D) and consequently firm productivity increases faster than with isolated international investment. Depending on the success-probability and the correlation between the various investment possibilities, even small firms (low productivity) invest in FDI.

To model firm behaviour we use a monopolistic competition framework with uncertain firm productivity in combination with a dynamic investment approach over a finite investment horizon. There are three countries, home and two foreign countries. The firms are located in the home country and decide to invest via FDI or FPI in the foreign countries. Thereby, they face uncertainty about their future productivity and returns on the respective investment. In particular, firm productivity is endogenous and follows a Poisson process. The productivity of the different investment opportunities are correlated with each other. Differences in correlation between FDI and home production account for different forms of FDI.⁶

The reminder of this paper is organized as follows. In section 2, we examine the different definitions of FDI and FPI and explain the motivations for firms to invest in FDI or FPI on the existing literature. Section 2 presents a overview of the recent literature. Section 3 outlines the theoretical framework and derives the optimality conditions for the various investments strategies. Following this, we present the numerical solution of the model and discuss the results in section 4. Finally, section 5 concludes.

⁶Aizenman and Marion (2004) as well as Markusen and Maskus (2001) show that horizontal FDI is established in countries similar in size and endowments, while vertical FDI is the preferred investment in countries with different characteristics as the source country.

2 Why should firms diversify their risk and why should they combine FDI and FPI?

The first problem that arises is to distinguish between the two investment instruments FDI and FPI. FDI and FPI consist of different kinds of foreign equity interests like equity shares, securities or derivatives.⁷ Most of all, the explanation of FDI and the accompanying activities are not unambiguous. There is the macro view which counts FDI as a specific capital flow between various countries. It measures FDI in Balance of Payments Statistics. On the other hand, the micro view examines the motivations of foreign direct investment from the investors' point of view. This view concentrates especially on the consequences resulting from investment for the investor, the host and the home country as well as on the firm activities. The main emphasize is that the motivation as well as the consequences of FDI arise from the investors' - the investing firms' control and influence on the management of the foreign investment or affiliate. However, the definition of FDI does not only change with the underlying theory but also has changed in time and still changes with institution. A prominent and widely accepted definition is the IMF (1993) concept. It is stated relatively vague that a direct investment is international investment that comprises a long-term interest in the relationship between the investing and the foreign firm. Furthermore, the investing firm clearly possesses a significant influence on the management or production process of the foreign firm. In addition to this rather loose concept, the IMF exemplifies a specific recommendation of 10% share-ownership at which FDI and a corresponding degree of control is identified. The IMF FDI perception complies with the Balance of Payment Statistics view of FDI. The micro view is more represented by the FDI definition of the United Nations System of National Accounts. In this concept, the main emphasis is placed on the investors' control on the foreign firm. The threshold for control and a perceptible influence on the foreign firm is at 10 - 50% or more of shares owned by the investing firm. The precise share depends on the individual country definition of foreign control.⁸ These are only two examples for the differing concepts and definitions of FDI. Based on these diverging perceptions it is difficult to distinguish explicitly between FDI and FPI. Moreover, the ambiguous definitions do not really provide a clear cut between FDI and FPI: Lipsey (1988) quotes an example where previous portfolios flows were converted into direct investment flows. Hence, there is no unambiguous distinction of FDI and FPI from the composition of the respective capital flow or by a definite control threshold. Consequently, in the present paper we will emphasize the various characteristics like control and volatility of the two investment instruments to distinguish between them. Furthermore, we will examine what the motivations of a firm are to invest in FDI or FPI and whether there are any gains from

⁷FPI in a broader sense also can include bonds, money market instruments, financial options and debt securities.

⁸For a detailed discussion of this concept see Inter-Secretariat Working Group on National Accounts (1993). For more various views on FDI and its definition see Lipsey (1999, 2001).

combining both investments.

The second question arising concerns mainly the motivations for a firm to invest in FPI. Whereas firms' motivations to invest in FDI are widely explored. the reasons for FPI are rather unexplored or just ignored. An arising strand of literature highlights the increasing interest and necessity in firms' risk hedging. This constitutes a more than appropriate motivation for firms engaging in FPI. An accepted reason for multinational firms to engage in risk sharing via financial markets is exchange rate volatility. Mello et al. (1995) for example show that the production choice and the competitive position of firm depend strongly on an appropriate financial risk hedging strategy with respect to exchange rate risk. One of the first approaches to combine financial hedging and corporate diversification is made by Ding and Kouvelis (2007). Still, they justify a firms' hedging necessity solely on exchange rate risk and price uncertainties. Lim and Wang (2007) show that operational risks arising inside the firm like for example the uncertainty of firm-specific investments by non-financial stakeholders may also require firms to engage in risk diversification. Furthermore, they argue that it is not only possible to combine financial and corporate diversification to hedge external and internal risks more efficiently. Actually, they find that the combination of the two hedging instruments complement each other. Lim and Wang argue that financial and corporate diversification hedges different types of risk. Movements of the market or industry as a whole can be hedged with financial instruments but not with corporate diversification. On the other hand, it is not possible or extremely costly to reduce firm-specific risk via financial markets. Corporate diversification is the appropriate instrument to hedge this idiosyncratic risk. If a firm engages in financial risk sharing and thus reduces its systematic risk then the share of idiosyncratic risk increases. Consequently, reducing firm-specific risk by corporate diversification becomes more valuable. These considerations confirm our assumption that a combination of FPI and FDI may enhance the value international direct investment for a firm.

A third controversial is the question, why a firm should engage in financial hedging. The common view is that a firm should emphasize its operational activities and risk diversification should be managed by the respective shareholder itself. However, the empirical evidence shows that firms indeed engage in risk diversification.⁹ There are several reasons why risk diversification by a firm cannot be substituted by shareholder portfolio diversification. First of all, financing costs can be reduced and tax benefits can be realized if financial risk diversification is undertaken by the firm instead of each individual shareholder.¹⁰ A further advantage of firms' risk hedging activities is the reduction of risk for not fully diversified managers and investors. Nevertheless, a crucial point is the protection of firm-specific investment of non-financial stakeholders. These investments are a function of a firms' total risk and hence financial

 $^{^9 \}rm See$ for example Bodnar et al. (1998) or for more recent empirical evidence Gates (2006) and Nocco and Stulz (2006).

 $^{^{10}}$ Stulz (1984) offers an extensive survey on the reasons for corporate risk management. Froot et al. (1993) emphasize on the reduced financing costs and Graham and Smith (1999) examine the tax benefits of firms risk hedging activities.

risk diversification enhances firm-specific investments. Again, this supports our assumption of additional gains for firm by combining FDI and FPI.

Finally, we link the information based trade-off literature between FDI and FPI by Goldstein and Razin (2005) (RG) and Albuquerque (2003) with the firmlevel Export and FDI approaches by Grossman, Helpman and Szeidl (2006) and Helpman, Melitz and Yeaple (2003). RG analyse the investors' decision between FDI and FPI under asymmetric information in a static model.¹¹ As a result of the information asymmetry the project revenue from FDI is higher than from FPI. In the case of FDI the investor is also the manager of the foreign firm. Hence, he has a higher control over the production processes and can ensure that the firm is run accordingly to the investors' interests. If the investor chooses FPI the investor has no control over the foreign production process and the expected return is lower. We use these different characteristics shown by RG to motivate the costs, flexibility and return of the different investment possibilities in the present paper. Additionally, we consider the findings of Chuhan, Perez-Quiraz and Popper (1996). They provide an empirical analysis on the different characteristics of short term and long term capital flows.¹² Furthermore, in contrast to RG, we introduce a long-term investor in a dynamic setting. This investor has the possibility to adjust his portfolio periodically with rigidity in FDI-shares. Hence, we also account for the different grades of flexibility of both investments.

Alburquerque (2003) analyses from a country perspective the risk-sharing character of FDI and non-FDI capital flows for countries with different degrees of financial constraints. Thereby, non-FDI flow adjustments arise from shocks in the receiving country. One result is that for financially constrained countries FDI is less volatile than non-FDI flows. With perfect enforcement, the difference in volatility diminishes. We modify this approach by taking the firm perspective and consider shocks on firm level as well as on host country level. Actually, we always find a higher volatility of non-FDI flows (FPI) than FDI flows in our firm-level perspective. The firm reacts to any short-term environment change by adjusting FPI. Precisely, FPI has the main function to smooth risk whereas FDI mainly exploits gains from technology transfers.

Uncertain firm productivity is decisive for the results of our model. This leads to the literature around Melitz (2003) or Grossman, Helpman and Szeidl (2006). They motivate the firms' choice to export or engage in FDI with differing firm productivity. Melitz (2003) shows that with heterogeneous firms only the large firms (with higher productivity) export. Small firms serve the domestic market only. Furthermore, Helpman, Melitz and Yeaple (2003) extend this and find that firms with higher productivity use higher integrated organisational production structures. They show that less productive firms only serve the domestic market, with increasing productivity firms start to export and finally the most productive firms engage in FDI. In contrast to this literature, in the present paper firm productivity is endogenous. Firms can push their pro-

¹¹See also Razin, Mody and Sadka (2002) and Razin (2002).

 $^{^{12}\}mathrm{Lipsey}$ (2001) also emphasize differences in volatility as a distinction between FDI and FPI.

ductivity by investing in research and development (R&D). The success of the R&D-investment is uncertain. Moreover, we extend these models by introducing FPI as a new form of investment possibility.

3 Theoretical Framework

The dynamic methodology in the model follows roughly the models of Abel (1973) and Holt (2003).

Firms optimize their investment decisions in a continuous-time model.¹³ Inspired by Melitz (2003), the model is based on monopolistic competition with stochastic firm productive. Domestic demand is exogenous and the consumers have Dixit-Stiglitz preferences. There are three countries. Two of these countries are northern countries West (home country) and East (foreign). The third country is a southern country (foreign). In the eastern country, as a result of the factor endowments production and cost structure are similar to the home country. Additionally, these countries are also based on a close cultural background. Hence, we assume a positive correlation of the productivities between the East and the home county. On the opposite, the South has different production, cost structure and cultural background than the home and the eastern country. Hence we assume negative productivity correlations between South and home or South and East.

We consider a setting in which a representative firm faces a choice between performing activities at home (production and R&D-investment) and engaging in two alternative foreign investments: foreign portfolio investment (FPI) or foreign direct investment (FDI). The initial position of each firm is home production and home R&D investment. Based on these home activities the firm can additionally choose to invest internationally. The firm's specific productivity θ is the crucial factor for the international investment decision of the firm. In particular, the firm can increase its specific productivity by investing in domestic research and development (R&D). Whether R&D-investment increases the firm's productivity is uncertain. The change of θ through R&D-investment follows a Poisson-Process

$$d\theta = \left[(1-\tau) \, \frac{\phi_t}{K_t} \right] \theta_t dq. \tag{1}$$

In (1) ϕ_t is the capital invested in R&D and K_t is the total stock of capital available to the firm in period t.¹⁴ As obsolete technologies have to be replaced, patent laws are renewed etc., even in case of successful R&D-investment, the growing rate of θ is smaller than the invested rate of capital. These costs correspond to a constant depreciation and are depicted by $\tau \in [0, 1]$. Finally q is a random variable that equals 1 with probability λ and 0 otherwise. Hence, if R&D-investment is successful, θ increases by $\left[(1-\tau)\frac{\phi}{K}\right]\theta$. With probability $(1-\lambda)$ R&D-investment fails and θ stays unchanged.

¹³Time runs from 0 to T.

 $^{{}^{14}}t \in [0,T].$

As every firm, no matter whether it engages in FDI, FPI or not, produces at home and serves the home market, we start with the analysis of the home country.

3.1 Home

Production The firm uses a single factor, capital, to produce output at home x^h

$$x_t^h(\theta_t) = \theta_t \left(k_t^h\right)^\alpha.$$
⁽²⁾

The superscript h states that these are the values in the isolated "Home"-scenario.¹⁵ According to (1), firms also can use capital to invest in R&D and increase their productivity

$$K_t = k_t^h + \phi_t. \tag{3}$$

As a consequence of monopolistic competition, firms choose the profit maximisingprice

$$p_t = \frac{1}{\varphi \theta_t}.\tag{4}$$

Where the rent for capital is set equal to one, $\frac{1}{\varphi}$ is the profit maximizing mark up and $\frac{1}{\theta_t}$ are the marginal costs of a firm with productivity θ_t . Furthermore, the firm has fixed costs of home production equal to f_t^h and costs of R&Dinvestment equal to ϕ_t . Hence, the profit of the firm at home in period t is

$$\pi_t \left(\theta_t \right) = p_t \theta_t \left(k_t^h \right)^\alpha - \left(f_t^h + \frac{x_t^h}{\theta_t} + \phi_t \right), \qquad 0 < \alpha < 1.$$
(5)

The first term on the right hand side equals the revenue from production and sales at the home market, r_t^h . The second term on the right hand side summarizes the costs of home production and R&D investment.

The expected value of firm profits over the whole time horizon is

$$V^{h}\left(\theta_{t}\right) = \max_{k_{s}^{h}, \phi_{s}} E_{t} \int_{t}^{t} \pi_{s}\left(\theta_{s}\right) e^{-\rho(s-t)} ds.$$

$$\tag{6}$$

subject to (2) - (4). Modification of (6) yields:

$$\rho V^{h}\left(\theta_{t}\right) dt = \max_{k_{t}^{h}, \phi_{t}} \pi_{t}\left(\theta_{t}\right) dt + E_{t}\left(dV^{h}\right) \tag{7}$$

which states that the mean required return of a firm equals the expected return. In period t, the expected return consists of the maximized profit at t and the expected gain or loss of the future profit flow.

To calculate the expected capital flow, we substitute (1) into dV^h :

$$E_t \left(dV^h \right) = \lambda \left[V^h \left(\gamma_t \theta_t \right) - V^h \left(\theta_t \right) \right] dt \tag{8}$$

with $\gamma_t \equiv (1 - \tau) \frac{\phi_t}{K_t}$.¹⁶ Equation (8) is the expected capital flow. The expected

 $^{^{15}}$ The following scenarios with isolated FPI, FDI and the combined investments are identified by the superscripts p, d, and c respectively.

¹⁶For a detailed derivation see Appendix A.

capital flow is a perpetual flow of the difference between the capital flow in case of successful R&D investment $V^h(\gamma_t \theta_t)$ and without successful R&D investment $V^h(\theta_t)$ weighted with the success-probability. Substituting (8) back into (7) and divide by dt yields:

$$\rho V^{h}\left(\theta_{t}\right) = \max_{k_{t}^{h}, \phi_{t}} \left\{ \pi_{t}\left(\theta_{t}\right) + \lambda \left[V^{h}\left(\gamma_{t}\theta_{t}\right) - V^{h}\left(\theta_{t}\right)\right] \right\}.$$
(9)

There are two important features about (9) which one should keep in mind thorough the following analysis. Firstly, all important information about the past concerning current or future decisions is summarized in θ . How the firm reached the present productivity does not matter at all. Secondly, choosing the optimal production and R&D-investment strategy with respect to the problem starting at the current productivity level θ that results from the initial firm strategies, is the optimal strategy no matter what the initial strategy of the firm was.

Optimality Conditions for R&D-Investment and Production Strategies From (9) we can derive the optimality conditions for firm-strategies for R&D-investment and home production.

R&D Investment Deriving the marginal valuation of R&D-investment from (9) yields¹⁷

$$\pi_{\phi}\left(\theta\right) + \lambda V_{\phi}^{h}\left(\gamma\theta\right) = 0. \tag{10}$$

The second part of the brackets of (9) disappears, as $V^{h}(\theta)$ does not depend on the current ϕ . Rearranging (10) delivers:¹⁸

$$V_{\phi}^{h}(\gamma\theta) = \frac{1}{\lambda} \left[1 - \frac{r_{\phi}^{h}(\theta)}{\omega} \right].$$
(11)

The marginal valuation of R&D-investment is a perpetual flow equal to one minus the revenue changes caused by ϕ , discounted by the probability of successful R&D-investment. The return decreases in the additional R&D investment because available capital for the domestic production is reduced. Thus (11) is positive. However, R&D investment increases the productivity and hence the output produced with one unit capital. Consequently, the valuation of additional corresponds to the decreased return caused by the R&D investment.

¹⁷For simplification, the time indices are dropped.

¹⁸For mathematical details see Appendix.

Home Production Differentiating the right hand side of (9) with respect to k^h , we obtain

$$\pi_{k^{h}}(\theta) + \lambda \left[V_{k^{h}}^{h}(\gamma\theta) - V_{k^{h}}^{h}(\theta) \right] = 0$$

$$\frac{r_{k^{h}}(\theta)}{\omega} + \lambda \left[V_{k^{h}}^{h}(\gamma\theta) - V_{k^{h}}^{h}(\theta) \right] = 0$$

$$V_{h}^{h}(\gamma\theta) = V_{h}^{h}(\theta) - \frac{1}{\lambda} \left[\frac{r_{h}^{h}(\theta)}{\omega} \right].$$
(12)

The subscripts unequal to t stand for the partial derivation. For simplicity, in the following cases the derivation subscripts are shortened to h for the derivation with respect to capital invested in home production instead of k^{h} .¹⁹ The marginal valuation of production-investment, in the case of successful R&Dinvestment equals the marginal valuation of production-investment with no R&D investment minus the marginal revenue stream resulting from increased capital in production - discounted with the probability of successful R&Dinvestment. It is $V_h^h(\theta)$ minus the revenue stream, as the valuation of k^h in case of additional investment in R&D is examined. Capital is divided between R&D investment and domestic production. If R&D investment is successful, the valuation of domestic production decreases relatively to unsuccessful R&D because there is an alternative use for capital with a high valuation. Analysing just the valuation of k^h without the increased productivity would be $V_h^h(\gamma\theta)$ plus the revenue stream.

An optimal strategy requires that the marginal valuation of investment in production equals the marginal valuation of R&D-investment. We can derive an explicit marginal valuation for investment in production by equating (12) and (11), namely

$$V_h^h(\theta) = \frac{1}{\lambda} \left[1 + \frac{r_h^h - r_\phi^h}{\omega} \right].$$
(13)

Similar to (11) the marginal valuation of investment in production equals a flow consisting of one plus the difference between the revenue change caused by the two investment decisions. Again, this flow is discounted by the probability of successful R&D investment.²⁰

Equation (13) reflects the trade-off between investing in R&D or not. First of all, investing in R&D reduces the capital available to invest in domestic production. This effect is negative. But secondly, R&D-investment increases productivity and higher productivity enforces the output of the employed production-capital and decreases the variable production costs $\frac{x}{\theta}$. Hence, there is also a positive effect of R&D-investment on the marginal valuation of capital invested in home-production. These considerations are reflected in the second part of (13).

 $^{^{19}\,{\}rm The}$ subscripts will be analogue p for investment in FPI and d for investment in FDI instead of k^p and $k^d.$

²⁰Consequently, $V(\gamma\theta) = -\frac{1}{\lambda}\frac{r_{\phi}}{\omega}$. The intuition is that with increased ϕ the return decreases directly, $r_{\phi} < 0$. The valuation of domestic production in total is positive as with foregone R&D investment k_h increases and thus increases the additional return.

3.2Home and Foreign Portfolio Investment

Now, we analyse the investment decision of the firm and allow for an additional investment alternative, namely foreign portfolio investment (FPI). With FPI equation (3) changes to

$$K_t = k_t^h + k_t^p + \phi_t. \tag{14}$$

This shows that the total capital available to a firm can be used to invest in domestic production, R&D-investment (the same as in the scenario above) and additionally k^p is the capital invested in FPI. As the firm invests in FPI, it gains ownership on a foreign firm. But the domestic firm has no - or only infinitely small - possibility to exert control over the foreign production and management process. Thus the domestic firm cannot directly influence the foreign revenue and the gained dividend

$$r_t^p = \mu_t \left(k_t^p \right). \tag{15}$$

 μ_t is the return rate from FPI (or the productivity of capital invested in FPI).^21 It varies with

$$\frac{d\mu}{\mu} = \sigma_{\mu} dz_{\mu} \tag{16}$$

where dz is a Wiener process with mean zero and unit variance. Following (15) and (16), the only impact the home firm has on the foreign investment, is the decision of how much capital to invest in FPI.

Investment in FPI requires buying assets, time to select the appropriate assets, additional administration systems and efforts etc. All these efforts are summarized as fixed costs f_t^p for this investment. Yet the profit function for the firm (5) changes to

$$\pi_t \left(\theta_t \right) = p_t \theta_t \left(k_t^h \right)^\alpha - \left\{ f_t^h + \frac{x_t^h}{\theta_t} + \phi_t \right\} + r_t^p - f_t^p.$$
(17)

Following the steps in the home-scenario we get the multi-period optimization problem for the firm

$$\rho V^p\left(\theta_t\right) dt = \max_{k_t^h, k_t^p, \phi_t} \pi_t\left(\theta_t\right) dt + E_t\left(dV^p\right) \tag{18}$$

subject to (2), (4) and (14) - (16). As the firm is now in the FPI-scenario, the superscript changed to p and there is one more control variable, namely k_t^p . The expected future capital flow depends on two state variables θ_t and μ_t ²²

$$dV^{p} = V^{p}_{\theta}d\theta + V^{p}_{\mu}d\mu + \frac{1}{2}V^{p}_{\mu\mu}(d\mu)^{2} + V^{p}_{\mu\theta}(d\mu)(d\theta).$$
(19)

Thus in case of FPI investment, the expectation of the change in the expected capital flow consists of three parts

$$E(dV^{p}) = \lambda \left[V^{p}(\gamma\theta) - V^{p}(\theta) \right] dt + \left[\frac{1}{2} \mu^{2} \sigma_{\mu}^{2} V^{p}_{\mu\mu} \right] dt + \left[V^{p}_{\mu\theta}(\gamma\theta) (\sigma_{\mu}\mu) \eta^{p} \right] dt.$$
(20)

 $^{{}^{21}\}mu \in R^+$. 22 For simplification, the time indices are dropped.

The first part is similar to the expected capital flow in the Home-scenario. Additionally, the variations of the foreign return impacts V^p . This impact occurs in the second term. Finally, the third term accounts for common variations of home productivity and foreign productivity that can result from global or industry shocks. The direction of this correlation depends on $\eta^p \equiv (dq) (dz_{\mu}) \neq$ 0. If the firm invests FPI in the East, η^p is positive. η^p is negative with FPI in the southern country.

In case of FPI, the present value of the firm profit flows is

$$\rho V^{p}\left(\theta\right) = \max_{k_{s}^{h}, k_{t}^{p}, \phi_{s}} \left[\pi^{p}\left(\theta\right) + \lambda \left[V^{p}\left(\gamma\theta\right) - V^{p}\left(\theta\right)\right] + \varepsilon\right]$$
(21)

with $\varepsilon \equiv \varepsilon^a + \varepsilon^b$, $\varepsilon^a \equiv \frac{1}{2}\mu^2 \sigma^2_{\mu} V^p_{\mu\mu}$ and $\varepsilon^b \equiv V^p_{\mu\theta} (\gamma\theta) (\sigma_{\mu\mu}) \eta^p$. The uncertain foreign productivity influences the present value of the profit flows twice. Firstly, the isolated variation of the foreign productivity ε^a enters the capital flows and secondly, the common variation of home and foreign productivity ε^b changes the capital flows. Whereas, the home productivity change is a discrete shock and ε is continuous. Similar to (9), all necessary information for any decision are summarised in θ and μ . Further, any optimality of future decision on FPI, home production or R&D-investment is independent of the firms' initial decision.

Optimality Conditions with FPI

$$V^{p}_{\phi}\left(\gamma\theta\right) = \frac{1}{\lambda} \left[1 - \frac{r^{h}_{\phi}}{\omega} - \Phi\right]$$
(22)

where $\Phi = \frac{\partial \left(V_{\mu\mu}^p \frac{1}{2}\mu^2 \sigma_{\mu}^2\right)}{\partial \phi} + \frac{\partial \left(V_{\mu\theta}^p (\gamma\theta)(\sigma_{\mu}\mu)\eta^p\right)}{\partial \phi}$. FPI does not have any direct impact on the R&D-investment. In comparison to the pure Home-scenario, the marginal valuation of R&D-investment is reduced by Φ . This effect arises through the common variation of the home and foreign productivities. If the firm invests into closely related industries or even in the same industry (eastern country, $\eta^p > 0$) then the own risk is not reduced. Thus Φ is positive and reduces the marginal valuation of R&D-investment slightly but never completely compensates it. Contrary, with investment in a dissimilar industry (South, $\eta^p < 0$) the risk of R&D failure is diversified. Φ is negative and increases the valuation of R&D-investment.

Home Production The direct valuation of home production is unchanged

$$V_{h}^{p}(\gamma\theta) = V_{h}^{h}(\theta) - \left[\frac{1}{\lambda}\frac{r_{h}^{h}(\theta)}{\omega}\right].$$
(23)

Following the optimality principle, we can equate the marginal valuation of investment in home production with the marginal valuation of R&D-investment

and get

$$V_{h}^{p}\left(\theta\right) = \frac{1}{\lambda} \left[1 + \frac{r_{h}^{h} - r_{\phi}^{h}}{\omega} - \Phi\right].$$
(24)

Similar to (22), the valuation changes by Φ . Analogue to (22), the change depends on the industry invested in.

FPI Optimality requires that the marginal valuation of FPI also equals the marginal valuation of investment in home production and R&D-investment. Therefore we differentiate (21) with respect to k^p rearranging delivers

$$V_p^p(\gamma\theta) = V_p^p(\theta) - \frac{1}{\lambda} \left[r_p^p + \varepsilon_p \right].$$
(25)

Valuation of FPI is lower with investment located in the East (similar production and cost structure, $\varepsilon_p > 0$) than with investment located in the South (different factor endowment, production and cost structure $\varepsilon_p < 0$). Obviously, the diversification of the risk increases the valuation of the investment abroad.

FPI vs Home The results from deriving all optimality conditions for FDI are summarized in Table (1):

R&D	$V^p_{\phi}\left(\gamma\theta\right) = \frac{1}{\lambda}$	$\left[1 - \frac{r_{\phi}^{h}}{\omega} - \Phi\right]$
Home	$V_{h}^{p}\left(\gamma\theta\right) = \frac{1}{\lambda}$	$\left[1 + \frac{\omega r_p^p - r_h^h - r_\phi^h}{\omega} - \Phi + \varepsilon_p\right]$
FPI	$V_p^p\left(\gamma\theta\right) = \frac{1}{\lambda}$	$\left[1 + \frac{r_h^h - \omega r_p^p - r_\phi^h}{\omega} - \Phi - \varepsilon_p\right]$

Table 1: Optimality Conditions with FPI

Table (1) shows that the effect of FPI on the marginal valuation of investment in home production and R&D is twofold. Additional capital invested abroad reduces capital available for domestic production and R&D-investments. A further effect arises through the exploitation of risk diversification possibilities, Φ . Investment into countries with closely related industries (East) diminishes the valuation of domestic production, $\Phi > 0$. Similar sources of risks are added. Investments in dissimilar countries (South) push the valuation slightly up, $\Phi < 0$. In this case, FPI constitutes a hedging instrument for the existing R&D-risk.

Finally, the additional variation of a further unit capital invested in FPI, ε_p , impacts the valuation of home production. At the same time, ε_p affects the valuation of FPI in the opposite direction. The marginal valuation of home production increases with further FPI in the East, $\varepsilon_p > 0$ and decreases with additional southern FPI. Eastern FPI delivers additional variation and risk. Home production is valued higher as it is a more secure source of future capital flows.²³

²³FPI valuation decreases through the same effect.

FPI in the South hedges existing home risk and consequently the valuation of home production decreases, $\varepsilon_p < 0.^{24}$ Additional southern FPI dampens the R&D risk and enforces further R&D investments. The firm withdraws capital from home production and invests the available capacities into southern FPI.

Hence, with isolated investment possibilities the firm will engage in southern FPI.

3.3 Home and Foreign Direct Investment

In the case of FDI, the home firm takes ownership as well as control over the foreign firm and thus can influence the profit of its direct-investment. In the present paper, the firm only transfers capital to the foreign firm. No intermediate goods are traded. However, the choice of the FDI receiving country has a significant impact on the valuation of FDI.

If the home firm decides for FDI it also transfers intangible assets, as for example managerial skills, technology..., to the foreign firm. As a side effect of this asset transfer, a part of the home productivity directly enters the return of FDI

$$r_t^d = \psi_t^2 \theta_t^{\frac{1}{a}} \left(k_t^d \right); \qquad 0 < a < 1.$$
 (26)

Home productivity θ does not impact the foreign investment to the same extent, than home production. This can be caused by country specific conditions or incomplete mobility of some home skills.²⁵ ψ is the foreign productivity which is stochastic and varies with²⁶

$$d\psi = \psi \sigma_{\psi} dz_{\psi}.$$
 (27)

Again, dz_{ψ} is a Wiener process with mean zero and unit variance. The amount of capital invested in FDI is k^d . Hence equation (3) becomes

$$K_t = k_t^h + k_t^d + \phi_t. \tag{28}$$

Further, FDI requires some specific up-front costs like country and market research, a merger or building a new plant. All these activities are costly and summarized in f^d , as the fixed costs arising from FDI. Now the modified profit function of the home firm is

$$\pi_t \left(\theta_t \right) = p_t \theta_t \left(k_t^h \right)^\alpha - \left\{ f_t^h + \frac{x_t^h}{\theta_t} + \phi_t \right\} + \frac{r_t^d}{\omega} - f_t^d.$$
(29)

It is important to keep in mind, that the FDI fix costs, f^d exceed the FPI fix costs, f^p .

The dynamic optimization problem of the home firm is

$$\rho V^d\left(\theta\right) dt = \max_{k_t^h, k_t^d, \phi_t} \left[\pi_t^d\left(\theta_t\right) dt + E_t\left(dV^d\right)\right].$$
(30)

²⁴In this case, the valuation of FPI increases.

²⁵With $a \to \infty$, the FDI scenario would be the same than the FPI scenario.

 $^{2^{6}\}psi \in R^{+}$

Equation (30) is a function of the state variables home productivity θ as well as foreign productivity ψ . The control variables are the three investment purposes, k^h, k^d, ϕ . The derivation of the functional equation from (30) is analogue to the steps in the FPI-scenario. Thus, we get

$$\rho V^{d}\left(\theta\right) = \max_{k_{t}^{h}, k_{t}^{d}, \phi_{t}} \left[\pi_{t}^{d} + \lambda \left[V^{d}\left(\gamma_{t}\theta_{t}\right) - V^{d}\left(\theta_{t}\right)\right] + \kappa\right]$$
(31)

with $\kappa \equiv \kappa^a + \kappa^b$, $\kappa^a \equiv \frac{1}{2}\sigma_{\psi}^2 \psi^2 V_{\psi\psi}^d$, $\kappa^b \equiv V_{\psi\theta}^d (\psi\sigma_{\psi}) (\gamma\theta) \eta^d$ and $\eta^d \equiv (dz_{\psi})(dq)$. Analogue to the FPI scenario, the uncertainty of the foreign productivity has two impacts on the present value of the profit flows: the variation of the foreign productivity κ^a and the common variation of the foreign and the home productivity κ^b . All necessary information for any decision is included in θ and ψ .

Optimality Conditions with FDI

R&D Investment Following the same steps as in the two previous scenarios we get the marginal valuation of additional R&D-investment²⁷

$$V_{\phi}^{d}(\gamma\theta) = \frac{1}{\lambda} \left[1 - \frac{r_{\phi}^{h} - \omega r_{\phi}^{d}}{\omega} - \varkappa \right].$$
(32)

First, there is a additional impact of FDI on the marginal valuation of R&Dinvestment. It is a very small positive effect through a slight increase in the foreign revenue. In comparison to the isolated home-scenario, this marginal change in r^d again increases the marginal valuation of R&D-investment.

Secondly, the influence of ϕ on the foreign productivity is included in $\varkappa \equiv \frac{\partial \kappa^a}{\partial \phi} + \frac{\partial \kappa^b}{\partial \phi}$. The sign of \varkappa is not definite. The degree $(\frac{1}{a})$ of the home productivity influence on foreign revenue is decisive for \varkappa .

Proposition 1 If a > 1 (low control over foreign firm - low impact of θ on r^d) then in both cases, eastern and southern FDI, $\varkappa > 0$ holds. If a < 1 and additional R&D exceeds the revenue losses caused by reduced capital input in FDI, then with FDI in the East $\varkappa > 0$ holds and $\varkappa < 0$ holds for southern FDI.²⁸, ²⁹

 $^{^{27}\}mathrm{Again},$ the time indices are dropped for the simplification of the equations.

 $^{^{28}}$ If the additional R&D investment increases productivity less than it reduces the additional return by reducing the available capital input for production, \varkappa changes its sign according to the respective FDI location. However, in the case with low control (a > 1) the impact of \varkappa on the R&D valuation stays unchanged.

²⁹A different assumption about the impact of foreign productivity ψ changes these effects. If additional productivity shows decreasing additional effects, ψ^{β} with $\beta < 1$ then even with low impact of the domestic firm on the foreign firm, $(a > 1), \varkappa < 0$ with eastern and southern FDI. Thus FDI increases the valuation of domestic R&D. Based on the same assumption but with high impact on the foreign firm, (a < 1), in the case of eastern FDI x > 0 and $\varkappa < 0$ for FDI in the South. In particular, R&D valuation increases with FDI in the South as technology transfer is facilitated and FDI in the East does not yield additional returns for further R&D investment.

With low impact on the foreign firm the valuation of domestic R&D-investment deceases with FDI. It does not matter whether FDI would be located in the East or in the South. If the domestic firm has a high impact on the foreign firm, southern FDI enhances the R&D valuation. FDI in the East does not change its impact on the R&D valuation. The technology transfer with horizontal FDI in countries with differing production and cost structures is rather complicated and depends strongly on the cost structure of the different countries.³⁰ Therefore, the implementation of new technologies - developed for domestic production in the South is only possible with a strong control position or a high impact of the domestic productivity on the foreign firm. Only based on these conditions, additional R&D investment induces additional valuation in the case of southern FDI. FDI in the East does not increase the R&D valuation neither with a high nor with low impact on the foreign firm. In this case additional capital is rather invested in FDI production directly than into domestic R&D-investment. As Home and East are very similar countries, additional R&D investment accounts for an investment similar to investment in FDI. This FDI implies additional productivity by adding the foreign productivity to the already existing domestic productivity. Such a productivity push caused by additional FDI increases the valuation of FDI and decreases the R&D valuation.

Home Production As expected from the previous section, home production stays unchanged again

$$V_{h}^{d}(\gamma\theta) = V_{h}^{d}(\theta) - \frac{1}{\lambda} \left[\frac{r_{h}^{h}(\theta)}{\omega} \right].$$
(33)

Substituting equation (32) into the marginal valuation of investment in home production delivers

$$V_{h}^{d}(\theta) = \frac{1}{\lambda} \left[1 + \frac{r_{h}^{h} - r_{\phi}^{h} - \omega r_{\phi}^{d}}{\omega} - \varkappa \right].$$
(34)

The changes in ϕ affect directly the FDI revenue and indirectly the variations of the productivity of FDI. The reduction of the marginal valuation of the investment in home production is not as high as under FPI. In the current case, R&D-investment does not only diminish the capital available for FDI, it also increases the productivity of capital invested in the foreign firm. Further, the sign of \varkappa depends on the FDI location.

FDI To derive the optimality condition for FDI, we differentiate (31) with respect to k^d . This yields

$$V_d^d(\gamma\theta) = V_d^d(\theta) - \frac{1}{\lambda} \left[r_d^d + \kappa_d \right].$$
(35)

 $^{^{30}}$ Grossman, Helpman and Szeidl (2003) show that under different cost structures in the observed countries, firm strategies changes from horizontal to vertical FDI and vice versa.

Equation (35) shows that the marginal valuation of FDI in case of successful R&D-investment depends again on the FDI location. If the firm invests in eastern FDI then the term in the brackets remains positive and hence reduces the valuation. On the other hand, if the firm undertakes southern FDI the sign of κ changes. But the indirect effect through κ is weaker than the direct effect of the changed revenue. Thus the valuation is still reduced but not as much as in the case of FDI in the East. Generally, we find a decreasing marginal product of capital either invested in domestic production or invested in foreign production. However, a negative correlation between domestic and foreign productivity at hand, the decrease of the marginal product invested in FDI is damped.

R&D	$V^{d}_{\phi}\left(\gamma\theta\right) = \tfrac{1}{\lambda}$	$\left[1 + \frac{\omega r_{\phi}^d - r_{\phi}^h}{\omega} - \varkappa\right]$	
Home	$V_{h}^{d}\left(\gamma\theta\right)=\tfrac{1}{\lambda}$	$\left[1+rac{\omega \left(r_{d}^{d}+r_{\phi}^{d} ight)-r_{h}^{h}-r_{\phi}^{h}}{\omega}-arkappa+\kappa_{d} ight]$	
FDI	$V_d^d\left(\gamma\theta\right) = \frac{1}{\lambda}$	$\left[1 + \frac{r_h^h + \omega \left(r_\phi^d - r_d^d\right) - r_\phi^h}{\omega} - \varkappa - \kappa_d\right]$	

FDI vs Home Table (2) summarizes the optimality conditions with FDI:

Table 2:	Optimality	Conditions	with	FDI

As discussed above, FDI impacts the marginal valuation of R&D-investment (first row of Table (2) up. This effect depends on the impact degree of the domestic productivity on the foreign revenue. With low domestic impact on r^d FDI decreases the valuation of R&D in either location. High domestic impact on r^d changes the impact of southern FDI on R&D-valuation. The results of domestic R&D-investment are easily transferred to the foreign firm. R&D does not only increase the domestic productivity but also boosts the foreign return by increasing the transferred productivity. Further, these effects carry over to the valuation of investment in home-production with respect to R&Dinvestment. From the second row of Table (2), it is obvious how the valuation of capital invested in home production depends on the different effects of FDI. Additional capital invested abroad decreases the marginal revenue of FDI no matter whether the firm undertakes southern or eastern FDI. This effect increases the valuation of investment in domestic production. Similar to R&D, FDI impacts the valuation of domestic production indirectly by the common variation of foreign and home productivity \varkappa .

The last parameter in the home valuation stands for the variation of one additional unit capital invested in FDI. Analogue to the FPI scenario, FDI in the East adds additional variations. The valuation of home production increases with further eastern FDI. In this case, home production is a very close substitute for FDI and even a more secure source for future capital flows. FDI in the south adds variations not common to the home variations. Thus, home production is not a close substitute for southern FDI as additional - even though only minor - gains on risk diversification arise with southern FDI.

The marginal valuation of FDI with respect to R&D (row three, Table (2)) equals a perpetual flow of the difference of changed revenues through additional capital invested in FDI and R&D, discounted by the probability of successful R&D-investment. The hedging components impact the FDI valuation in the same way as the R&D-valuation.

Finally, with isolated FDI the preferred location depends on the impact degree of home productivity on foreign revenue - or rather on the control degree of the investor. High control investors prefer southern FDI and low control investors are indifferent between eastern or southern FDI.

3.4 Home and Combined International Investment

Finally, we analyse a combined international investment strategy for the firm. Besides the usual home activities of the firm, it invests in FPI as well as in FDI at the same time. Because there are four different investment alternatives for capital, (3) changes to

$$K_t = k_t^h + k_t^p + k_t^d + \phi_t. {(36)}$$

The return functions of the international investments are similar to the return functions under isolated international investment. Hence, the firms' profit function with combined international investment is³¹

$$\pi_t^c(\theta_t) = p_t \theta_t \left(k_t^h\right)^\alpha - \left\{ f^h + \frac{x_t^h}{\theta_t} + \phi_t \right\} + r_t^p - f_t^p + r_t^d - f_t^d.$$
(37)

and the dynamic firm problem is:³²

$$\rho V^{c}\left(\theta\right) dt = \max_{k^{h}, k^{p}, k^{d}, \phi} \left[\pi^{c}\left(\theta\right) dt + E_{t}\left(dV^{c}\right)\right].$$
(38)

The control variables in the dynamic combined optimization problem are the various investment purposes: investment in domestic production k^h , R&D-investment ϕ and the two international investment alternatives FPI k^p and FDI k^d . Further, in the combined scenario the present value of the firms capital flows is a function of the three state variables: home productivity θ , productivity of the portfolio investments μ and the productivity of the direct investment ψ . These three variables summarize all the necessary information for an optimal investment-decision in the present period. We need the functional equation of the optimizing problem (38) to derive the optimality conditions. Again, the steps are very similar to the isolated investment strategies and therefore, we neglect them and directly turn to the functional equation

$$\rho V^{c}\left(\theta\right) = \max_{k_{t}^{h}, k_{t}^{p}, k_{t}^{d}, \phi_{t}} \left[\pi_{t}^{c} + \lambda \left[V^{c}\left(\gamma_{t}\theta_{t}\right) - V^{c}\left(\theta_{t}\right)\right] + \varepsilon + \kappa + \xi\right]$$
(39)

where $\xi \equiv V_{\mu\psi}^c (\mu\sigma_{\mu}) (\psi\sigma_{\psi}) \eta^c$ and $\eta^c \equiv (dz_{\mu}) (dz_{\psi})$. In (39) we have the investment effects of the isolated international strategies combined. Additionally, the common variation of the two international investments is included through ξ .

³¹We have to keep in mind that $f^d > f^p$ still holds.

 $^{^{32}}$ We will keep the detailed transforming-steps very short as the necessary steps for the transformation are similar to the steps undertaken in the previous isolated section.

Optimality Conditions with Combined International Investment (CII)

R&D Investment Following (39), the optimality condition for R&D-investment changes slightly in comparison to the isolated scenarios:³³

$$V_{\phi}^{c}(\gamma\theta) = \frac{1}{\lambda} \left[1 - \frac{r_{\phi}^{hc} - \omega r_{\phi}^{dc}}{\omega} - \Phi - \varkappa - \xi_{\phi} \right].$$
(40)

The first part of the bracket stays unchanged. Also, the isolated effects of the different investment possibilities, Φ and \varkappa , are the same as above. But the interaction of FPI and FDI changes the impact of the isolated investment effects. The only new term is ξ_{ϕ} . Its impact depends on the international investment interaction, too. Table (3) summarizes the effects from the isolated strategies and adds the common effects in case of CII.

	impact FPI	impact FDI	impact FDI	common
		low control	high control	impact
eastern FPI / eastern FDI	$\Phi > 0$	$\varkappa > 0$	$\varkappa > 0$	$\xi_{\phi} > 0$
southern FPI / southern FDI $$	$\Phi < 0$	$\varkappa > 0$	$\varkappa < 0$	$\xi_{\phi} > 0$
eastern FPI / southern FDI	$\Phi > 0$	$\varkappa > 0$	$\varkappa < 0$	$\xi_{\phi} < 0$
southern FPI / eastern FDI	$\Phi < 0$	$\varkappa > 0$	$\varkappa > 0$	$\xi_{\phi} < 0$

Table 3: Impact of different International Investment Possibilities

From Table (3), we can emphasize two cases. The first case is a domestic firm with low influence on the foreign revenue (or low productivity). According to Table (3), there is no directly dominant strategy with respect to FDI. FDI in the East has the same impact on R&D-valuation than FDI in the South. However, combining FDI and FPI affects on the R&D-valuation shows that the combination with FPI in a southern country and FDI in the East has a slightly higher positive impact on the R&D valuation. With FPI in an unrelated country, the firm secures risk diversification. Isolated FDI in the East is not better than isolated FDI in the South but in combination with southern FPI both investment possibilities are negatively correlated and this pushes the marginal valuation of R&D-investment additionally.

The second case is a firm with high influence on the foreign revenue (or high productivity). The preferred FPI location stays unchanged; whereas FDI switches to the South. Now, technology transfer is easily possible via FDI. As in the former case, FPI still serves as diversification instrument for domestic risk. It does not hedge FDI-location risk anymore. But with the increasing domestic productivity and its higher impact on foreign revenue, the remaining share of FDI location specific risk diminishes.

³³For simplification time indices are dropped.

Home Production Analogue to the isolated investment possibilities, the impact of home production does not change

$$V_{h}^{c}(\gamma\theta) = V_{h}^{c}(\theta) - \frac{1}{\lambda} \left[\frac{r_{h}^{hc}(\theta)}{\omega} \right].$$
(41)

In combination with the marginal valuation of R&D-investment, the impact of CII on the home production valuation becomes clear:

$$V_{h}^{c}(\theta) = \frac{1}{\lambda} \left[1 + \frac{r_{h}^{hc} - r_{\phi}^{hc} - r_{\phi}^{dc}}{\omega} - \varkappa - \Phi - \xi_{\phi} \right].$$

$$(42)$$

We see from Table (3), that the optimal investment combinations with respect to R&D-investment are the optimal combinations with respect to the valuation of home production in combination with R&D. But we still cannot generalize this optimal investment combination.

CII First, we have to examine the effects on the various international investments and the combination of all effects. As they are all derived similarly to the isolated strategies, Table (4) just summarizes the results

R&D	$V^{c}_{\phi}\left(\gamma heta ight)=rac{1}{\lambda}\left[1-rac{r^{hc}_{\phi}-r^{dc}_{\phi}}{\omega}-\Phi-arkappa-\xi_{\phi} ight]$
Home	$V_{h}^{c}\left(\gamma\theta\right) = \frac{1}{\lambda} \left[1 + \frac{r_{d}^{dc} + r_{\phi}^{dc} - r_{h}^{hc} - r_{\phi}^{hc}}{\omega} - \varkappa - \Phi + \varepsilon_{p} + \kappa_{d} - \xi_{\phi} + \xi_{d} \right]$
FDI	$V_d^c\left(\gamma\theta\right) = \frac{1}{\lambda} \left[1 + \frac{r_h^{hc} - r_\phi^{hc} + \omega \left(r_\phi^{dc} - r_d^{dc}\right)}{\omega} - \Phi - \varkappa - \kappa_d - \xi_\phi - \xi_d \right]$
FPI	$V_{p}^{c}\left(\gamma\theta\right) = \frac{1}{\lambda} \left[1 + \frac{r_{h}^{hc} - r_{\phi}^{hc} + \omega\left(r_{\phi}^{dc} - r_{p}^{pc}\right)}{\omega} - \varkappa - \Phi - \varepsilon_{p} - \xi_{\phi} - \xi_{p} \right]$

Table 4: Optimality Conditions with CII

From Table (4), we see that each marginal valuation increases with negative correlation of the home industry and the chosen industry for FPI ($\Phi < 0$). The risk of unsuccessful R&D-investment at home can be propped up by the short term portfolio-investment.

To detect the preferred FDI location, again we have to distinguish two cases: low and high impact on the foreign firm. With low productivity and low control, $\varkappa > 0$ reduces the respective valuation. The positive sign for \varkappa arises under eastern as well as southern FDI. Overall, there is no facilitated technology transfer under eastern or southern FDI. However, FDI in the East is negatively correlated with the chosen FPI location. This variation effect dampens the direct negative FDI impact on the respective valuations. Hence, both international investments are mostly favoured with FPI in South and FDI in the eastern country. For FPI, the risk diversification is the stronger effect with the highest impact on the firm decision. In particular, FPI is the more flexible investment and can be adjusted with only minor costs. Therefore, it is the appropriate instrument to diversify a firm's risk. On the other hand, FDI reacts more sensitive to productivity changes and thus, is the favourable instrument to exploit productivity gains internationally. The additionally arising negative correlation between the two international investments pushes all valuations slightly up.

With high domestic control over the foreign firm the preferred FDI location switches from the East to the South. High control (low *a*) reduces the share of location or industry specific risk and facilitates the technology transfer from home to the South.³⁴ FPI looses its function of direct-hedging FDI location specific risk. But, FPI still works as hedging instrument for R&D risk. With the increasing domestic control - and therefore higher impact of θ on the FDI revenue, this role even gains importance. Higher domestic productivity requires higher R&D-investments and this in turn stipulates a more intensive risk hedging. Concluding, FPI loses its impact as direct hedging instrument with respect to FDI but with respect to domestic production and thus indirectly to FDI, the hedging necessity increases.

In CII, FPI can prop up the risk from home production and FDI. The relations between the home country and the recipient countries are unchanged to the isolated investment scenarios for FPI as well as for FDI. Hence, we expect in CII the share of FPI to adjust to short-term environment changes whereas FDI stays unchanged. Because of the complexity of the problem there is not possibility to derive an explicit analytical solution for the respective international investment shares. The definite shares of FPI and FDI will be derived numerically.

4 Optimal Investment Strategies

As for both FDI investor scenarios - low and high control on the foreign firm, the results emphasize that FPI works as diversification instrument and the firm uses FDI as a technology transfer channel. These findings are valid for the isolated strategies as well as for the combined strategy. To prove or reject these findings clearly in the following analysis we consider FDI in the East and FPI in the South.

Unfortunately, the problem has no tractable closed form solution. Hence, the solution must be approximated by numerical methods. We break the model down into many one-period decisions. We use recursive policy function iteration.³⁵ From a given capital stock K = 10 for every period we set the choice for investment in domestic production k^h . The remaining decision variables are a result of the optimality conditions. We repeated this procedure with various values for k^h . The initial value of θ is set to one and changes according to the R&D-investment decision. Additionally, we examined different cost structures. They changed the results in terms of their value but they never had an impact

 $^{^{34}\}mathrm{A}$ high home productivity θ has equivalent consequences.

 $^{^{35}}$ For detailed discussion and mathematical background see Adda and Cooper (2003), Judd (1998) and Dixit and Pindyck (1994).

on the bottom line of the results. We tested the model with different time horizons: 10, 20, 30, 40, 50 periods. There are some small variations in the absolute investment values but the length of the time horizon does not change the main results either. The results are only sensitive to productivity strength and correlation of the international investments. This is discussed in the following sections.

The first run computes the solution for the isolated international investment strategies and determines the cut-offs at which the firm changes from one strategy to another (home, FPI or FDI). In the second run, we repeat the same steps for the combined international investment strategy. Precisely, with CII the firm changes its strategy only once: from isolated home production to FPI and FDI at the same time.

We derive a benchmark case with a depreciation of $\tau = 0.3$. A higher depreciation pushes the start of international activity backwards in time and a lower depreciation pulls it forward. The general results stay the same. Further, the probability for successful R&D-investment λ varies and shows a significant impact on the firms' decision to invest internationally or not.

4.1 Isolated International Investment

Start of International Activity The first international activity of the firm is FPI. As expected, FPI requires lower cut-off productivity than FDI. However, the firm starts investing in FPI not until the probability of successful R&D is 0.3 or higher. The figures (1) and (2) show for example the investment choices and changes for a R&D-success probability of $\lambda = 0.3$ and $\lambda = 0.5$. Even with $\lambda = 0.5$, the international activity starts very late in time. With increasing R&D-probability, the firm undertakes international investment at an earlier stage. This is very intuitive, as with high success-probability the productivity increases more quickly. All these results confirm the findings in the recent literature. Firms with low productivity stary isolated at the home market. With a slight increase in productivity the first small international steps are made and finally, firms with a remarkable high productivity invest in FDI.



Figure 1: Isolated investmentshares of all four possible investments depend on time and domestic produdicitvity. R&D success probability $\lambda = 0.3$.

Furthermore, the figures (1) and (2) show that not only with a high productivity FDI is undertaken, there is also a reversed relationship. As soon as the firm invests in FDI the R&D investment increases and this in turn boosts the productivity θ . Thus with the additional investment abroad, investment in R&D is more valuable than without the direct investment. The incentive to invest in R&D increases because a higher domestic productivity now pushes not only the domestic output and return but also the return of the foreign direct investment. Anyway, this effect diminishes with time. Each additional investment in R&D adds less productivity for each unit capital invested in home and foreign production. Consequently, the firm will draw capital from R&D investment and invest it in additional FDI and later on even in additional home production. Again, the figures (1) and (2) show the increasing share of FDI and the late increase in home production in the firms' total investment.



Figure 2: Isolated investmentshares of all four possible investments depend on time and domestic productivity. R&D success probability $\lambda = 0.5$.

Variation of Foreign Productivity Firstly, we examine changes in the FPI productivity. Table (5) shows that neither the productivity cut offs nor the cutoff time change with variations in FPI productivity. One might have expected that with higher foreign productivity the firm engages earlier in international investment. This is not the case. The firm first secures the home production process and then goes abroad.

	Switch-Period	Productivity	Share of	Switch-Period	Productivity	Share of
	FPI	of FPI	FPI	FDI	of FDI	FDI
$\lambda = 0, 3$	5	1,087	0,352	6	1,36	6,0
low fpi	x	x	x	х	x	x
high fpi	5	1,087	0,84	6	1,36	6,0
$\lambda = 0, 4$	4	1,086	0,36	5	1,38	6,0
low fpi	x	x	x	x	x	x
high fpi	4	1,086	0,85	5	1,38	6,0
$\lambda = 0, 5$	3	1,07	0,41	4	1,35	6,0
low fpi	x	x	x	х	x	x
high fpi	3	1,07	0,92	4	1,35	6,0
$\lambda = 0, 6$	3	1,09	0,37	4	1,59	5,0
low fpi	х	x	x	х	x	x
high fpi	3	1,09	0,86	4	1,59	5,0
$\lambda = 0, 7$	2	1,05	0,5	3	1,21	7,0
low fpi	x	x	x	х	x	x
high fpi	2	1,05	1,04	3	1,21	7,0
$\lambda = 0, 8$	2	1,06	0,48	3	1,36	6,0
low fpi	x	x	x	x	x	х
high fpi	2	1,06	1,01	3	1,36	6,0

Table 5: Productivity cut-offs and changing investments with varying FPI productivity. Switch-Period is the period in which the firm engages in FPI or FDI for the first time.

Further, the firm does not reduce or increase its share in FDI. Only the FPI shares increase with higher FPI productivity. This might seem intuitive, as only the FPI-productivity changes. Hence, the FDI shares are independent of the FPI productivity. Let's take a closer look on FDI-productivity changes, to see whether this independence also hold in the opposite direction and we can confirm FPI as the more flexible instrument.

Table (6) compares the firms' international investment with high and low FDI productivity. It shows that the firm engages in international investment earlier in time with a high FDI-productivity than with a lower FDI-productivity. Further, the productivity cut-off is lower than with the benchmark productivity. The only exceptions are the cases with a very high success probability of R&D investment. For these cases the cut-offs are the same as for the benchmark case and the high FDI-productivity.

	Switch-Period	Productivity	Share	Switch-Period	Productivity	Share
	FPI	of FPI	of FPI	FDI	of FDI	of FDI
$\lambda = 0, 2$	х	x	х	х	х	х
low fdi	x	x	x	х	х	x
high fdi	2	1,01	0,6	3	1,28	8,0
$\lambda = 0, 3$	5	1,087	0,352	6	1,36	6,0
low fdi	x	x	x	х	х	x
high fdi	2	1,02	0,58	3	1,04	8,0
$\lambda = 0, 4$	4	1,086	0,36	5	1,38	6,0
low fpi	х	x	x	х	х	x
high fpi	2	1,03	0,56	3	1,06	8,0
$\lambda = 0, 5$	3	1,07	0,41	4	1,35	6,0
low fpi	х	x	x	х	х	x
high fpi	2	1,04	0,54	3	1,07	8,0
$\lambda = 0, 6$	3	1,09	0,37	4	1,59	5,0
low fpi	x	x	x	х	х	x
high fpi	2	1,04	0,52	3	1,09	8,0
$\lambda = 0, 7$	2	1,05	0,5	3	1,21	7,0
low fpi	x	x	x	х	х	x
high fpi	2	1,05	0,5	3	1,21	7,0
$\lambda = 0, 8$	2	1,06	0,48	3	1,36	6,0
low fpi	x	x	x	x	x	x
high fpi	2	1,06	0,6	3	1,36	6,0

Table 6: Productivity cut-offs and changing investment shares with varying FDI productivity. Switch-Period is the period in which the firm engages in FPI or FDI for the first time.

Finally, with varying FDI-productivity both international shares change in comparison to the benchmark case. In particular, the FPI shares do not only vary in comparison to the benchmark case. They also change between the various cases of high FDI-productivity while the FDI shares stay almost the same. Again, only with the high R&D-probability the FDI shares change between the different cases, but they do not change with respect to the benchmark case. So, we find again FPI as the flexible instrument adjusting to short-term changes while FDI reacts more sluggishly. Note that, these are only results for the isolated investment scenario.

4.2 Combined International Investment

In contrast to the isolated international investment strategy, the firm starts its international activities with both investment alternatives FPI and FDI at the same time. Figure (3) shows that even with a low R&D-probability, the firm engages in its first international investment. However, we have to distinguish between the first international investment and the investment in FDI. In both cases, the first international activity under CII (CII FDI and FPI vs isolated FPI) takes place at an earlier date in time than the first international firm activity under an isolated international investment strategy. Additionally, the first international activity at all requires a lower R&D success probability under CII than for the isolated international investments.

At a moderate probability, the firm switches from home to international investment (isol. FPI and combined FPI-FDI respectively at the same time). With increasing probability the isolated investment even dominates the combined strategy in time. It is important to keep in mind, that in the current situation a firm starting isolated FPI is compared with a firm starting combined FPI and FDI at the same time.



Figure 3: The first international activities of both investment scenarios are compared. For the isolated investment strategy the first international activity is FPI. For the combined investment strategy the first international activity is FDI. Both international investments are compared in dependence of the R&D success probability.

Now, we turn to the comparison of isolated FDI and the combined international investment. For the switch to FDI the picture changes as shown in Figure (4). Under CII the firm switches from home production to international investment at a lower R&D-probability and at an earlier stage in time. Further, with increasing success-probability, CII still dominates the isolated investments in time.



Figure 4: The first investment in FDI for the isolated and the combined international investment are compared. Both investments are depicted in dependence of the R&D success probability.

However, CII does not always dominate isolated FDI in productivity. Particularly, the productivity cut-offs for FDI under CII do not always lie below the cut-offs for isolated FDI. The productivity cut-offs are analysed according to the value of θ at which the firm switches from one strategy (for example home) to another strategy (for example FDI). We compare the marginal impact of θ on the different discounted capital flows under a given productivity level. The figures (5) and (6) show the different productivities for FDI cut off with isolated FDI (iso) and CII-FDI (cii).



Figure 5: Producitivity of the first FDI for the isolated and the combined international investment strategy are compared. The R&D success probability varies and a positive correlation between FDI and FPI is assumed.

The domination of the isolated investment strategy might be unexpected. In the case of the positively correlated foreign productivities, FPI cannot prop up the FDI variations directly. But FPI absorbs the variations of the domestic productivity. The firm only invests in southern FDI if it has a high impact on the foreign productivity, hence θ influences significantly the foreign direct return. This in turn transfers the domestic variations into the foreign productivity. These variations constitute a high share of the foreign variation as the impact of θ is high. Since FPI is an effective instrument to dampen the domestic variation resulting from θ , FPI indirectly smoothes variations in the foreign direct return. The incentive to invest in domestic R&D is enhanced by this mechanism and the productivity cut-off is higher than without the combined investment possibility.



Figure 6: Producitivities of the first FDI for the isolated and the combined international investment are compared. The R&D success probability varies and a negative correlation between FDI and FPI is assumed.

In case of the negative correlation the productivity cut-offs of both investment strategies are as expected. The isolated cut-off productivity is always higher than the cut-off for the combined international investments. Obviously, FPI props up the FDI variations as well as the variations resulting from domestic productivity uncertainty.

One may not neglect, that with positive as well as with negative correlated international investments the combined international investment starts not only earlier in time than the isolated international investment but also at a lower R&D success-probability. Thus, the possibility to use FPI as a financial hedging instrument boosts the value of corporate diversification via FDI. This confirms the results in Table (4).

Variation of Foreign Productivity First of all, minor changes in the foreign productivities relation may diminish any international investment under isolated strategy completely. If both productivities are very low or at least the FPI-productivity is very low then the firm does not invest abroad. On the other hand, these changes do not reduce international investment under CII totally. The productivity cut-off changes as both foreign productivities drift apart (negatively correlated) or move together (positively correlated). The following figures (7) and (8) show the variation of FPI and FDI shares in dependence on their productivity relation.



Figure 7: Investment shares of FPI in the combined international investment scenario are depicted for a negative as well as positive correlation between FPI and FDI.



Figure 8: Investment shares of FDI in the combined international investment scenario are depicted for a negative as well as positive correlation between FPI and FDI.

Overall, the share of FPI varies more thorough the changed productivities than the FDI shares. The latter are more stable than the former. Additionally, FPI shares under CII fluctuate even more than under isolated international investment. Hence with CII, the firm reacts to short-term changes in its environment by adjusting FPI and keeping FDI stable. Thus, FPI does not necessarily increase with FDI, but adjusts according to R&D-probability, depreciation and variation in home and both foreign productivities. These results confirm again the risk-adjusting task of FPI and the more sluggish technology transfer FDI instrument.

5 Conclusion

This paper shows that even though the empirical distinction between FDI and FPI is rather complicated a differentiation of a firms' motivation to use these investment instruments is possible.³⁶ We examine in a dynamic investment setting that the relation of FPI and FDI is rather strategic complementary. Isolated FPI and FDI investments are compared to combined FPI and FDI investments. The combined investment strategy dominates the isolated investments always in time. Further, CII comprises a higher incentive to invest in R&D. The risk diversifying effect from additional FPI pushes the marginal valuation of R&D investment above the valuation with isolated investment strategies. As a consequence, home productivity increases much faster and without smaller relative opportunity costs than under isolated investment strategies. Finally, this leads to a higher productivity cut off for FDI but at an earlier date in time. The significant higher CII R&D investment than isolated FDI R&D investment confirms this observations. Surprisingly, this is not only the case with a combination of horizontal FDI in a country with similar structure and FPI in a country with dissimilar structure than the home country, but also with both international investments in a dissimilar country structure than the home structure.

Furthermore, we also find that firms adjust to short-term changes via FPI and keep FDI stable. FPI can prop up small and medium sized changes and therefore, the valuation of FDI with combined FPI is higher than of isolated FDI. Hence, a combined FPI and FDI investment strategy increases the firms' flexibility. A combination of both investment instruments increases the valuation of the respective instruments.

³⁶Empirical data for firms' FPI is very hard to obtain. Aggregate portfolio investment data is available on firm level. The sector or industry of the portfolio investment is not possible to locate because there is no notification requirement for firms.

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7 Appendix

Derivation of Expected Capital Flow

The value of the firm in the case without international investment is a function of the state variable θ (productivity).

$$dV^h = V^h_\theta d\theta \tag{43}$$

The state variable follows a Poisson process with q = 1 with prob. λdt and q = 0 with probability $(1 - \lambda dt)$:

$$\Rightarrow d\theta = \left[(1-\tau) \frac{\phi}{K} \right] \theta dq \tag{44}$$

$$\Rightarrow E\left(dV^{h}\right) = \underbrace{\lambda\left[V^{h}\left(\left[\left(1-\tau\right)\frac{\phi}{K}\right]\theta\right) - V\left(\theta\right)\right]dt}_{(45)}$$

change of capital flow caused by increased θ weighted with the probability

$$\underbrace{\left(1 - \lambda dt\right) \left[V^{h}\left(\theta\right) - V^{h}\left(\theta\right)\right]}_{(46)}$$

change of capital flow in the case of unchanged θ weighted with respective probability $\Rightarrow E(dV^{h}) = +\lambda \left[V^{h}(\gamma \theta) - V^{h}(\theta) \right] dt$ (47)

with

+

$$\gamma \equiv \left[\left(1 - \tau\right) \frac{\phi}{K} \right] \tag{48}$$

For a general discussion of Poisson processes in continuous time see Dixit and Pindyck (1994).

Derivation of the Profit Function with Variable Revenue

Domestic consumers have Dixit-Stiglitz preferences for differentiated goods with elasticity of substitution $\vartheta = \frac{1}{1-\varphi} > 1$. The price index for the home country is

$$P = \left[\int_{j \in J} p\left(j\right)^{1-\vartheta} dj \right]^{\frac{1}{1-\vartheta}}$$
(49)

and the demand level is

$$A = \left[\int_{j \in J} x\left(j\right)^{\varphi} dj \right]^{\frac{1}{\varphi}}.$$
 (50)

From (49) and (50) we derive the demand function

$$x_i = A p_i^{-\vartheta} \tag{51}$$

for each good variety produced by firm i. In the following the firm index i is neglected, as we just analyse one representative firm.

According to (5) the profit of the firm in period t equals

$$\pi_t \left(\theta_t \right) = r_t^h - f_t^h - \frac{x_t^h}{\theta_t} - \phi_t.$$
(52)

Revenue equals supply multiplied by the price we can rearrange (52) to

$$\pi_{t}(\theta_{t}) = r_{t}^{h} - f_{t}^{h} - \frac{px_{t}^{h}}{\theta_{t}} \frac{1}{p} - \phi_{t}$$

$$\pi_{t}(\theta_{t}) = r_{t}^{h} - f_{t}^{h} - \frac{r_{t}^{h}}{\theta_{t}} \frac{1}{\frac{1}{\varphi\theta_{t}}} - \phi_{t}$$

$$\pi_{t}(\theta_{t}) = r_{t}^{h} (1 - \varphi) - f_{t}^{h} - \phi_{t}$$

$$\pi_{t}(\theta_{t}) = \frac{r_{t}^{h}}{\omega} - f_{t}^{h} - \phi_{t}.$$
(54)