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Coordination: Bernd Hayo • Philipps-University Marburg  
Faculty of Business Administration and Economics • Universitätsstraße 24, D-35032 Marburg  
Tel: +49-6421-2823091, Fax: +49-6421-2823088, e-mail: [hayo@wiwi.uni-marburg.de](mailto:hayo@wiwi.uni-marburg.de)

# Agglomeration, Congestion, and Regional Unemployment Disparities

Ulrich Zierahn

University of Kassel and  
Hamburg Institute of International Economics (HWWI)  
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## Abstract

Regional labor markets are characterized by huge disparities of unemployment rates. Models of the New Economic Geography explain how disparities of regional goods markets endogenously arise but usually assume full employment. This paper discusses regional unemployment disparities by introducing a wage curve based on efficiency wages into the New Economic Geography. The model shows how disparities of regional goods and labor markets endogenously arise through the interplay of increasing returns to scale, transport costs, congestion costs, and migration. In result, the agglomeration pattern might be catastrophic or smooth depending on congestion costs. The transition between both patterns is smooth.

**JEL Classification:** J64, R12, R23

**Keywords:** regional unemployment, New Economic Geography, core-periphery, wage curve, labor migration

# 1 Introduction

Regional labor markets are characterized by huge disparities. Since the seminal paper of Krugman (1991) and his core-periphery-model, a new literature - the New Economic Geography - emerged discussing how disparities between regional economies endogenously arise. The common feature of models in this literature is that they compare the relative strength of centrifugal and centripetal forces (Krugman; 1998). Whereas centrifugal forces tend to disperse economic activity, centripetal forces strengthen the agglomeration of economic activity in a single or few regions. Yet, models of the New Economic Geography differ in how these forces are modeled and which mechanisms form the basis for these forces.

Depending on how these forces are modeled, different agglomeration patterns emerge. Such agglomeration patterns are visualized by bifurcation diagrams. The probably most prominent one is the catastrophic agglomeration pattern of Krugman (1991). Pflüger and Südekum (2004) label his corresponding bifurcation diagram a “bang bang” one, since either a full agglomeration or symmetry between the two regions under observation might result depending on transport costs. There is a certain level of transportation costs that decides whether the agglomeration is stable or not - the sustain-point. Similarly, there is a certain level of transportation costs that decides whether the symmetry is stable or not - the break-point. Sustain- and break-point lie at different levels of transportation costs so there is a certain range of transportation costs where both, the agglomeration-periphery and the symmetry, are stable. The actual situation of the two-region-system then depends on initial conditions. Small changes in the distribution of the labor force hence might result in a catastrophic change from symmetry in both regions to full agglomeration in one region and emptying out of the other region.

Another prominent agglomeration pattern is represented by the “bubble-shaped” bifurcation diagram, such as Pflüger and Südekum (2004). Here, there is no range of transport costs where both - agglomeration and symmetry - are stable equilibria. Instead sustain- and break-point coincide. When transport costs decline at some point a level of transport costs is reached where symmetry becomes instable. This is exactly the point where agglomeration becomes stable and economic activity agglomerates in one region. However, this pattern is smooth rather than catastrophic since the agglomeration slowly becomes larger with decreasing transport costs. When transportation costs decline further, the agglomeration shrinks until the system returns to symmetry. Therefore Pflüger and Südekum (2004) call this type of agglomeration pattern (or bifurcation diagram) “bubble-shaped”.

There exists another agglomeration pattern, especially when it comes to such models that integrate labor market frictions based on job matching into the New Economic Geography - such

as the model of Epifani and Gancia (2005). Here, the agglomeration pattern is catastrophic since there is a certain range of transportation costs where both, agglomeration and symmetry are stable. Then the distribution of economic activity depends on initial conditions. Though, this is not a “bang bang” agglomeration pattern since there is no full agglomeration and some economic activity remains in the periphery. Furthermore, at some level of transportation costs the agglomeration starts to shrink with declining transport costs and smoothly becomes the symmetry, similar to the “bubble-shaped” agglomeration pattern.

Usually these different agglomeration patterns result from different underlying centrifugal and centripetal forces (and hence different models).<sup>1</sup> In contrast, the present model shows, based on congestion costs, how these three agglomeration pattern might result from the same model and hence from the same centrifugal and centripetal forces. The idea is to construct a more universal model which is able to discuss, under what circumstances which of these patterns result. When congestion costs change, these three agglomeration patterns smoothly fade into each other. The point is than the relative strength of centrifugal and centripetal forces change, when congestion costs change whereas the underlying mechanisms of the centrifugal and centripetal forces remain the same. With such a model we discuss disparities of regional economies without restricting ourselves to a single agglomeration pattern. We extend this model to cover frictions of regional labor markets in order to discuss regional labor market disparities.

The economic literature has already presented several models to discuss regional labor markets. According to Elhorst (2003) the probably most encompassing model of regional labor markets is delivered by the seminal paper of Blanchard and Katz (1992). The authors discuss how regional labor markets adjust to shocks in labor demand through migration, participation, and unemployment and how disparities in these variables evolve in time as a consequence of such shocks. Overman and Puga (2002) show that indeed, labor demand is the reason for regional labor market disparities. Yet, the model of Blanchard and Katz (1992) is unable to show why disparities in labor demand endogenously arise.

It therefore stands to reason to introduce labor market frictions into the New Economic Geography in order to explain, how disparities of regional labor demand endogenously arise and how these result in regional labor market disparities. There already exist different models combining labor market frictions and the New Economic Geography. Epifani and Gancia (2005) for example introduce job-matching into the New Economic Geography. They show how disparities of regional economies endogenously arise and how these result in disparities of regional

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<sup>1</sup>There are of course exceptions: Pflüger and Südekum (2010) for example show how the catastrophic agglomeration pattern of the core-periphery-model becomes a smooth transition from symmetry to agglomeration when the Cobb-Douglas upper tier utility function is replaced by a CES utility function and how this transition depends on the corresponding elasticity of substitution.

labor markets. They especially show how this results in disparities of regional unemployment rates. Francis (2009) extends their model to cover endogenous job-destruction. Still, Epifani and Gancia (2005) cover only frictions in job-matching and conclude that further research needs to discuss frictions in wage setting within the New Economic Geography.

Südekum (2005) in turn introduces frictions in wage setting based on efficiency wages into an agglomeration model. Nevertheless, since he focuses exclusively on centripetal forces, full agglomeration is prevented only by relying on a home bias for regional migration. He therefore does not refer to his model as a New Economic Geography model, due to the omission of centrifugal forces.

Egger and Seidel (2008) combine a wage curve based on a fair-wages-approach with the New Economic Geography. In their model the work effort of low qualified is influenced by the fairness of their wages. This leads to a link between wages and unemployment. However, only low qualified may become unemployed. Furthermore, low qualified are (in contrast to high qualified) inter-regionally immobile. In addition, the wage of low qualified is fixed to one in both regions. Thus a wage curve exists only in the sense that the unemployment rate of low qualified is linked to the wage of high qualified.

The present model instead introduces a wage curve based on efficiency wages into the New Economic Geography in order to discuss frictions in wage setting and consequences for regional labor market disparities. In contrast to Egger and Seidel the present model shows a link between wage and unemployment of the same labor market group and unemployed migrate between the regions.

In addition, the present model discusses regional labor market disparities with a more universal form of agglomeration patterns, since three different patterns are unified in a single model. Labor market disparities might arise in the form of a “bang bang” or “bubble shaped” agglomeration pattern, or in an intermediate form comparable to Epifani and Gancia (2005). Whether the agglomeration pattern is catastrophic or smooth depends on congestion costs. The agglomeration pattern is accompanied by corresponding disparities in regional labor markets.

The rest of the paper is organized as follows: In section 2 the basic setup of the model is presented. The equilibrium and its dependency on migration are discussed in Section 3. Section 4 deals with the stability of the equilibria. In section 5 the results are interpreted. Conclusions are drawn in the final chapter.

## 2 Basic Model

The present paper develops a New Economic Geography model with labor market rigidities based on efficiency wages (and thus a wage curve). The model is constructed to discuss how disparities of regional labor markets endogenously arise. The New Economic Geography part of this model is depicted from Fujita et al. (1999). Their household model is extended to disutility of work effort which is basic for modeling efficiency wages. Efficiency wages are based on the approach of Shapiro and Stiglitz (1984). The goods market in turn is based on Fujita et al. (1999). However, the assumption of full employment is dropped and unemployment results as a consequence of efficiency wages.

There exist two regions,  $r$  and  $s$ , as well as two sectors, agriculture  $A$  and manufacturing  $M$ . Agriculture is characterized by perfect competition on both, goods and labor market. Manufacturing instead is characterized by monopolistic competition on the goods market and efficiency wages on the labor market. Labor is inter-sectoral immobile but labor in manufacturing is interregional mobile. Labor in agriculture is interregional immobile.

### 2.1 Households

Households live in regions  $r$  and  $s$ . They receive utility by the consumption of agricultural goods  $C_A$  and by the consumption of manufacturing goods.  $C_M$  represents a composite index of manufacturing goods. Utility is lowered by work effort  $e$ . Furthermore congestion costs  $H$  influence utility.

$$U = H \left( C_M^\mu C_A^{1-\mu} - e \right) \quad (1)$$

The congestion costs are modeled similar to Ricci (1999). That is, given the share of the manufacturing labor force  $\lambda$  in region  $r$ , the congestion cost  $H$  for regions  $r$  and  $s$  are:

$$H_r = h^{1-\frac{\lambda}{1-\lambda}} \quad (2)$$

$$H_s = h^{1-\frac{1-\lambda}{\lambda}} \quad (3)$$

Here,  $h$  is a congestion costs parameter. For  $h > 1$  increasing agglomeration in one region decreases utility of households in the agglomeration, but increases utility of households in the other region since the labor force is fixed at the national level. Agglomeration occurs only through the division  $\lambda$  of the fixed manufacturing labor force between the regions. Congestion costs are

therefore introduced in an analogy to the iceberg transportation costs. Both are constructed in a rather simple fashion in order to keep track of how the agglomeration pattern changes when transport costs and congestion costs change. The congestion costs can be interpreted as a local fix supply of a good such as land or housing with positive and decreasing marginal utility.

The composite index of manufacturing goods is a CES utility function:

$$C_M = \left[ \int_0^m C_i^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \quad (4)$$

The number of firms is given by  $m$  and the elasticity of substitution between the varieties of the manufacturing goods is  $\theta > 1$ . Households maximize their utility in two stages. They decide upon the optimum division of their income on agricultural and manufacturing goods. In addition they decide upon the optimum composition of the varieties of the manufacturing good. The budget constraint of household  $j$  is:

$$GC_{Mj} + P_A C_{Aj} = I_j \quad (5)$$

Household  $j$  uses all of her income  $I_j$  for consumption of agricultural goods  $C_A$  at price  $P_A = 1$  and for consumption of the composite index of manufacturing goods  $C_M$  at price index  $G$ . Due to the standardization  $P_A = 1$ , the prices of the manufacturing goods (and all wages) are measured relative to agricultural prices. Inserting the budget constraint into the utility function delivers:

$$U_j = H \left( C_{Mj}^\mu (I_j - GC_{Mj})^{1-\mu} - e \right) \quad (6)$$

Utility maximization ( $\partial U_j / \partial C_{Mj} = 0$ ) leads to the consumption expenditure shares of agricultural and manufacturing goods in income. Note that the result of the utility maximization is independent of congestion costs and work effort:

$$C_{Mj} = \mu \frac{I_j}{G} \quad (7)$$

$$C_{Aj} = (1 - \mu) I_j \quad (8)$$

The optimum division of expenditures for manufacturing goods on the individual varieties results from utility maximization over the varieties. This is equal to minimizing the expenditures for the varieties (Shepards Lemma):

$$\min \int_0^m P_i C_i di \text{ s.t. } \left[ \int_0^m C_i^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} = C_M \quad (9)$$

This leads to the CES manufacturing price index  $G$ :

$$G = \left[ \int_0^m P_i^{1-\theta} di \right]^{\frac{1}{1-\theta}} \quad (10)$$

In the two-region-case with identical firms and iceberg transport costs  $\tau \geq 1$  this leads to the manufacturing goods price index  $G_r$  in region  $r$ :

$$G_r = \left[ m_r P_r^{1-\theta} + m_s (\tau P_s)^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (11)$$

whereas  $m_r$  and  $m_s$  represent the number of firms (=varieties) in the corresponding region. The demand for variety  $i$  by household  $j$  follows from minimizing expenditures for the varieties:

$$C_{ij} = \left( \frac{P_i}{G} \right)^{-\theta} C_{Mj} = \left( \frac{P_i}{G} \right)^{-\theta} \mu \frac{I_j}{P} \quad (12)$$

## 2.2 Labor Market

The labor market is modeled within the efficiency wage framework of Shapiro and Stiglitz (1984). Here the derivation of their wage curve is based on Zenou and Smith (1995).<sup>2</sup> Only the derivation of the wage curve for region  $r$  is presented. The wage curve for region  $s$  follows analogously. Employees (which are equal to households) receive utility  $v(w_r)$  through the wage  $w_r$  in region  $r$ , which is equal to the household income  $I_j$ . The income of unemployed households is zero. Households suffer from disutility of work effort  $e$  and their utility is lowered by congestion cost factor  $H_r$ , analogous to the utility function (1):

$$v(w_r) = H_r C_M^\mu C_A^{1-\mu} \quad (13)$$

Thus, the term  $v(w_r)$  is only a means to abbreviate the derivation of the wage curve. Due to disutility of work effort, employees have an incentive to shirk and hence to avoid work effort. The utility of a non-shirking employee in region  $r$  is  $U_r^{ns} = v(w) - H_r e$ , the utility of a shirking employee is  $U_r^s = v(w)$  and unemployed do not receive any utility  $U_r^u = 0$ . The employment status of the households are subject to a time-homogeneous Markov process with status 0 for unemployed and status 1 for employed. The transition probabilities  $P_t(i, j)$  at time  $t$  depend on the current status, the endogenous job generation rate  $\delta_r$ , the exogenous job destruction rate  $\psi$

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<sup>2</sup>Zenou and Smith (1995) construct a two-city-model with intra- and inter-city migration. They derive the Shapiro and Stiglitz (1984) efficiency wage model by using Markov processes to model the transitions from and to unemployment. For the purpose of the present model the inter-city migration is adopted to the two-region case.

	Description
$C_A$	consumption of agricultural goods
$C_M$	composite index of manufacturing goods
$C_i$	consumption of manufacturing good $i$
$e$	disutility of work effort
$G$	CES manufacturing price index
$H$	congestion costs
$h$	congestion costs factor
$I$	income
$L_{A,M}$	agricultural/manufacturing employees
$M$	difference in expected life-time utilities
$m$	number of manufacturing firms (varieties)
$N_{A,M}$	agricultural/manufacturing labor force
$P_A$	price of agricultural goods
$P_i$	price of manufacturing good $i$
$P_t(i, j)$	transition probability from status $i$ to status $j$ at time $t$
$q_i$	manufacturing output of firm $i$
$()_{r,s}$	regions $r$ and $s$
$s$	additional labor input due to shirking
$U_j$	utility of household $j$
$U^{ns}, U^s, U^u$	utility of (non-)shirking employees / unemployed
$U_{r,s}$	unemployment rate in region $r$ ( $s$ )
$v(w)$	utility resulting from the wage
$V^{ns}, V^s, V^u$	expected life-time utility of (non-)shirking employees / unemployed
$w$	wage rate
$\beta$	fix labor input
$1 - \gamma$	detection probability of shirking
$\delta$	endogenous job creation rate
$\theta$	elasticity of substitution between manufacturing varieties
$\lambda$	share of manufacturing employees in region $r$
$\mu$	expenditure share of manufacturing goods
$\pi$	yield/profit
$\rho$	discount rate of utility
$\tau$	transport costs
$\phi$	variable labor input
$\psi$	exogenous job destruction rate

Table 1: List of Variables and Parameters

and the detection probability of shirking  $1 - \gamma$ .<sup>3</sup> The transition probabilities of non-shirking and shirking employees are then described by (see Zenou and Smith (1995) for a detailed derivation of this result):

$$P_{t,r}^{ns}(0, 1) = \frac{\delta_r}{\psi + \delta_r} - \frac{\delta_r}{\psi + \delta_r} e^{-t(\delta_r + \psi)} \quad (14)$$

$$P_{t,r}^{ns}(1, 1) = \frac{\delta_r}{\psi + \delta_r} - \frac{\psi}{\psi + \delta_r} e^{-t(\delta_r + \psi)} \quad (15)$$

$$P_{t,r}^s(0, 1) = \frac{\delta_r}{\psi + \delta_r + 1 - \gamma} - \frac{\delta_r + 1 - \gamma}{\psi + \delta_r + 1 - \gamma} e^{-t(\delta_r + \psi + 1 - \gamma)} \quad (16)$$

$$P_{t,r}^s(1, 1) = \frac{\delta_r}{\psi + \delta_r + 1 - \gamma} - \frac{\psi + 1 - \gamma}{\psi + \delta_r + 1 - \gamma} e^{-t(\delta_r + \psi + 1 - \gamma)} \quad (17)$$

The parameter  $\rho$  is the discount rate of utility. The lifetime utilities of shirking and non-shirking employees are then derived similar to Zenou and Smith (1995):

$$\begin{aligned} V_r^{ns} &= \int_0^\infty [P_t^{ns}(1, 0)U_r^u + P_t^{ns}(1, 1)U_r^{ns}] e^{-\rho t} dt \\ &= \frac{(\delta_r + \rho)U_r^{ns} + \psi U_r^u}{\rho(\delta_r + \psi + \rho)} \\ &= \frac{(\delta_r + \rho)(v(w_r) - H_r e)}{\rho(\delta_r + \psi + \rho)} \end{aligned} \quad (18)$$

$$\begin{aligned} V_r^s &= \int_0^\infty [P_t^s(1, 0)U_r^u + P_t^s(1, 1)U_r^s] e^{-\rho t} dt \\ &= \frac{(\delta_r + \rho)U_r^s + (\psi + 1 - \gamma)U_r^u}{\rho(\delta_r + \psi + \rho + 1 - \gamma)} \\ &= \frac{(\delta_r + \rho)(v(w_r))}{\rho(\delta_r + \psi + \rho + 1 - \gamma)} \end{aligned} \quad (19)$$

The labor market equilibrium is given by two conditions. First, employers pay efficiency wages to prevent shirking at the margin. Therefore the wages are set at the level required to equalize utilities of shirking and non-shirking employees ( $V_r^{ns} = V_r^s$ ):

$$v(w_r) = H_r e \frac{\rho + \psi + \delta_r + 1 - \gamma}{1 - \gamma} \quad (20)$$

Second, in equilibrium the inflow to unemployment  $\psi L_{Mr}$  is equal to the outflow of unemployment  $\delta(N_{Mr} - L_{Mr})$  (where  $L_{Mr}$  is the number of manufacturing employees and  $N_{Mr}$  is the

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<sup>3</sup>Those employees who are detected shirking at work are laid off.

manufacturing labor force in region  $r$ ). Therefore the endogenous rate of job creation is given by:

$$\delta_r = \psi L_{Mr} / (N_{Mr} - L_{Mr}) \quad (21)$$

Taking into account the definition for  $v(w_r)$  and the definition of the unemployment rate  $U_r = L_{Mr} / (N_{Mr} - L_{Mr})$  delivers the wage curve for region  $r$  (the wage curve for region  $s$  is constructed in the same way).

$$w_r = \frac{G_r^\mu}{K} e \left[ 1 + \frac{\rho}{1 - \gamma} + \frac{\psi}{(1 - \gamma)U_r} \right] \text{ where } K = \left( \frac{1 - \mu}{\mu} \right)^{1 - \mu} \mu \quad (22)$$

Equation (22) directly links the wage to the unemployment rate and represents the wage curve resulting from efficiency wages. It represents the wage firms pay in order to prevent shirking at the margin.

Now, migration takes place. Individuals who migrate are unemployed in the immigration-region at first, due to search unemployment. An individual decides to migrate when her expected life-time utility as an unemployed is larger abroad than in the current status at home. However, to monitor whether migration takes place it is sufficient to compare expected life-time utilities of unemployed in both regions. The reason is that the expected life-time utility of employees is always larger than that of unemployed:  $V_r^{ns} > V_r^u$ . Therefore, migration takes place when  $V_r^u < V_s^u$ . This is true as long as we are interested in whether someone migrates instead of who (employees or unemployed) migrates. For observing migration we therefore compare expected life-time utilities of unemployed in both regions. The expected life-time utility of an unemployed in region  $r$  is given by:<sup>4</sup>

$$\begin{aligned} V_r^u &= \int_0^\infty [P_t^{ns}(0, 0)U_r^u + P_t^{ns}(0, 1)U_r^{ns}] e^{-\rho t} dt \\ &= \frac{\delta_r U_r^{ns} + (\psi + \rho)U_r^u}{\rho(\delta_r + \psi + \rho)} \\ &= \frac{\delta_r(v(w_r) - H_r e)}{\rho(\delta_r + \psi + \rho)} \end{aligned} \quad (23)$$

Taking account of the definition for  $v(w_r)$  delivers:

$$\rho V_r^u = \frac{\delta_r}{\rho + \psi + \delta_r} H_r \left[ w_r \frac{K}{G_r^\mu} - e \right] \quad (24)$$

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<sup>4</sup>See Zenou and Smith (1995).

Emigration (immigration) takes place when the expected life-time utility of unemployed in the neighboring region is larger (lower) than the expected life-time utility of unemployed in the home region.

### 2.3 Goods Market

The goods market is based on the core-periphery model of Fujita et al. (1999) and is separated into agriculture and manufacturing. The agricultural sector produces a homogeneous good under perfect competition, and trade between regions is free and costless, i.e. a single price results. The labor market of the agricultural sector is characterized by perfect competition as well, leading to full employment. Labor input  $L_A$  and output in agriculture  $C_A$  are linked through the production function  $C_A = L_A$ . Due to marginal productivity payment in the agricultural labor market, the price of agricultural goods is equal to 1:  $P_A = \partial C_A / \partial L_A = w_A = 1$ . Prices and wages in agriculture are fixed to 1 and serve as reference for prices and wages in manufacturing.

Firms in manufacturing instead produce under increasing returns to scale and monopolistic competition. There is trade of manufacturing goods at iceberg transport costs  $\tau$ . The production function in manufacturing is:

$$L_{Mi} = \beta + \phi q_i + s_i \quad (25)$$

For production firm  $i$  needs a fixed labor input  $\beta$ , a variable labor input  $\phi$  per unit of output  $q_i$  and an additional labor input  $s_i$  due to shirking employees. Labor demand  $L_{Mi}$  of firm  $i$  is the sum of these three components. Due to efficiency wages there is no shirking and hence no additional labor input is needed:  $s_i = 0$ . Since there is love for variety, no combination of firms exists producing the same variety. The yield of firm  $i$  is given by:

$$\pi_i = P_i q_i - w_r (\beta + \phi q_i) \quad (26)$$

Firms maximize their profit through prices and ignore their influence on the price index  $G$ . This leads to the price setting rule for the regional price  $P_r$  which is identical for all firms of a region (due to identical wages within a region and due to identical firms):

$$\frac{\partial \pi_i}{\partial P_i} = 0 \Rightarrow P_r = \frac{\theta}{\theta - 1} w_r \phi \quad (27)$$

The number of firms is endogenous. New firms enter the market until the profits decrease to zero (zero-profit condition):

$$\pi_i = \frac{\theta}{\theta - 1} w_r \phi q_i - w_r \left( \beta \frac{\theta - 1}{\theta} \phi q_i \right) = 0 \Rightarrow 0 = w_r \left( \frac{\phi q_i}{\theta - 1} - \beta \right) \quad (28)$$

Then production and employment of a firm in equilibrium are given by:

$$q_i = \frac{\beta(\theta - 1)}{\phi} \quad (29)$$

$$L_{Mi} = \beta + \phi \frac{\beta(\theta - 1)}{\phi} = \beta\theta \quad (30)$$

Production and employment per firm in equilibrium are constant and equal for all firms irrespective of their region. This leads to the number of firms in a region:

$$m_r = L_{Mr} / L_{Mi} = L_{Mr} / (\beta\theta) \quad (31)$$

The labor input per unit of output is standardized to  $\phi \equiv (\theta - 1)/\theta$ , so that price and production reduce to:

$$P_r = w_r \quad (32)$$

$$q_i = \theta\beta = L_{Mi} \quad (33)$$

The fix labor input is standardized to  $\beta \equiv \mu/\theta$ . Then the number of firms (=varieties) in a region as well as the production of a firm are given by:

$$m_r = L_{Mr} / \mu \quad (34)$$

$$q_i = L_{Mi} = \mu \quad (35)$$

In equilibrium the production of a firm is equal to the sum of regional demand for the variety of the firm and import demand of the neighboring region for the firms variety (taking into account iceberg transport costs  $\tau$ ).<sup>5</sup>

$$q_i = \mu I_r P_r^{-\theta} G_r^{\theta-1} + \mu I_s (\tau P_r)^{-\theta} G_s^{\theta-1} \tau \quad (36)$$

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<sup>5</sup>If one unit of the manufacturing good is transferred to the neighboring region, only  $1/\tau$  units arrive. Therefore  $\tau$  units have to be sent when one unit shall arrive.

$$w_r = \left[ I_r G_r^{\theta-1} + I_s G_s^{\theta-1} \tau^{1-\theta} \right]^{\frac{1}{\theta}} \quad (37)$$

The latter equation represents the goods market equilibrium in form of a price setting function. It represents the wage at which the condition of zero profits is fulfilled and no firms enter or leave the market. For lower wages, the profit of an additional firm is greater than zero so that new firms enter the market. This results in increasing employment, decreasing unemployment and increasing shirking. To prevent shirking, firms increase wages (wage curve). This process continues until the wage fulfilling the zero-profit condition is equal to the wage preventing shirking.

### 3 Equilibrium and Migration

The simultaneous equilibrium in both regions is defined by the price indexes, price setting functions, incomes and wage curves of both regions (only equations for region  $r$  are presented, equations for region  $s$  are constructed analogous):

$$G_r = \left[ \frac{1}{\mu} \left( L_{Mr} w_r^{1-\theta} + L_{Ms} (\tau w_s)^{1-\theta} \right) \right]^{\frac{1}{1-\theta}} \quad (38)$$

$$w_r = \left[ I_r G_r^{\theta-1} + I_s G_s^{\theta-1} \tau^{1-\theta} \right]^{\frac{1}{\theta}} \quad (39)$$

$$I_r = w_r L_{Mr} + L_{Ar} \quad (40)$$

$$w_r = \frac{G_r^\mu}{K} e^{\frac{\rho + \psi + \delta_r + 1 - \gamma}{1 - \gamma}} \quad (41)$$

From (38), it follows that the region with the larger number of manufacturing employees has a lower price index. This is because a larger number of manufacturing employees results in a larger number of varieties produced, increasing competition. Then the demand for any individual variety is lower, its price and corresponding revenues decrease, leading to a lower price index. Furthermore transport costs are lower in the agglomeration, which further reduces the price index in the agglomeration.

The price setting equation (39)<sup>6</sup> represents the wage (=price) at which firms reach their break-even point (i.e. where profits are zero). The higher incomes and prices and the lower transport costs are, the higher is this wage. Regions with a higher income have a higher purchasing power and the break-even point of firms lies at a higher wage. An increase of income in a region leads to a lower or higher increase of employment, depending on the wage elasticity of labor supply.

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<sup>6</sup>This equation is labeled "wage equation" by Fujita et al. (1999).

When the increase in employment is larger, centripetal forces dominate: A region that once manages to gain a higher income will be able to use this advantage for attracting new firms, income and demand, enforcing an agglomeration process. This process endogenously leads to agglomeration and regional disparities.

The region with the larger number of manufacturing employees thus has higher nominal wages (backward linkage) so that this region is more attractive for firms due to its higher purchasing power. This region is further characterized by a larger number of varieties and thus a lower price index and is therefore more attractive for immigration (forward linkage). These forward and backward linkages establish the centripetal forces leading to endogenous agglomeration. These are opposed to centrifugal forces resulting from the demand by the agricultural employees.

Equation eq:simGGincome defines the income in region  $r$  and (41) represents the wage curve, which is a key extension of this paper to the core-periphery model. The wage curve is the link between employment and wages, leading to unemployment. It represents the wage set by firms to prevent shirking.

For a compact illustration of the model the labor force (as a sum of agricultural and manufacturing labor force) is standardized to one. This labor force is separated into agriculture ( $N_A$ ) and manufacturing ( $N_M$ ) according to the expenditure shares of agricultural and manufacturing goods in income ( $\mu$ ). The agricultural labor force is equal in both regions whereas the labor force in manufacturing is divided between the regions according to  $\lambda$ . Due to full employment in agriculture the corresponding labor force is equal to employment in both regions ( $N_{Ar} = L_{Ar}$  and  $N_{As} = L_{As}$ ).

$$L_{Ar} = \frac{1 - \mu}{2} \quad (42)$$

$$L_{As} = \frac{1 - \mu}{2} \quad (43)$$

$$N_{Mr} = \mu\lambda \quad (44)$$

$$N_{Ms} = \mu(1 - \lambda) \quad (45)$$

The simultaneous equilibrium in the short term depends on the parameters disutility of work effort ( $e$ ), probability to observe shirking ( $1 - \gamma$ ), job destruction rate ( $\psi$ ), share of expenditures for manufacturing ( $\mu$ ), elasticity of substitution between manufacturing goods varieties ( $\theta$ ) and discount rate ( $\rho$ ). The model is intractable and results are derived by simulation, which is standard practice in New Economic Geography.

In the long term unemployed are allowed to migrate between the regions. Unemployed compare

their expected utility in both regions and decide to migrate when their utility is higher in the neighboring region. Their utility depends on their real wages,<sup>7</sup> chances to find employment,<sup>8</sup> and congestion costs in both regions. The migration behavior is thus given by the difference in expected life-time utility of unemployed between both regions ( $M$ ) (based on equation (24)):<sup>9</sup>

$$M = \frac{\delta_r}{\rho + \psi + \delta_r} H_r \left[ w_r \frac{K}{G_r^\mu} - e \right] - \frac{\delta_s}{\rho + \psi + \delta_s} H_s \left[ w_s \frac{K}{G_s^\mu} - e \right] \quad (46)$$

$$\begin{aligned} \dot{\lambda} &> 0 \text{ for } M > 0 \\ \dot{\lambda} &= 0 \text{ for } M = 0 \\ \dot{\lambda} &< 0 \text{ for } M < 0 \end{aligned} \quad (47)$$

In case of symmetry ( $\lambda = 0.5$ ) there is no migration since the endogenous variables are equal in both regions. When there is no symmetry ( $\lambda \neq 0.5$ ), the endogenous variables can differ between both regions and migration might occur depending on these differences. For any given  $0 < \lambda < 1$ , a short term equilibrium exists. If the utility of unemployed differs between the regions in the short term equilibrium, unemployed migrate until a long-term equilibrium is reached where there is no incentive to migrate. In the long term equilibrium the expected utility of unemployed is equal in both regions and therefore there is no incentive to migrate.

Figure 1 displays the difference between the expected life-time utility of unemployed in region  $r$  minus the expected life-time utility of unemployed in region  $s$  ( $M$ ) for different constellation of the parameters (the parameter constellations of all figures are summarized in table 2). Qualitatively, three different situations can be compared. In situation A the expected life-time utility of unemployed is always lower in the larger region. Unemployed migrate back to the smaller region until the symmetrical simultaneous equilibrium in both regions is reached at  $\lambda = 0.5$ . Then, the symmetry is the only stable equilibrium. Differently, in situation B the expected life-time utility of unemployed is larger in the larger region for intermediate levels of  $\lambda$  and is smaller in the larger region for very small or very large  $\lambda$ . That is, once a region becomes larger than the other region, an advantage for this region results and an agglomeration process sets in.

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<sup>7</sup>The real wage in region  $r$  is:  $w_r \frac{K}{G_r^\mu}$

<sup>8</sup>The chance to find employment depends on the endogenous job creation rate which is closely linked to the unemployment rate.

<sup>9</sup>This definition of migration behavior is motivated by optimal migration decisions based on static expectations on the differences in real wages, unemployment and congestion costs between both regions (Baldwin et al.; 2003, Appendix 2.B.4). It further extends the underlying logic of the basic efficiency wage model to the migration-case: In the basic model the equilibrium is reached when the expected life-time utilities of shirking and non-shirking employees are equal. Analogously the long-term equilibrium is reached when the expected life-time utilities of unemployed in both regions are equal.

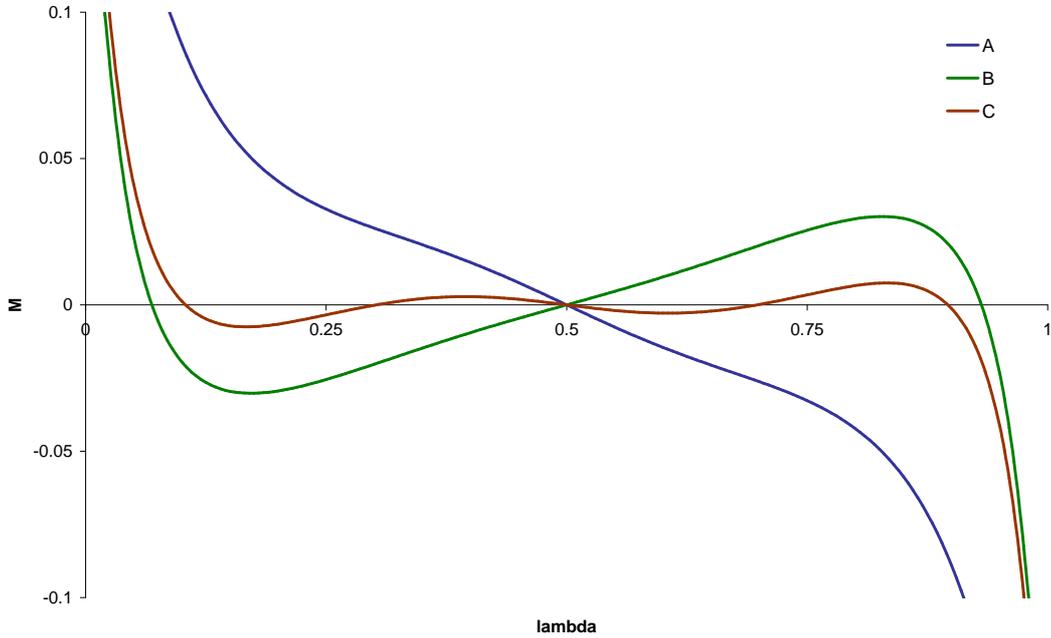


Figure 1: Equilibria and Migration

Nevertheless, the agglomeration does not attract all labor since large agglomerations suffer from congestion costs. Therefore in this situation there are two stable equilibria, both of which are agglomerations. The symmetry is instable. Finally, C three stable equilibria exist in situation C: Symmetry and agglomeration both may result, depending on at which  $\lambda$  the system starts.

## 4 Stability

Multiple equilibria exist and three basic situations arise. The stability characteristics of the system – illustrated by the above situations – depend on the transport costs  $\tau$  and the congestion costs  $h$ . As in Fujita et al. (1999) the stability characteristics of the system can be described by the break- and sustain-points. However, since these points now depend on both, transport costs  $\tau$  and congestion costs  $h$ , the stability characteristics are more complex. As illustrated by Figure 1, both symmetry and agglomeration might be stable or instable, depending on transport costs and congestion costs. The break-point then describes the point where the symmetry

Figure	$\mu$	$\theta$	$\tau$	$e$	$\rho$	$\psi$	$\gamma$	$h$	$\lambda$
1 A	0.6	4	3.2	0.2	0.05	0.3	0.2	1.12	/
1 B	0.6	4	2.5	0.2	0.05	0.3	0.2	1.04	/
1 C	0.6	4	3.2	0.2	0.05	0.3	0.2	1.04	/
2	0.6	4	/	0.2	0.05	0.3	0.2	1.04	0.5
3	0.6	4	2	0.2	0.05	0.3	0.2	/	0.5
4	0.6	6	/	0.2	0.05	0.3	0.2	1	1
5	0.6	4	/	0.2	0.05	0.3	0.2	/	/
6	0.6	4	/	0.2	0.05	0.3	0.2	/	/
7	0.6	4	/	0.2	0.05	0.3	0.2	/	/

Table 2: Parameter constellations of the figures

changes from being instable to being stable, whereas the sustain-point describes the point where the agglomeration changes from being instable to being stable, both with regard to changes in transport costs and congestion costs.

## 4.1 Break-Point

The break-point describes the situation, where a change in transport or congestion costs leads to the symmetric equilibrium changing from instable to stable (or vice versa). To illustrate this point, consider the following: If a marginal deviation from symmetry (i.e. a marginal increase/decrease of  $\lambda$ ) leads to a larger expected life-time utility of unemployed in the marginally larger region, then symmetry is instable and an agglomeration endogenously arises. Hence the derivative of the difference in utility  $M$  against  $\lambda$  is larger than zero. In contrast, if this derivative is smaller than zero, symmetry is stable. In this case the break-point lies exactly at that symmetrical equilibrium where the derivative of the differences in utilities of unemployed is zero ( $dM = 0$ ).

To calculate this point,  $\lambda$  is set to 0.5 and we can utilize the fact that all endogenous variables are equal in both regions. Furthermore the change of one variable in a region is equal to the negative change of the same variable on the other region. Therefore the system can be expressed in units of region  $r$  and the index for regions is dropped. Consequently the break-point is expressed by the system of equations (48) to (58):

$$G = \left[ \frac{1 + \tau^{1-\theta}}{\mu} (L_M w^{1-\theta}) \right]^{1/(1-\theta)} \quad (48)$$

$$w = \left[ (1 + \tau^{1-\theta}) I G^{\theta-1} \right]^{1/\theta} \quad (49)$$

$$I = wL_M + L_A \quad (50)$$

$$w = \frac{G^\mu}{K} e^{\frac{\rho + \psi + \delta + 1 - \gamma}{1 - \gamma}} \quad (51)$$

$$\delta = \frac{RL_M}{\lambda N - L_M} \quad (52)$$

$$dG = \frac{G^\theta(1 - \tau^{1-\theta})}{\mu} \left[ \frac{w^{1-\theta}}{1 - \theta} dL_M + L_M w^{-\theta} dw \right] \quad (53)$$

$$dw = \frac{w^{1-\theta} G^{\theta-1} (1 - \tau^{1-\theta})}{\theta} \left[ dI + (\theta - 1) I \frac{dG}{G} \right] \quad (54)$$

$$dI = wdL_M + L_M dw \quad (55)$$

$$dw = \mu \frac{w}{G} dG + \frac{G^\mu}{K} \frac{e}{1 - \gamma} d\delta \quad (56)$$

$$d\delta = \frac{\psi}{\mu \lambda N - L_M} dL_M + \frac{\psi L_M}{(\mu \lambda N - L)^2} (\mu N d\lambda - dL_M) \quad (57)$$

$$\begin{aligned} \frac{dM}{2} = 0 &= \frac{\delta}{\rho + \psi + \delta} \left[ \left[ w \frac{K}{G^\mu} - e \right] dH + \frac{HK}{G^\mu} dw - \frac{HwK}{G^{1+\mu}} dG \right] \\ &+ \left( \frac{1}{\rho + \psi + \delta} - \frac{\delta}{(\rho + \psi + \delta)^2} \right) H \left[ \frac{wK}{G^\mu} - e \right] d\delta \end{aligned} \quad (58)$$

By finding the solution to this system we can identify all combinations of  $\tau$  and  $h$ , where a break-point exists. Assume the congestion cost factor  $h$  is given. Then figure 2 illustrates the behavior of the symmetry for changes in  $\tau$ . For  $h > 1$ , we see that  $dM < 0$  for very small  $\tau$ . That is, a marginal deviation of the system from symmetry goes along with incentives to migrate back into the symmetry and symmetry is stable. For further increases in  $\tau$ , this situation flips and emigration out of the symmetry leads to a self reinforcing agglomeration process since emigration is accompanied by gains in utility of the emigrating people. For small transport costs a marginal deviation from symmetry leads to a higher nominal wage and a smaller price index in the larger region. Therefore the real wage (unemployment rate) is higher (smaller) in the larger region and immigration into the larger region sets in. Symmetry then is instable. However, this is only true if the agglomeration advantages (higher real wages, lower unemployment) over-compensate the higher congestion costs in the agglomeration. For very low transport costs the agglomeration advantages are too small and symmetry is stable.

For large  $\tau$  in turn, the symmetry again becomes stable. When transport costs are large, a marginal deviation from symmetry does not allow the marginally larger region to export its

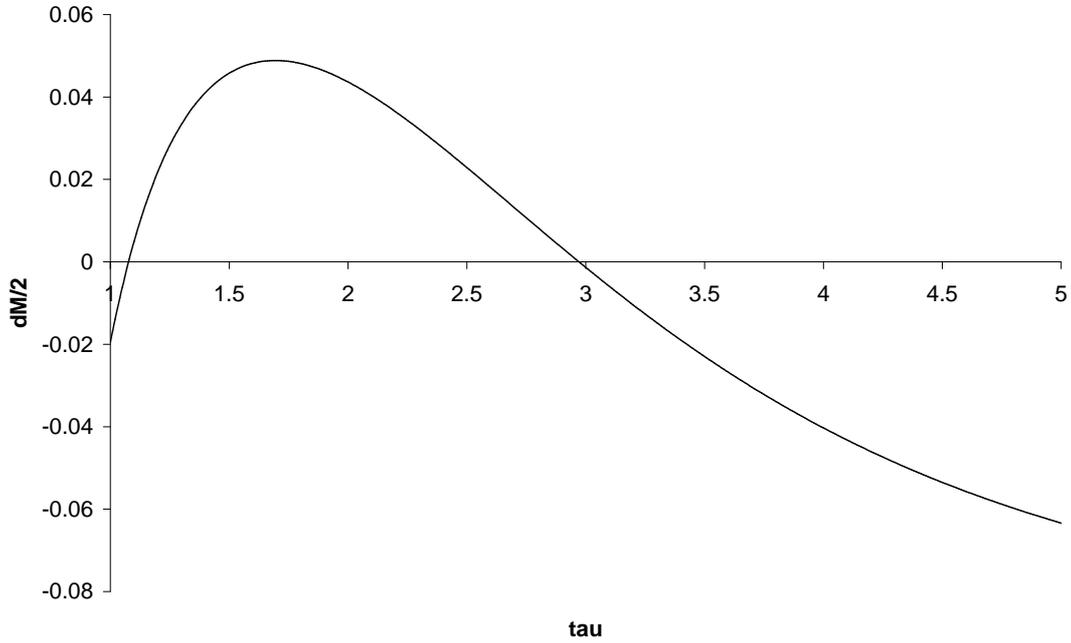


Figure 2: Migration at symmetry for different  $\tau$

production due to high transport costs. The effect of the larger manufacturing employment in the larger region on income in that region cannot offset the negative effects of higher congestion costs in the agglomeration and the negative effects of decreased manufacturing employment on income and import demand in the smaller region. The utility of unemployed is therefore smaller in the agglomeration and re-immigration into the smaller region sets in, resulting in symmetry. Thus, there exist two break-points at very low and at large transport costs for any given level of  $h$ .

If we instead focus on the congestion costs  $h$  and assume fix transport costs  $\tau$ , there only exists one break-point. Figure 3 illustrates the behavior of the symmetry for changes in  $h$ . We see that for very low congestion costs  $h$  emigration to the marginally larger region is accompanied by a gain in utility. Then the symmetry is instable. However, when congestion costs increase further, symmetry becomes stable since the agglomeration advantages are offset by congestion costs.

Figure 5 combines the information gathered so far: For all transport costs  $\tau$  and congestion costs  $h$  the break-point-line illustrates where the break-points lie. Outside the field defined by

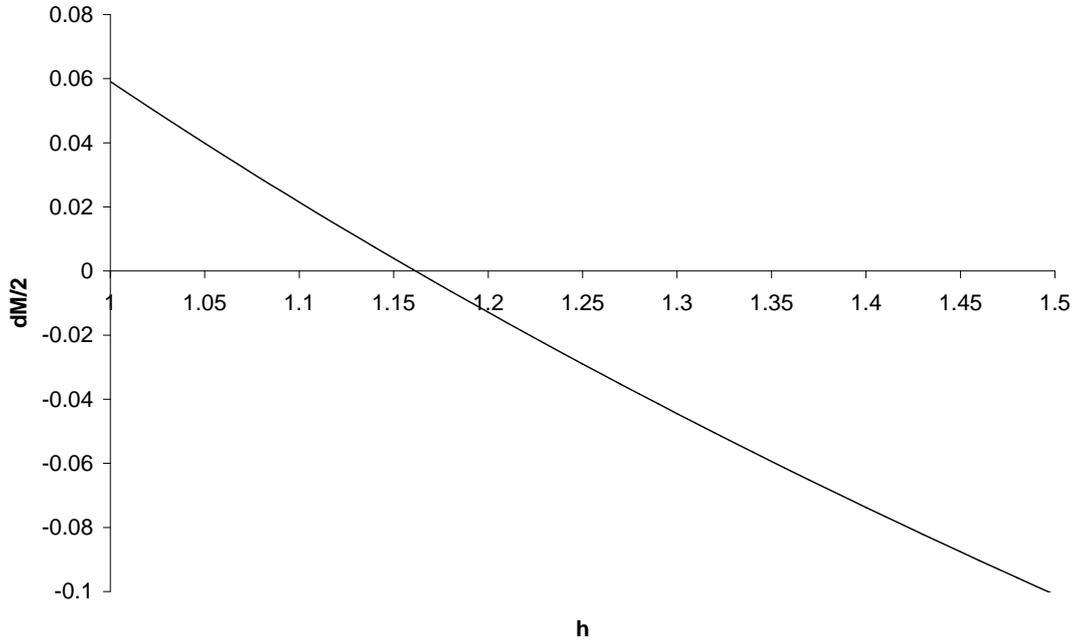


Figure 3: Migration at symmetry for different  $h$

this curve and the line  $h = 1$ , symmetry is stable – and accordingly symmetry is instable inside this field.

## 4.2 Sustain-Point

Similar to the break-point, the sustain-point describes the situation, where a change in transport or congestion costs results in a change of the situation where the agglomeration is instable to a situation where the agglomeration is stable (or vice versa). To illustrate this point, consider the following: If a marginal deviation from agglomeration (i.e. a marginal increase/decrease of  $\lambda$ ) leads to a larger expected life-time utility of unemployed in the periphery, then the agglomeration is instable and collapses. Then the derivative of the differences in utility ( $dM$ ) against  $\lambda$  is smaller than zero (assuming that  $r$  is the agglomeration and thus  $\lambda > 0.5$ ). In contrast, if this derivative is larger than zero, agglomeration is stable. The sustain-point then lies exactly at that agglomeration where the derivative of the differences in expected life-time utilities of unemployed

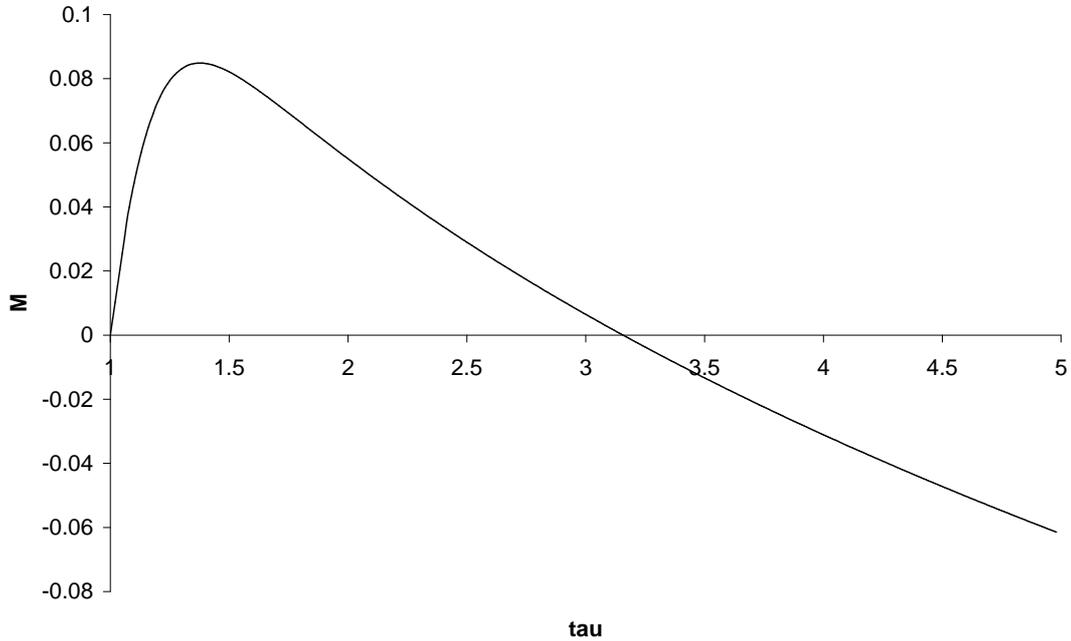


Figure 4: Migration at full agglomeration for different  $\tau$

is zero. Consider figure 1: In situation C there is a local maximum at  $\lambda > 0.5$  and to the right of this maximum there is a stable agglomeration. The sustain-point then is the situation, where this maximum is tangent to the line  $M = 0$ .

It is apparent that in contrast to the break-point the situation is now more complex since  $\lambda$  is not given in advance (by  $\lambda = 0.5$ ) but rather is endogenous. The sustain-point is defined by the situation where the derivative of the difference in utilities  $dM$  against  $\lambda$  is zero and where at the same time the difference in utilities  $M$  is zero, too. These two conditions are always fulfilled at the break-point. However, these two conditions are fulfilled for  $\lambda \neq 0.5$  for some constellations of  $\tau$  and  $h$  (we will discuss the interpretation of this in more detail in the subsequent chapter). A mathematical definition of this point is attached to the appendix.

To understand the behavior of the system around the sustain-point, it is helpful to consider the case of  $h = 1$ . When  $h = 1$ , no congestion costs exist and if there is a stable agglomeration, this is always a full agglomeration (i.e.  $\lambda = 0$  or  $\lambda = 1$ ). In this case it is sufficient to monitor the difference in utility ( $M$ ) for different  $\tau$ , which is done in figure 4.

When there is full agglomeration, increasing transport cost lead to an increase of the price index in the periphery and the periphery becomes less attractive (the agglomeration becomes more stable). However, at the same time increasing transport costs lead to a decrease in the level of the wage at which firms reach their break-even point in the periphery. Thus, the periphery becomes more attractive for firms when transport costs increase (the agglomeration pattern becomes more instable). For low transport costs the first effect dominates and the agglomeration becomes more stable. For high transport costs, the second effect dominates and the agglomeration pattern becomes instable. The net effect is illustrated in Figure 4 by showing the development of the difference in utility  $M$  against transport costs. Thus, agglomeration forces are strongest for intermediate transport costs and are low both, for large and for low transport costs. When congestion costs are larger than zero ( $h > 1$ ), then  $\lambda$  is endogenous and we cannot draw this figure, neither can we draw the difference in expected life-time utilities against congestion costs for the same reason. However, the characteristics of the agglomeration forces with regard to transport costs remain the same:  $h$  only influences the level of these forces (since it influences the migration decision, but not the goods market equilibrium, similar to the case of the break-point).

Since we know the conditions for the sustain-point, we can calculate all combinations of  $\tau$  and  $h$ , where there is a sustain point and arrange them on a map. This is done in figure 5. Inside the field defined by the sustain-point-curve and the line  $h = 1$ , the agglomeration is stable – and instable outside, accordingly.

### 4.3 Agglomeration Pattern

Combining the information on the break and sustain point delivers a picture of the systems behavior. This is done in figure 5. For any level of  $h$ , one can depict from figure 5 at which levels of  $\tau$  there are sustain- and break-points. Consider the case of Fujita et al. (1999), where there are no congestion costs so that  $h = 1$ . Then there is a break-point at  $\tau = 3.26$  and a sustain-point at  $\tau > 5$ . Additionally there is a simultaneous break- and sustain-point at  $\tau = 1$  - this is because for  $\tau = 1$  there are no transportation costs and the regions are not economically distinct (see Fujita et al. (1999) for a more detailed discussion).

From figure 5 it becomes obvious, that when  $\tau$  is large and decreases, then there is a level of  $h$  at which sustain- and break-point coincide. To understand what happens here we must go back to figure 1. Qualitatively there are three different situations. In A only symmetry is stable. In figure 5 this is the region outside the fields marked by the two curves and the ( $h = 1$ )-line. In situation B only the agglomeration is stable. This is the region inside the break-point-curve and

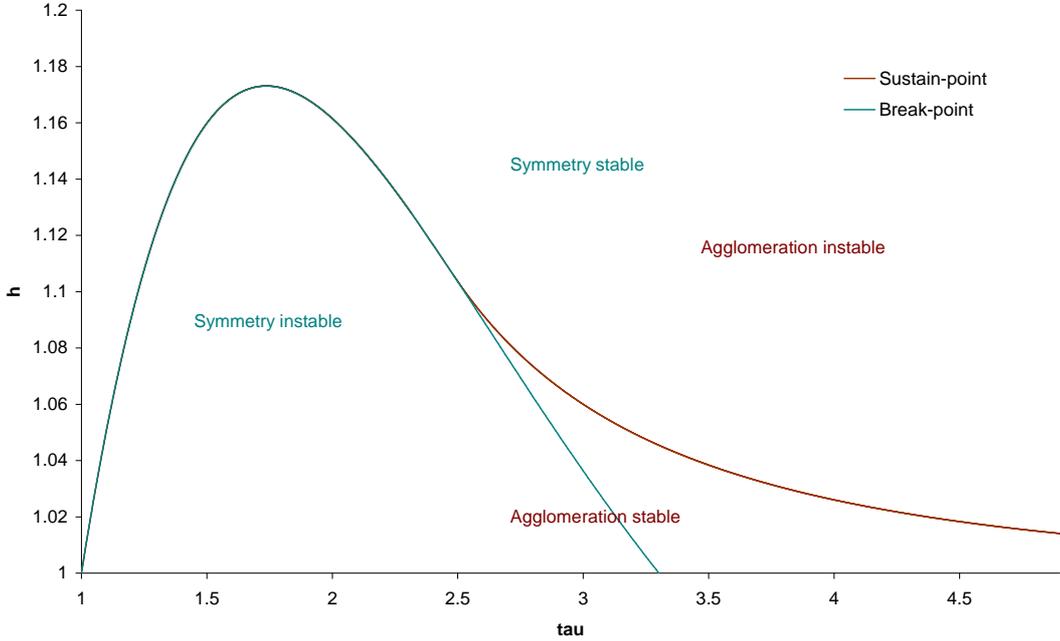


Figure 5: Sustain- and break-points in the  $\tau$ - $h$ -space

the ( $h = 1$ )-line of figure 5. Finally, in situation C both, break- and sustain-point are stable. This is the field marked by the sustain-point-line and the ( $h = 1$ )-line minus the field marked by the break-point-line and the ( $h = 1$ )-line.

In situation C there is a local maximum in figure 5, which is above the ( $M = 0$ )-line and thus enables both, agglomeration and symmetry to be stable. This situation arises when  $h$  is small and  $\tau$  is large (but not too large). At this level of transport costs (and congestion costs) agglomeration forces are strong enough to reinforce the agglomeration only when  $\lambda$  is large (or small) enough. When  $\lambda$  is close to 0.5, then agglomeration forces are not strong enough and symmetry is stable.

At the margin, this maximum is tangent to the ( $M = 0$ )-line, representing the sustain-point. Now, imagine how the transport costs decrease. To remain on the sustain-point-line, congestion costs need to increase (since for decreasing transport costs agglomeration forces increase). However, this means that the sustain-point in figure 1, which is the maximum of the curve of figure C as a tangent to the ( $M = 0$ )-line, moves towards the symmetrical equilibrium at  $\lambda = 0.5$ . This

means that the sustain-point approaches the break-point. Then only situations A and B can result near to this point, so that the sustain point becomes the break point at the margin.

To understand why this happens recall figures 2 and 4. When there are no transport costs ( $\tau = 1$ ), then an increase of the transport costs in figure 2 destabilizes the symmetry and at the same time stabilizes the agglomeration in figure 4 (since agglomeration forces increase). The slope of the M-curve in figure 1 then is monotonous for zero congestion costs. The stability of the system now depends on congestion costs only. An increase of the congestion costs leads to a monotonous increase of the agglomeration disadvantages (instead of a non-monotonous decrease as in the case of transport costs). Therefore the slope of the M-curve in figure 1 remains monotonous, but the sign of the slope depends on the congestion costs. Since the slope is monotonous, only symmetry *or* agglomeration can be stable.

However, when transport costs increase further the agglomeration advantages decrease in figure 2, but agglomeration remains stable for even larger values of  $\tau$  in figure 4. That is, the agglomeration forces are not monotonous in transportation costs anymore for sufficient large levels of  $\tau$ . In this case the strength of the agglomeration forces depend on the level of  $\lambda$  – agglomeration forces are only strong enough to reinforce agglomeration, when the agglomeration is large enough. Then the sustain-point is different from the break-point and the sustain- and break-point-lines in figure 5 divide from each other.

## 5 Interpretation and Discussion

By introducing unemployment and congestion costs into the model of Fujita et al. (1999), the agglomeration pattern changes considerably and conclusion for regional labor market disparities can be drawn since unemployment disparities arise. This section deals with the interpretation and discussion of these two issues.

With figure 5 in mind we can distinguish qualitatively four different agglomeration patterns for different levels of the congestion costs parameter  $h$ . (1) When  $h = 1$ , then there is one sustain- and one break-point (for  $\tau > 1$ ). (2) For low congestion costs an agglomeration pattern arises where agglomeration and symmetry might be stable simultaneously. (3) For intermediate congestion costs either agglomeration or symmetry is stable, but not both simultaneously. (4) Finally, for high congestion costs symmetry is always stable and agglomeration always instable. Case (4) is trivial. Case (1) has been analyzed by Zierahn (2011) in depth. Hence, cases (2) and (3) are of special interest here. Both cases are visualized as bifurcation diagrams for the share of manufacturing employees and for the difference in unemployment rates of both regions in figures 6 and 7.

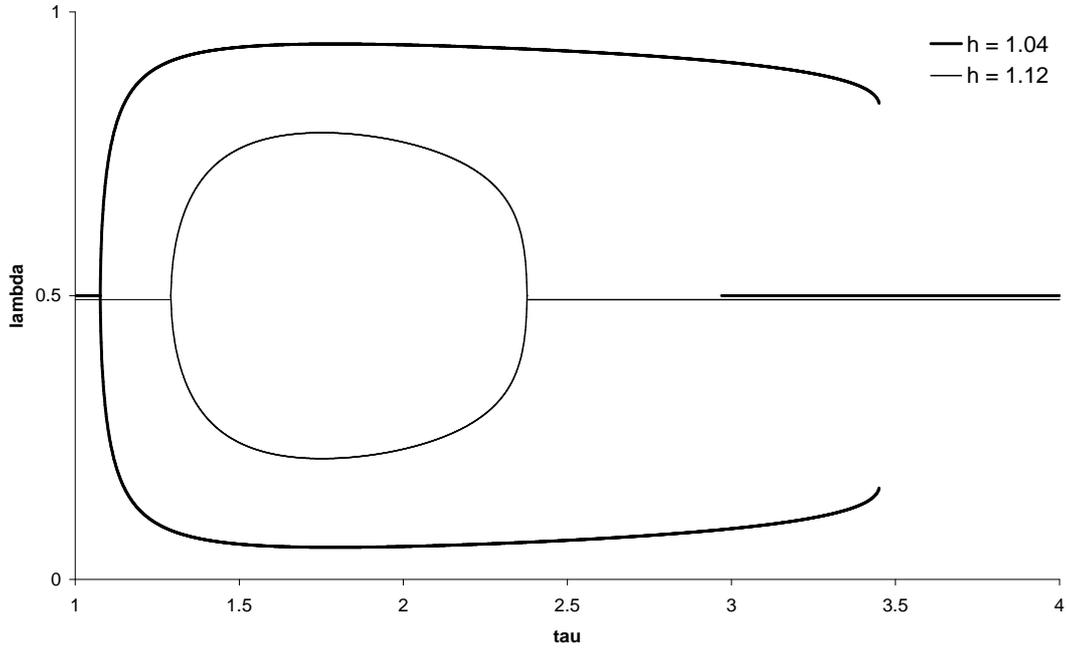


Figure 6: Bifurcation diagram for  $\lambda$

In case (2) there are low congestion costs (here:  $h = 1.04$ ). When transport costs are high, then only the symmetry is stable and there are no differences between the regions. However, when transport costs start to decline, a catastrophic agglomeration pattern arises: For a certain range of  $\tau$ , both – agglomeration and symmetry – are stable. Whether symmetry or agglomeration results, depends on the initial distribution of manufacturing employees. The agglomeration is marked by a smaller unemployment rate. Further decreases of the transportation costs lead to an even lower unemployment rate in the agglomeration compared to the periphery and at a certain level of  $\tau$ , symmetry becomes unstable. Agglomeration then is the only stable equilibrium. That is, for intermediate levels of transportation cost the previously described centripetal forces endogenously lead to agglomeration and result in unemployment disparities. When transportation costs decline further centripetal forces decline. Nevertheless, this does not lead to a catastrophic change of the agglomeration pattern but instead the agglomeration becomes smaller until both regions are equally large and symmetry results. This is because break- and sustain-point coincide. Simultaneously the unemployment rates converge.

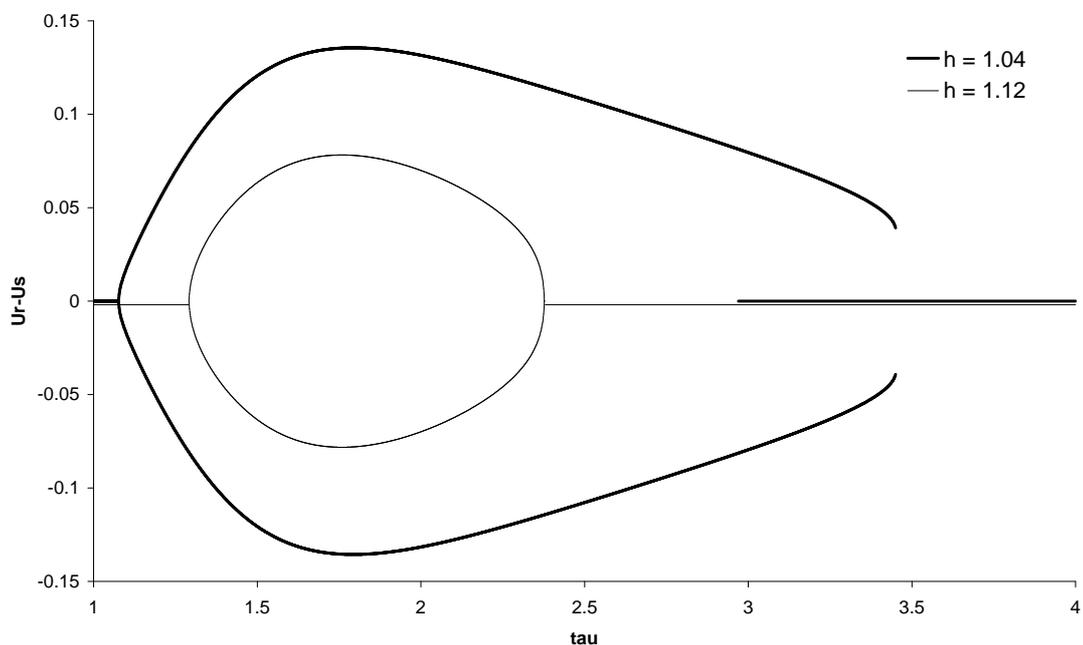


Figure 7: Bifurcation diagram for the difference in unemployment rates

In case (3) there are intermediate congestion costs (here:  $h = 1.12$ ). When transport costs are high, then only the symmetry is stable and there are no differences between the regions. However, when transport costs start to decline, symmetry becomes unstable and agglomeration results. This process is smooth, i.e. one of the regions becomes successively larger when transport costs decrease. This is accompanied by a decrease of the unemployment rate in the agglomeration relative to the unemployment rate in the periphery. There is no catastrophic change from symmetry to agglomeration since break- and sustain-point coincide. In a similar fashion further decreasing transport costs lead to a decline of the centrifugal forces and the agglomeration again approaches symmetry in a smooth process, both in terms of manufacturing employees and unemployment rates.

The results are comparable to those of Epifani and Gancia (2005). These authors base their regional labor market model on the model of Fujita et al. (1999) as well. Disparities in wages and unemployment endogenously arise through agglomeration in a similar way. However, they focus on labor market frictions in job matching processes and assume flexible wages, whereas

this paper focuses on frictions in wage setting. Thus, their approach and the model presented here complement each other by discussing different labor market frictions in the framework of the New Economic Geography. The present model further shows how the agglomeration pattern changes when congestion costs arise. Both, a catastrophic and a smooth agglomeration pattern might result and the transition between both pattern is smooth.

The results are further comparable to Südekum (2005). He discusses a wage curve based on efficiency wages within the framework of a regional goods market model. In contrast to the present paper, he exclusively focuses on centripetal forces to be able to solve the model analytically. In his model agglomeration patterns lead to higher wages and lower unemployment in the core compared to the periphery. Nevertheless, without any additional assumptions on migration full agglomeration necessarily results due to the lack of centrifugal forces. The present model instead discusses disparities of regional labor markets within the interplay of centrifugal and centripetal forces and is therefore able to distinguish under which circumstances disparities arise (or not) and how the agglomeration pattern depends on congestion and transport costs

## 6 Conclusions

Disparities of regional labor markets are a key characteristic in many countries. In the literature on the wage curve it is argued that there exists a negative relationship between wages and unemployment on regional level (Blanchflower and Oswald; 1994). However, this literature cannot explain how disparities in these variables endogenously arise. The New Economic Geography in turn explains how disparities of regional economies endogenously arise, but usually assumes full employment. The present model hence introduces efficiency wages into the New Economic Geography in order to explain how disparities of regional labor markets endogenously arise.

The literature on the New Economic Geography delivers a large number of different models. These have in common that they combine centrifugal and centripetal forces in order to explain under which circumstances agglomerations endogenously arise - or not. However, many different ideas and mechanisms underlying these two forces exist in the literature, delivering distinct models and different agglomeration pattern (described by bifurcation diagrams).

In order not to lean on a single agglomeration pattern but rather to rely on a more universal approach, the present paper comprises three different agglomeration patterns which arise out of the same model and the same underlying centrifugal and centripetal forces. Which of these patterns arises depends on the strength of congestion costs and the transition between these patterns is smooth with regard to changes in congestion costs.

First, an agglomeration pattern similar to the core-periphery-model of Krugman (1991) arises

when congestion costs are zero. This is a “bang bang” agglomeration pattern, as Pflüger and Südekum (2004) call it. That is, there is either full agglomeration or symmetry - depending on transport costs. Furthermore there is a certain range of transport costs where agglomeration and symmetry both are stable equilibria. Then a change in transport costs may lead to catastrophic behavior of the regional system. The implications of this situation for regional labor markets are discussed in detail by Zierahn (2011).

Second, when congestion costs are low (but positive), an intermediate pattern arises. In this case, when transport costs are large and decrease, there is a catastrophic change from symmetry to agglomeration: A certain range of transport costs exists where both agglomeration and symmetry are stable. Once the break-point is reached symmetry breaks down and one of the regions attracts a large fraction of economic activity. However, the agglomeration attracts not all economic activity due to congestion costs. When transport costs decrease further, the agglomeration shrinks and smoothly becomes the symmetry.

Third, when congestion costs are intermediate, a “bubble-shaped” (Pflüger and Südekum; 2004) agglomeration pattern might result. Then the transition of symmetry to agglomeration is smooth with regard to changes in transport costs. That is, when transport costs are high and decrease there is a point where symmetry breaks down and agglomeration arises. However, agglomeration only slowly becomes larger - instead of catastrophic in the “bang bang” pattern above. Further decreasing transport costs result in a shrinking agglomeration until symmetry returns.

A key result of the present model is that these three patterns arise through the same underlying centrifugal and centripetal forces and that the transition between these patterns is smooth with regard to changes of congestion costs.

Furthermore the present model is extended to cover unemployment. Whereas Epifani and Gancia (2005) introduce job-matching-frictions into the New Economic Geography, the present model rests upon frictions in wage-setting based on efficiency wages to cover unemployment in the New Economic Geography-framework. Similar to the model of Epifani and Gancia (2005) we observe higher wages and lower unemployment in the agglomeration compared to the periphery. However, the agglomeration pattern only is similar to their model when congestion costs are set at a low level (the second case above). Additionally to this pattern two other patterns might result. The model therefore shows how unemployment disparities arise in different agglomeration patterns.

## A Appendix: Definition of the sustain-point

The sustain-point is defined by the equilibrium conditions (38) to (41) for region  $r$  and for region  $s$  accordingly, the definition of the congestion costs (2) and (3), the no-migration-condition (46), and the derivative of the system against  $\lambda$  (only the derivative for region  $r$  is presented here, the equations for region  $s$  are calculated in the same way):

$$dG_r = \frac{G_r^\mu}{\mu} \left[ \frac{w_r^{1-\theta}}{1-\theta} dL_{Mr} + w_r^{-\theta} dw_r + \frac{(w_s \tau)^{1-\theta}}{1-\theta} dL_{Ms} + \frac{\tau^{1-\theta}}{w_s^\theta} dw_s \right] \quad (59)$$

$$dw_r = \left( \frac{w_r}{G_r} \right)^{1-\theta} \frac{1}{\theta} \left[ dI_r + (\theta - 1) \frac{I_r}{G_r} dG_r \right] + \left( \frac{w_r}{G_s} \right)^{1-\theta} \frac{\tau^{1-\theta}}{\theta} \left[ dI_s + (\theta - 1) \frac{I_s}{G_s} dG_s \right] \quad (60)$$

$$dI_r = w_r dL_{Mr} + L_{Mr} dw_r \quad (61)$$

$$dw_r = \mu \frac{w_r}{G_r} dG_r + \frac{G_r^\mu}{K} \frac{e}{1-\gamma} d\delta_r \quad (62)$$

$$dH_r = \left[ -\frac{1}{1-\lambda} - \frac{\lambda}{(1-\lambda)^2} \right] \ln(h) h^{1-\frac{\lambda}{1-\lambda}} d\lambda \quad (63)$$

$$\begin{aligned} dM = & \frac{\delta_r}{\rho + \psi + \delta_r} \left( \left[ w_r \frac{K}{G_r^\mu} - e \right] dH_r + H_r \frac{K}{G_r^\mu} dw_r - H_r \frac{w_r K}{G_r^{1+\mu}} dG_r \right) \\ & + \left( \frac{1}{\rho + \psi + \delta_r} - \frac{1}{(\rho + \psi + \delta_r)^2} \right) H_r \left[ w_r \frac{K}{G_r^\mu} - e \right] d\delta_r \\ & - \frac{\delta_s}{\rho + \psi + \delta_s} \left( \left[ w_s \frac{K}{G_s^\mu} - e \right] dH_s + H_s \frac{K}{G_s^\mu} dw_s - H_s \frac{w_s K}{G_s^{1+\mu}} dG_s \right) \\ & - \left( \frac{1}{\rho + \psi + \delta_s} - \frac{1}{(\rho + \psi + \delta_s)^2} \right) H_s \left[ w_s \frac{K}{G_s^\mu} - e \right] d\delta_s \end{aligned} \quad (64)$$

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