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Bayesian Estimation of a DSGE Model with Inventories

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This paper introduces inventories in an otherwise standard Dynamic Stochastic General Equilibrium Model (DSGE) of the business cycle. Firms accumulate inventories to facilitate sales, but face a cost of doing so in terms of costly storage of intermediate goods. The paper's main contribution is to present a DSGE model with inventories that is estimated using Bayesian methods. Based on U.S. data we show that accounting for inventory dynamics has a significant impact on parameter estimates and impulse responses. Our analysis also reveals that the contribution of structural shocks to variations in the observable variables changes significantly when we allow for inventories. Moreover, we find that inventories enter the Phillips curve as an additional and significant driving variable of inflation and make the inflation process less backward-looking.

KEYWORDS: Inventories, Bayesian Estimation, DSGE model, Business Cycles

JEL CLASSIFICATION: C13, E20, E30

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1. Introduction

Among the characteristic features of business cycles is the behavior of inventories. Inventory investment typically increases in boom phases and decreases in recessions. Moreover, the inventories to sales ratio is countercyclical. This pattern has been studied in a large family of business cycle models following the work of Bils and Kahn (2000). Most of these models, however, are in the tradition of the Real Business Cycle paradigm and lack many important frictions such as nominal rigidities and monopolistic competition. The modern workhorse model for business cycle analysis, the Dynamic Stochastic General Equilibrium (DSGE) model, in contrast, is characterised by a rich set of frictions and distortions, but is often silent about inventories.

Furthermore, those DSGE models that explicitly account for the behavior of inventories have not yet been estimated using the complete set of restrictions implied by the theoretical framework. An exception is the Bayesian estimation of a two-sector model of Iacoviello et al. (2010) who distinguish between input and output inventories. However, they focus mainly on the behavior of production and inventories and do not consider important shocks such as shocks to capital investment and labor supply.

Jung and Yun (2005) present an optimizing sticky price model that includes the accumulation of finished goods inventories. They are able to replicate the observed relationship between a monetary tightening and a fall in the ratio of stocks to available goods seen in the data. Their model, however, is estimated using a minimum distance approach to match empirical impulse response functions from a VAR in the tradition of Christiano et al. (2005). In a similar study Kryvtsov and Midrigan (2010) confirm the results of Jung and Yun (2005). However, they do not estimate the model but present simulations. Furthermore, both studies restrict the analysis on shocks to monetary policy. Also using a calibrated model with inventories Chang et al. (2009) examine the reaction of employment due to permanent changes in productivity. But neither the behavior of inventories nor the effects of other shocks are considered in their analysis.

In this paper we include inventories into an otherwise standard sticky-price business cycle model and use Bayesian methods to estimate the model. By introducing several shocks we can explore the relevance and influence of other shocks than monetary policy on variables of interest. While the model framework is taken from the benchmark Justiniano et al. (2010) model, the accumulation of inventories is modeled along the lines of Lubik and Teo (2010). Storing intermediate goods facilitates sales at a given price. However, we depart from previous research by assuming that accumulating a stock of inventories is costly since firms have to rent storage capacity from households in order to store goods.

We find that accounting for inventories has a significant impact on business cycle properties. Our results are threefold. First, the paper presents an estimated DSGE model with inventories and numerous shocks deemed important by the literature. We are able to obtain a full set of parameter estimates using Bayesian estimation techniques and examine the empirical results. Second, the model exhibits the countercyclical pattern of the inventories-sales ratio. Accounting for inventories greatly increases the persistence of business cycle dynamics following monetary policy shocks and markup shocks. Third, we show that accounting for inventories changes the functional form of the New Keynesian Phillips Curve (NKPC). The costs of inventory management enter the price setting problem and the marginal cost equation. As a result, inflation is not only driven by marginal costs from production, but also by the inventory-sales ratio. Moreover, when inventories are considered the degree of price indexation, i.e. our proxy for backward-looking price setting, falls significantly, thus making the Phillips curve more forward-looking. Accounting for inventories as an important feature of real-world cycles, therefore, makes it less necessary to resort to ad-hoc assumptions about backward-looking behavior.

The paper is closely related to the recent work by Lubik and Teo (2010). Apart from the assumption that the stock of inventories depreciates their model is similar. Moreover, we introduce costly storage areas needed to store inventories. In their study Lubik and Teo (2010) estimate only the NKPC resulting from their model using single-equation GMM. Our approach, instead, is to confront the complete model with the data. While they cannot find a significant role of inventories for inflation dynamics, our results suggest that accounting for inventories has a significant impact on the inflation process.

This paper is organised as follows. Section 2 includes inventories into an otherwise standard New Keynesian model. Details about the estimation strategy and the parameter estimates are presented in section 3. The main results of our empirical exercise as well as a comparison with the implications of an estimated model without inventories are discussed in section 4. Section 5 concludes.

2. A Model with Inventories

We present a standard New Keynesian model in the spirit of Smets and Wouters (2003) and Christiano et al. (2005) which we enhance with inventories. The modelling approach is based on Jung and Yun (2005) and Lubik and Teo (2009) who incorporate inventories into DSGE models in the manner of Bils and Kahn (2000). Storing goods boosts sales because it avoids shortages, e.g. due to unanticipated demand shifts, and market participants appreciate that their needs can be satisfied at any time. With inventories, demand can be satisfied either by current production or by the stock of goods previously produced. Firms must rent storage area if they want to transfer inventories to the next period.

In the following we present the firm sector of the model with inventories in which we distinguish between production, sales and stock of goods available. We proceed by describing the household sector and labor unions. The government sector as well as the resource constraint are presented at the end of this section.

2.1. Final Good Firms

Perfectly competitive final good firms produce the final good by purchasing differentiated intermediate goods. Goods of intermediate firms with a higher stock of available goods relative to the economy-wide average, $a_{i,t}/a_t$, are preferred. The idea is that firms with a higher level of stockkeeping have a lower probability of running out of goods and thus a final good firm faces a lower risk of not being able to compose its final good.

The Dixit-Stiglitz aggregator type function describing the technology is

$$s_t = \left[\int_0^1 \left(\frac{a_{i,t}}{a_t} \right)^{\theta \frac{\mu_t^p}{1+\mu_t^p}} (s_{i,t})^{\frac{1}{1+\mu_t^p}} \, \mathrm{d}i \right]^{1+\mu_t^p} . \tag{1}$$

Here, the variables s_t and $s_{i,t}$ denote aggregate and firm-specific sales, respectively. Cost minimization leads to the demand for the specific intermediate good

$$s_{i,t} = \left(\frac{p_{i,t}}{p_t} \right)^{-\frac{1+\mu_t^p}{\mu_t^p}} \left(\frac{a_{i,t}}{a_t} \right)^{\theta} s_t \, , \tag{2}$$

where θ is the elasticity of demand for an intermediate good of type i with respect to the stock of available goods intermediate good firm i holds in period t.

The economy-wide price index is defined as

$$p_t = \left[\int_0^1 \left(\frac{a_{i,t}}{a_t} \right)^{\theta_t} (p_{i,t})^{-\frac{1}{\mu_t^p}} \, \mathrm{d}i \right]^{-\mu_t^p} . \tag{3}$$

2.2. Intermediate Good Firms

In the monopolistically competitive intermediate goods market firms, indexed by $i \in [0,1]$, supply their specific intermediate good. Using capital, $k_{i,t-1}$, and labor services (denoted in the form of hours worked), $l_{i,t}$, the representative firm produces its output $y_{i,t}$ with the help of the technology

$$y_{i,t} = k_{i,t-1}^{\alpha} (z_t l_{i,t})^{1-\alpha} - z_t \phi , \tag{4}$$

where z_t is a variable indicating the level of Labor-augmenting technological progress. It is assumed that its growth rate is stochastic. We define $v_t \equiv z_t / z_{t-1}$. The law of motion of technological progress is formulated as

$$\log v_t = (1 - \rho_v) \log v + \rho_v \log v_{t-1} + \eta_t^v , \tag{5}$$

and η_t^v are innovations that are IID. As it is standard in the literature, we include fixed cost of production, parameterized by ϕ. We set ϕ such that firms' profits are zero in steady state.

In period t intermediate good firms own a stock of available goods $a_{i,t}$ stemming from inventories, i.e. the stock of available goods in period $t-1$ less goods sold in period $t-1$ $(a_{i,t-1} - s_{i,t-1})$, and produced goods in period t, $y_{i,t}$. We write this as

$$a_{i,t} = y_{i,t} + (a_{i,t-1} - s_{i,t-1}) . \tag{6}$$

An identical statement is

$$x_{i,t} = y_{i,t} - s_{i,t} + x_{i,t-1} , \tag{7}$$

where $x_{i,t} = a_{i,t} - s_{i,t}$ is the stock of inventories firm i holds at the end of period t. Naturally, the inventory stock rises if the production exceeds sales and vice versa. So far, intermediate good firms have an incentive to increase the stock of available goods by raising production in order to increase sales. On the other side, firms face cost of storing inventories which lowers inventory holdings and the stock of available goods.

Every intermediate good firm has to store its stock of available goods not sold by the end of each period in order to carry it over into the next period. More precisely, the inventory stock is stored in storage areas, h_t. They are rented from households at the current rental rate r_t^h. We assume that the relation between storage areas and inventories at the end of period t is given by

$$h_{i,t} = \psi(a_{i,t} - s_{i,t}) = \psi x_{i,t}\,, \tag{8}$$

where ψ is a constant. As can be seen, the elasticity of storage area demand with respect to inventories is unity since we make the assumption that all goods require the same amount of storage area independent of volume and time.

The representative intermediate good firm maximizes the present discounted stream of future real profits

$$E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left\{ \frac{p_{i,t+\tau}}{p_{t+\tau}} s_{i,t+\tau} - w_{t+\tau} L_{i,t+\tau} - r_{t+\tau}^k k_{i,t+\tau-1} \right.$$
$$\left. - r_{t+\tau-1}^h h_{i,t+\tau-1} - \frac{\kappa_p}{2} \left(\frac{p_{i,t+\tau}}{\pi_{t+\tau-1}^{\gamma_p} \pi^{1-\gamma_p} p_{i,t+\tau-1}} - 1 \right)^2 s_{t+\tau} \right\}, \tag{9}$$

taking into account the demand for its specific good, (2), the production technology given in (4), the evolution of the stock of available goods as defined in (6) as well as the required storage room for its inventories, (8). Labor services $l_{i,t}$ are compensated by the hourly wage rate w_t denoted in real terms, i.e. W_t/p_t, while capital is rented from households at the current rental rate r_t^k. Note that in the model with inventories revenues depend on sales $s_{i,t}$ instead of output $y_{i,t}$. At the end of each period intermediate good firms realize how much storage room they need and rent the required amount of storage areas. In the following period they settle accounts, i.e. the owners of storage areas receive payments with a lag of one period.

Prices are set according to a mechanism à la Rotemberg (1982). Each intermediate good producer decides every period about the optimal price for his specific good, taking into account that adjusting the price induces cost if the ratio between the current price and that one period before, $p_{i,t}/p_{i,t-1}$, differs from the economy-wide gross inflation rate realized one period before, i.e. $\pi_{t-1} = p_{t-1}/p_{t-2}$, and steady state inflation π. Here, γ_p is an indexation parameter. In addition, κ_p is a parameter that measures the degree of adjustment cost.

The optimal price $p_{i,t}$ satisfies the condition

$$\frac{p_{i,t}}{p_t}s_{i,t} + \mu_t^p \kappa_p \frac{p_{i,t}}{\pi_{t-1}^{\gamma_p}\pi^{1-\gamma_p}p_{i,t-1}}\left(\frac{p_{i,t}}{\pi_{t-1}^{\gamma_p}\pi^{1-\gamma_p}p_{i,t-1}} - 1\right)s_t$$
$$= E_t\beta\mu_t^p\frac{\lambda_{t+1}}{\lambda_t}\kappa_p\frac{p_{i,t+1}}{\pi_t^{\gamma_p}\pi^{1-\gamma_p}p_{i,t}}\left(\frac{p_{i,t+1}}{\pi_t^{\gamma_p}\pi^{1-\gamma_p}p_{i,t}} - 1\right)s_{t+1}$$
$$+ E_t\beta\frac{\lambda_{t+1}}{\lambda_t}\left[mc_{t+1} - \psi r_t^h\right](1+\mu_t^p)s_{i,t}\,, \quad (10)$$

where marginal cost mc_t are given by

$$mc_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\left(r_t^k\right)^\alpha\left(\frac{w_t}{z_t}\right)^{1-\alpha}. \quad (11)$$

Without cost of price adjustment ($\kappa_p = 0$), each intermediate firm sets its price as a markup over expected marginal cost next period minus the cost of stockkeeping.

As in Justiniano and Primiceri (2008), the price markup μ_t^p is assumed to be autocorrelated of order one and driven by an exogenous IID disturbance η_t^p, formally

$$\log\mu_t^p = (1-\rho_p)\log\mu^p + \rho_p\log\mu_{t-1}^p + \eta_t^p\,. \quad (12)$$

Furthermore, optimization yields that marginal cost evolve according to

$$mc_t = \theta_t\frac{p_{i,t}}{p_t}\frac{s_{i,t}}{a_{i,t}} + E_t\left[1-\theta_t\frac{s_{i,t}}{a_{i,t}}\right]\beta\frac{\lambda_{t+1}}{\lambda_t}\left[mc_{t+1} - \psi r_t^h\right], \quad (13)$$

Obviously, costs of stockkeeping enter both the price setting equation and the marginal cost equation. In (13), the representative intermediate good firm faces a trade-off between today's marginal cost of production plus marginal cost of inventory holding, i.e. ψr_t^h, and expected marginal cost of production tomorrow. This calculus in turn affects the choice of the optimal price in (10).

2.3. Households

The economy consists of a mass of households indexed by $j \in [0,1]$. Households purchase consumption and investment goods, supply labor and are members of labor unions which set their wages. Every household offers a specific type of labor service to intermediate good firms through labor unions. Living endlessly, each household maximizes the utility function

$$E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[\epsilon^c_{t+\tau} \log\left(c_{t+\tau} - b c_{t+\tau-1}\right) - \frac{\left(l_{j,t+\tau}\right)^{1+\sigma_l}}{1+\sigma_l} \right], \tag{14}$$

where utility depends positively on consumption c_t and negatively on hours worked $l_{j,t}$.[1] The parameter b measures the degree of habit persistence in consumption and σ_l is the inverse of the Frisch elasticity of labor supply. We adopt a logarithmic utility in consumption in order to ensure that the steady state of the model features a balanced growth path.

Furthermore, household's preference for consumption is affected by a consumption shock ϵ^c_t with mean unity that follows

$$\log \epsilon^c_t = \rho_c \log \epsilon^c_{t-1} + \eta^c_t, \tag{15}$$

where the innovations η^c_t are IID with mean zero.

Each household faces the budget constraint

$$c_t + i^k_t + i^h_t + \frac{B_t}{p_t} = \frac{r^m_{t-1} B_{t-1}}{p_t} + \frac{W_{j,t}}{p_t} l_{j,t} + r^k_t u_t \bar{k}_{t-1} - a(u_t)\bar{k}_{t-1} + r^h_{t-1} h_{t-1} + div_{j,t}. \tag{16}$$

Here, p_t denotes the economy-wide price level. In period t, the household buys government bonds B_t which yield a return of r^m_t in period $t+1$. Finally, the nominal hourly wage rate is denoted by $W_{j,t}$ and $div_{j,t}$ captures the net flow of dividends from intermediate good firms, membership fees to labor unions and lump-sum taxes paid to the government.

Households are owner of the capital stock which they rent to intermediate good firms at the current rental rate r^k_t. Furthermore, they decide how intensively the physical capital stock is used by setting the rate of capital utilization u_t, coming at a cost of $a(u_t)$ multiplied by the stock of physical capital $\bar{k}_{t-1}$. We assume that in steady state $a(1) = 0$ as well as that $\sigma_a = a''(1)/a'(1) > 0$, with a steady state capital utilization rate of unity. By $\bar{k}_t$ we denote the end-of-period t stock of physical capital. Furthermore, $\bar{k}_{t-1}$ is related to capital k_{t-1} by

$$k_{t-1} = u_t \bar{k}_{t-1}. \tag{17}$$

To keep the capital stock from deteriorating the household purchases capital investment goods i^k_t. Each period, a constant share δ^k of the physical capital stock depreciates. As a

[1] Note that we exploit that in equilibrium every household chooses the same level of consumption. This is guaranteed through the purchase of state contingent securities.

result, at the end of period t the physical stock of capital is given by

$$\bar{k}_t = (1-\delta^k)\bar{k}_{t-1} + \epsilon^k_t \left(1 - S\left(\frac{i^k_t}{i^k_{t-1}}\right)\right) i^k_t \,, \tag{18}$$

where $S\left(\frac{i^k_t}{i^k_{t-1}}\right)$ are cost associated with changes in the level of investment. We assume that $S(\cdot) = S'(\cdot) = 0$ and $S''(\cdot) > 0$ in steady state. The variable ϵ^k_t is a shock to the efficiency of transforming investment goods into new physical capital, and it is assumed that it follows the stochastic process

$$\log \epsilon^k_t = \rho_k \log \epsilon^k_{t-1} + \eta^k_t \,, \tag{19}$$

with η^k_t being IID innovations.

Beside the stock of capital, households own storeage areas that they lend to intermediate good firms. For this service they earn a rent of r^h_t. Furthermore, storeage areas depreciate by $\delta^h \in (0,1)$ every period (e.g. erosion of storehouses due to environmental influences). In period t, households receive payments $r^h_{t-1}h_{t-1}$ from lending the end-of-period $t-1$ stock of storage areas to intermediate good firms.

Households can acquire storage area investment goods in order to build up the undepreciated stock of storage aras. Storage areas evolve according to

$$h_t = (1-\delta^h)h_{t-1} + \epsilon^h_t \left(1 - S\left(\frac{i^h_t}{i^h_{t-1}}\right)\right) i^h_t \,. \tag{20}$$

Similar to investments in the physical stock of capital, investing in storage areas causes adjustment cost amounting to $S\left(\frac{i^h_t}{i^h_{t-1}}\right)$ with steady state properties $S(\cdot) = S'(\cdot) = 0$ and $S''(\cdot) > 0$. In (20), ϵ^h_t is a shock to the transformation of storage area investment goods into new and rebuilt storage areas. Its law of motion is given by

$$\log \epsilon^h_t = \rho_h \log \epsilon^h_{t-1} + \eta^h_t \,. \tag{21}$$

Here, η^h_t are disturbances with IID normal distribution. A positive storage area investment shock leads to a rise in supply of storage areas. Ceteris paribus, this leads to a fall in the storage area rental rate and therefore firms face lower cost for their stored goods. Thus we refer to ϵ^h_t as an unexplained variation in the cost of inventory holding.

2.4. Labor Unions

The specific types of labor services supplied by households are bundled into one homogenous labor input, l_t. The technology used is described by the Dixit-Stiglitz function

$$l_t = \left[\int_0^1 \left(l_{j,t} \right)^{\frac{1}{1+\mu_t^w}} \, \mathrm{d}j \right]^{1+\mu_t^w} . \tag{22}$$

Cost minimization then yields that the demand for labor type j is given by

$$l_{j,t} = \left(\frac{W_{j,t}}{W_t} \right)^{-\frac{1+\mu_t^w}{\mu_t^w}} l_t \, , \tag{23}$$

where W_t is the aggregate nominal wage rate

$$W_t = \left[\int_0^1 \left(W_{j,t} \right)^{-\frac{1}{\mu_t^w}} \, \mathrm{d}j \right]^{-\mu_t^w} . \tag{24}$$

Households are members of labor unions that set the nominal wage rate and the amount of working hours. More precisely, each household is represented by exactly one labor union that corresponds to its labor type. Labor unions receive a membership fee from households to finance quadratic cost of wage adjustment. Costs depend on the growth rate of hourly wages relative to inflation and technology growth last period and in steady state. A labor union optimizes the objective function

$$\begin{aligned} & E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \Bigg\{ - \frac{\left(l_{j,t+\tau} \right)^{1+\sigma_l}}{1+\sigma_l} \\ & + \lambda_{t+\tau} \left[\frac{W_{j,t+\tau}}{p_{t+\tau}} l_{j,t+\tau} - \frac{\kappa_w}{2} \left(\frac{W_{j,t+\tau}}{\left(\pi_{t+\tau-1} v_{t+\tau-1} \right)^{\gamma_w} \left(\pi v \right)^{1-\gamma_w} W_{j,t+\tau-1}} - 1 \right)^2 \frac{W_{t+\tau}}{p_{t+\tau}} l_{t+\tau} \right] \Bigg\} , \end{aligned} \tag{25}$$

subject to the demand for the differentiated labor service as derived in (23). In (25), λ_t is the Lagrange multiplier in the household's optimization problem associated with the budget constraint and equals marginal utility of consumption. The parameter κ_w determines the size of wage adjustment cost and γ_w is a parameter that measures the degree of indexation to past inflation and technology growth.

Finally, optimization yields the result

$$
\begin{aligned}
&(1+\mu_t^w)\left(l_{j,t}\right)^{1+\sigma_l} + E_t \frac{\mu_t^w \beta \kappa_w \lambda_{t+1} W_{j,t+1}}{\left(\pi_t v_t\right)^{\gamma_w} (\pi v)^{1-\gamma_w} W_{j,t}} \left(\frac{W_{j,t+1}}{\left(\pi_t v_t\right)^{\gamma_w} (\pi v)^{1-\gamma_w} W_{j,t}} - 1\right) \frac{W_{t+1}}{p_{t+1}} l_{t+1} \\
&= \lambda_t \frac{W_{j,t}}{p_t} l_{j,t} + \frac{\mu_t^w \kappa_w \lambda_t W_{j,t}}{\left(\pi_{t-1} v_{t-1}\right)^{\gamma_w} (\pi v)^{1-\gamma_w} W_{j,t-1}} \left(\frac{W_{j,t}}{\left(\pi_{t-1} v_{t-1}\right)^{\gamma_w} (\pi v)^{1-\gamma_w} W_{j,t-1}} - 1\right) \frac{W_t}{p_t} l_t \,.
\end{aligned} \tag{26}
$$

Without adjustment cost, the real wage multiplied by marginal utility of consumption would be a markup over the disutility of work. The markup μ_t^w evolves according to

$$
\log \mu_t^w = (1-\rho_w) \log \mu^w + \rho_w \log \mu_{t-1}^w + \eta_t^w \,, \tag{27}
$$

whereas η_t^w are the IID innovations.[2]

2.5. Government and Market Clearing

The monetary authority sets the nominal interest rate r_t^m according to a generalized Taylor rule. More precisely, monetary policy is described by the formula

$$
\frac{r_t^m}{r^m} = \left(\frac{r_{t-1}^m}{r^m}\right)^{\rho_m} \left[\left(\frac{\pi_{t-1}}{\pi}\right)^{\varphi_\pi} \left(\frac{y_{t-1}}{z_{t-1} y^*}\right)^{\varphi_y}\right]^{1-\rho_m} \left(\frac{\pi_t}{\pi_{t-1}}\right)^{\varphi_{\Delta\pi}} \left(\frac{y_t}{v_t y_{t-1}}\right)^{\varphi_{\Delta y}} e^{\eta_t^m} \,. \tag{28}
$$

Note that y^* is the steady state value of stationarized output. In linearized terms, (28) becomes

$$
\hat{r}_t^m = \rho_m \hat{r}_{t-1}^m + (1-\rho_m)\left(\varphi_\pi \hat{\pi}_{t-1} + \varphi_y \hat{y}_{t-1}^*\right) + \varphi_{\Delta\pi}(\hat{\pi}_t - \hat{\pi}_{t-1}) + \varphi_{\Delta y}(\hat{y}_t^* - \hat{y}_{t-1}^*) + \eta_t^m \,, \tag{29}
$$

where y_t^* is the stationarized output level. A hat above a variable denotes its percentage deviation from steady state. We add an IID shock η_t^m to the interest rule to allow the actual federal funds rate to deviate from the formulated Taylor rule.

Furthermore, we assume that the ratio of government spending, g_t, to final sales varies

[2] As for the model in Justiniano et al. (2010), in our linearized model the wage markup shock has the same effect as a shock that would affect household's disutility of labor in (14). Including such a 'labor supply' shock would also require a decision between an autocorrelated and an IID shock, since two autocorrelated shocks in the wage setting equation bring up identification issues. Therefore we omit the labor supply shock.

over time depending on an exogenous shock to governmental economic activity, i.e.

$$g_t = \left(1 - \frac{1}{\epsilon_t^g}\right) s_t \,. \tag{30}$$

where the evolution of the government spending shock ϵ_t^g is given by (letting η_t^g be IID disturbances)

$$\log \epsilon_t^g = (1 - \rho_g) \log \epsilon^g + \rho_g \log \epsilon_{t-1}^g + \eta_t^g \,. \tag{31}$$

With inventories and storage specific investment goods that accompany the existence of storage areas the aggregate resource constraint becomes

$$c_t + i_t^k + i_t^h + g_t + a(u_t)\bar{k}_{t-1} + \frac{\kappa_p}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\gamma_p}\pi^{1-\gamma_p}} - 1\right)^2 s_t = s_t \,. \tag{32}$$

In this economy total investment, i_t, is the sum of the two specific investment goods, i.e. $i_t = i_t^k + i_t^h$.

3. Estimation

3.1. Bayesian Approach

In recent years Bayesian estimation of DSGE models has become popular for various reasons. It is a system-based estimation approach that offers the advantage of incorporating assumptions about the parameters, coming from either economic theory or previous micro- and macroeconomic studies. The assumptions can be nested comfortably in the econometric framework and reduce weak identification issues as well.

Bayesian estimation is based on Bayes' theorem. It states that the posterior distribution of the parameters can be computed from the likelihood function and the prior distribution. The prior distribution has to be specified by the researcher and reflects her beliefs about the true parameter values.

Let $p(\zeta)$ be the prior distribution and $p(\zeta|Y_t)$ be the posterior distribution of our model's parameter set, say ζ. By Y_t we denote the data set. Then, Bayes' theorem states that

$$p(\zeta|Y_t) = \frac{p(Y_t|\zeta)\, p(\zeta)}{p(Y_t)} \,, \tag{33}$$

where

$$p(Y_t) = \int p(Y_t|\zeta)\, p(\zeta)\, \mathrm{d}\zeta \tag{34}$$

is the marginal likelihood of the data conditional on the model. The marginal likelihood is a constant and therefore it plays no role for the maximization of the posterior. Thus, we can disregard the marginal likelihood and obtain the proportional term

$$p(\zeta|Y_t) \;\propto\; p(Y_t|\zeta)\, p(\zeta)\,. \tag{35}$$

Furthermore, it is well-known that the probability of the data given the parameters is equivalent to the likelihood function of ζ given Y_t, or formally: $p(Y_t|\zeta) \equiv \mathcal{L}(\zeta|Y_t)$. As a result, we obtain the formula

$$p(\zeta|Y_t) \;\propto\; \mathcal{L}(\zeta|Y_t)\, p(\zeta) \tag{36}$$

We build up the likelihood function with the help of filter techniques. First, the models' equilibrium conditions are log-linearized around the non-stochastic balanced growth path. When applicable, we detrend the variables by the current level of technology in order to make them stationary. Using a generalized Schur decomposition the system of equations is then transformed into its state space form where the observed (control) variables are linked to the predetermined (state) variables. Given the state space representation of our model, the Kalman filter is applied to generate optimal forecasts of and inference about the vector of unobserved state variables. With the results obtained by the Kalman algorithm we are able to evaluate the joint likelihood function of the observable endogenous variables.

The posterior distribution of the parameters, $p(\zeta|Y_t)$, is derived as follows: First, we numerically optimize (36) in logarithmic terms so as to obtain a maximum, called the posterior mode, and approximate standard errors, the latter based on the inverse Hessian evaluated at the posterior mode. Thereafter, the parameter values of the posterior mode as well as the Hessian are employed to simulate the posterior distribution which is derived numerically by applying Monte-Carlo Markov-Chains methods. In this way we can generate draws of the parameters in ζ, the realisations of which yielding the posterior distribution of ζ (according to (33) and (34)). As in most applications of Bayesian estimation with respect to DSGE models, we employ the Random-Walk Metropolis-Hastings algorithm.[3]

We estimate the model by running two chains of the Random-Walk Metropolis-Hastings algorithm with 160,000 iterations in each case. This is sufficient to let the algorithm converge. We drop the first 60,000 candidates and retain every 20th draw. Finally we keep 10,000 draws from which we calculate the posterior distribution of the parameters, the variance decompositions and the impulse responses. Autocorrelation and cross-correlation functions are obtained by generating 200 observations. This is done 100 times for each of the 500 parameter draws taken from the total of 10,000 draws.

3.2. Data and Priors

We employ quarterly U.S. data on real consumption, real investment, real compensation per hour, and real GDP, obtained by dividing nominal terms by the price index. The price index is calculated by the ratio of nominal to real GDP. Expenditures for durable consumption goods are attributed to investment expenditures. Furthermore, data on hours worked in nonfarm business sector, the federal funds rate and nonfarm inventories to final sales are used for estimation. When applicable we divide the mentioned time series by civilian noninstitutional population aged over 16.[4] The time series on hours worked is normalized such that its sample average is zero. Similar to Smets and Wouters (2007), our sample starts in 1957Q1, but we use observations up to 2006Q4. The first 10 years we use for the initialization of the Kalman Filter.

Several parameters are fixed during estimation. The depreciation parameters δ^h and δ^k are both set to 0.025, implying a depreciation of 10% at annual rate. We set α to 0.3. Furthermore, we choose a value of 0.2 for the steady state wage markup μ^w. Due to the assumption of adjustment cost à la Rotemberg (1982) we have to fix either μ^w or the adjustment cost parameter κ_w in order to ensure identification. The steady state ratios of consumption, investment and government spending to sales are set to 0.55, 0.25 and 0.2, respectively. The ratio of sales to available goods in steady state, s/a, is fixed to 0.29.[5] These values correspond to the average values in our sample.

[3] The Random-Walk Metropolis-Hastings algorithm was first used by Schorfheide (2000) and Otrok (2001), later in common articles such as Smets and Wouters (2003) as well as Adolfson et al. (2007), amongst others.

[4] Except for the inventories-to-sales ratio, which is extracted from NIPA tables of the Bureau of Economic Analysis, all data are taken from the FRED Database.

[5] We also tried to estimate the share of sales to available goods in steady state. Since it did not affect the estimation results and the estimated values were very close to the historical average we decided to fix this ratio to 0.29.

Table 1 shows the prior distributions for the estimated parameters. In the following we shortly comment our prior choice and name the corresponding studies. For a more extensive discussion the reader is refered to the mentioned literature.

The priors for υ, π, S'', ρ_m are taken from Smets and Wouters (2007). In addition, the autoregressive coefficients and standard deviations of shocks are similar to the ones in Smets and Wouters (2007), but with a standard deviation of unity for the shocks. The prior for habit consumption, b, captures the range of results in the business cycle literature (Justiniano et al. (2010) and Smets and Wouters (2007)) as well as of the results of micro studies (e.g. Ravina (2007)). The discount rate β as well as the parameters regarding indexation, γ_w and γ_p, resemble the priors in Justiniano et al. (2010) and Smets and Wouters (2007). As in Smets and Wouters (2007), hours worked in steady state, l_{stst}, are distributed normally around zero. The prior for the inverse Frisch elasticity, σ_l, is taken from Justiniano et al. (2010). Our prior for σ_a (elasticity of capital utilization) is less strict than the one formulated in Justiniano et al. (2010). For the adjustment cost parameters, κ_p and κ_w, we adopt the priors from Gerali et al. (2010). The Taylor rule parameters have priors similar to Smets and Wouters (2007) (φ_π) and Adolfson et al. (2007) (φ_y, $\varphi_{\Delta\pi}$, $\varphi_{\Delta y}$).

The prior for the elasticity of demand with regard to available goods, θ, is set to an intermediate value of the results in Jung and Yun (2005). With a mean of 0.6 and a standard deviation of 0.2 (normally distributed), 95% of the prior density lies between 0.2 and 1. Concerning the ratio of storage areas to inventories, ψ, we choose as prior a beta distribution with mean 0.4 and standard deviation of 0.2.

3.3. Posteriors

The estimated parameter results are shown in table 1. Technology growth in steady state is estimated to be 0.36 which is slightly smaller than assumed while steady state quarterly inflation is somewhat higher with a value of 0.67. Consumption habits are more relevant than in Smets and Wouters (2007), but with a median value of 0.84 still in the range of the estimates in Justiniano et al. (2010).

Our results for the inverse Frisch elasticity are fairly low. Nevertheless, the median value (1.09) is not significantly different from the one estimated in Smets and Wouters (2007) (0.91 to 2.78). French (2004) examines the response of labor supply to changes in wages during 1980 to 1986 and reveals values between -0.5 and 0.6 for the Frisch elasticity, corresponding to a value of 1.7 or higher for the inverse.

The results obtained for costs of changes in capital utilization fit almost perfectly the findings presented in Smets and Wouters (2007). Price and wage adjustment cost are slightly higher than expected, indicating a non-negligible degree of price and wage stickiness. On the other side, we obtain a remarkably low indexation to past inflation (and technology growth regarding wage changes) which corresponds to stronger forward-looking components in the Phillips curves.

Turning to the parameters regarding inventories we see that the demand elasticity of sales with respect to available goods, θ, is estimated to be 0.33, a value that is in accordance with the lower estimates in Jung and Yun (2005). Rather the trade-off between cost of production and storing goods than a demand effect determines the amount of available goods in each period. Finally, the ratio of storage areas to inventories is almost unity and significantly higher than the formulated prior.

With dynamic costs of stockkeeping, inventories and sales are part of the NKPC. More precisely, in linearized terms the inflation equation in our model is

$$\widehat{\pi}_t = \underbrace{\frac{\beta}{1+\beta\gamma_p}}_{0.9046} E_t\widehat{\pi}_{t+1} + \underbrace{\frac{\gamma_p}{1+\beta\gamma_p}}_{0.0938}\widehat{\pi}_{t-1} + \widehat{\widetilde{\mu}}_t^p$$
$$+ \underbrace{\frac{\theta\frac{s}{a}\left(\frac{v}{\beta}+\psi r^h\right)-\psi r^h}{\left(\frac{v}{\beta}-1+\psi r^h\right)\kappa_p\left(1+\beta\gamma_p\right)\left(1-\theta\frac{s}{a}\right)}}_{0.0281}\left[\widehat{mc}_t + \underbrace{\frac{\theta\frac{s}{a}\left(1-\theta\frac{s}{a}\right)\left(\frac{v}{\beta}-1+\psi r^h\right)}{\theta\frac{s}{a}\left(\frac{v}{\beta}+\psi r^h\right)-\psi r^h}}_{0.0352}\left(\widehat{x}_t-\widehat{s}_t\right)\right], \tag{37}$$

where we normalize the markup shock such that $\widehat{\widetilde{\mu}}_t^p = \frac{1}{(1+\mu^p)\kappa_p(1+\beta\gamma_p)}\widehat{\mu}_t^p$. The values assigned to the coefficients are medians calculated from the retained 10,000 parameter draws.

The elasticity of current inflation with respect to marginal cost is rougly 3%. This estimated value lies in the upper spectrum compared to other estimated DSGE models. While several studies obtained point estimates around 2% (e.g. Smets and Wouters (2007)), 2.5% (e.g. Justiniano et al. (2010)) or nearly 3% (e.g. Gertler et al. (2008)), most estimation results are centered around 1% (see for example Altig et al. (2011) or Adolfson et al. (2007)). Overall, marginal cost affect current inflation quite considerably (compared to the literature) and we estimate a stronger forward-looking component than generally observed.

Inventories and sales have an influence on current inflation that is only 3.5% of the one of marginal cost. Note that current marginal cost and inventories are related to future marginal cost by (13) which in turn affects (expected) future inflation. Using a single-equation GMM approach Lubik and Teo (2010) estimate values of 5.85% and 4.03% for the elasticity of current inflation with respect to marginal costs, depending on the calculation method of the marginal cost series. Their corresonding estimate for the sales and available goods coefficient corresponding to our notation is 1%. Notably, a significant difference is that the results of Lubik and Teo (2010) depend on the assumption that marginal cost consist solely of the wage rate. Furthermore, our inflation equation with inventories is more forward-looking: Lubik and Teo (2010) obtain an elasticity of current inflation with respect to expected future inflation of less than 80% and of about 20% regarding past inflation. With their inventory model estimated by impulse response matching Jung and Yun (2005) obtain a coefficient in front of marginal cost that is below 0.3% and changes in past or future inflation feed into a change of current inflation by 50%.

Concluding, our estimates for the elasticities of current inflation to marginal cost and the ratio of inventories to sales take values in the upper range of previous results of above mentioned studies. For this reason and a low indexation parameter inflation is comparatively flexible and it reacts relatively strongly to changes in marginal cost, the inventory-sales ratio and expected inflation for tomorrow.

4. Empirical Results

4.1. Empirical Fit

To examine the empirical fit of our model, we first discuss the cross-correlations between the endogenous variables as predicted by our model. Figure 1 presents the results for selected variables. Overall the model captures the empirical correlations quite well, i.e. the empirical correlations lie within the 90% confidence band. Especially for the inventories-to-sales ratio, $\frac{x_t}{s_t}$, we obtain reasonable results. Note that the model predicts a persistence that matches the observations perfectly. On the other side, the autocorrelation of output growth is unsatisfying since the data yield a significantly lower value of autocorrelation.

Similar to Justiniano et al. (2010), our model cannot claim to replicate the cross-correlation pattern between consumption growth and investment growth correctly. This does not hold for the output growth and investment growth series where observed and fitted cross-correlations are almost identical. Furthermore, for several variables the model predicts an autocorrelation coefficient higher than actually observed. Nevertheless, we can state that our model can compete with other models previously presented in the literature and does a good job in replicating the autocorrelation structure of the inventories-to-sales ratio.

4.2. Impulse Responses

In figure 2 we display the response to a technology shock. Output, sales, consumption, investments and the real wage increase on impact. Furthermore, we obtain the result that hours worked decrease initially but rise significantly above their steady state value after two periods. This is in line with the findings in Gertler et al. (2008) who estimate a labor-market search model without inventories. The stock of inventories increases gradually while the ratio of inventories to sales shrinks significantly over all time horizons due to the rise in sales. Note that this result coincides with the observed pattern of procyclical inventories and a countercyclical inventories-sales ratio.

The price markup shock (depicted in figure 3) leads to a strong rise in inflation. Therefore, sales and output fall while inventories rise. The higher price level reduces demand for consumption and capital investment goods as well as total investment. Higher labor supply combined with a fall in labor demand leads to a reduction in the real wage by more than 1% compared to its stedy state level. As for the technology shock, inventories rise gradually. Forced by the reduction in sales the ratio of inventories to sales rises significantly.

We now turn to the shocks that are associated with households' behavior. The responses to a wage markup shock are shown in figure 4. It can be seen that inflation rises significantly while output and sales fall. The same holds for consumption and investments. Inventories decline by nearly 0.7%. In the short run this leads to a rise in the inventories-to-sales ratio. In contrast to this the ratio declines in the long run as a result of a constant fall in the stock of inventories that outweighs the fall in demand.

Figure 5 depicts the consumption shock and its impact on selected variables. Households exploit the rise in marginal utility of consumption and lower their investment expenditures. As in Lubik and Teo (2009), the reaction of sales exceeds the response of output. The hump-shaped responses stem from rigidities such as habit consumption and adjustment cost in investment. Higher production corresponding to higher demand should result in higher factor prices. Remarkably, and in contrast to the results in Lubik and Teo (2009), real wages fall and cause a smaller inflation rate for several quarters. The rise in labor supply seems to exceed the one in labor demand. Inventories as well as the ratio of inventories to sales deviate significantly negative from their steady state values.

The impulse responses to a positive capital investment shock (shown in figure 6) are similar to those in Justiniano et al. (2010) with one exception: while the responses of capital investment are equal, the existence of storeage area investment in our model significantly dampens the reaction of total investment to a capital shock. In our estimated model, the more effective transformation of capital investment goods into new capital increases demand for investment goods and total demand. Higher output leads to higher labor demand, resulting in an increase in the real wage per hour. Sales react somewhat stronger than output and inventories are significantly below their steady state value for about four years. Thereupon demand for storage areas shrinks which leads to a reduction in storage area investment during the first periods after the shock. The peak of the reaction of total investment relative to capital investment is only 2/3 but its reaction is more persistent. Remarkably, the ratio of inventories to sales decreases significantly more than 1%, indicating that the capital investment shock is likely to explain much of the short run variations in the inventory-to-sales ratio.

This statement also holds for the government spending shock. The impulse responses are shown in figure 7. Fiscal stimulus financed through lump sum tax leads to similar reactions in the displayed variables as for the capital investment shock even though less hump-shaped. On the household side, higher public spending significantly crowds out both consumption and investments. Households increase labor supply such that the real wage falls slightly but significantly by 0.05% after two years. Note that inflation rises only by 0.02%. Furthermore, its reaction is insignificant. Our results correspond largely to findings of Smets and Wouters (2003), only the responses of some real variables such as output and investment are stronger. For the inventories-to-sales ratio we see that it decreases by 1.2% on impact due to excess demand and it returns to its steady state level only very slowly.

Several dynamic models have been developed to study the effects of monetary policy on inventories, e.g. Jung and Yun (2005) and Kryvtsov and Midrigan (2010). Figure 8 presents the responses of the variables to a positive shock to the nominal interest rate. The reaction of capital investment is quite strong and exceeds the response of consumption by a factor of ten. As in Jung and Yun (2005) who employ a minimum distance approach for their estimation, output, inflation and the sales-to-available goods ratio fall significantly (i.e.

the ratio of inventories to sales rises significantly).[6] Beyond that, their model with habit consumption and quadratic adjustment costs related to the sales-to-available goods ratio delivers almost identical responses in terms of magnitude and persistence. Contrary to this result, Kryvtsov and Midrigan (2010) find that the ratio of inventories to sales remains nearly unchanged given a small depreciation of the inventory stock and adjustment cost in output deviations from steady state. Regarding hours worked and the hourly real wage rate we obtain a significant negative deviation as in Gertler et al. (2008).

4.3. Variance Decompositions

The contribution of all shocks to the forecast error variances of selected variables on impact is shown in table 4. The main driving forces of variations in output growth are shocks to costs of inventory holding, government spending and capital investment. However, sales growth is only affected by capital investment and government spending shocks, storage area investment shocks are negligible. As a result, the forecast error variance of the ratio of inventories to sales in the very short run is entirely explained by capital investment and government spending shocks. But note that government spending shocks in our model also reflect unexplained variations in net exports.

Table 5 shows the variance decomposition after one year. The forecast error for the inventories-to-sales ratio is significantly explained only by capital investment shocks. In table 6 we see the forecast error variance after 2.5 years. Remarkably, the importance of the government spending shock regarding variations in the ratio of inventories to sales falls significantly as the time horizon becomes longer. The opposite holds for the price markup shock and inventory holding cost shock which become more important over time. For sales growth we observe that it is always predominantly driven by shocks to capital and storage area investment. In addition to the two specific investment shocks the forecast error variance of output growth is quite strongly explained by shocks to the cost of inventory holding. This can also be seen from the long run variance decomposition (table 3). Again, the forecast error variance of output growth is to a great extent attributed to unexpected changes in inventory holding cost.

In the short run we obtain the result that hours worked are closely related to output

[6]Linearized the inventories-to-sales ratio and the ratio of sales to available goods are related via the formula

$$\widehat{s}_t - \widehat{a}_t = -\left(1 - \frac{s}{a}\right)(\widehat{x}_t - \widehat{s}_t) \ .$$

growth meaning that the shocks that drive theses variables are the same. In the medium term, the forecast error for hours worked is attributed to several shocks equally. Notably, the monetary policy and the technology shock do not contribute to unexpected movements in the hours worked series. Looking at the real wage series we see that 90% of the variance in the forecast error is assigned to wage markup shocks (about 40%), price markup and technology shocks (25% each). On impact labor shocks are even the most important disturbances influencing real wage growth while technology shocks are insignificant. Obviously, while most of the variations in real wage growth can in general be explained by the three shocks mentioned above, hours worked are either mainly driven by investment and government shocks (short run) or similarly by all shocks except for shocks to technology and monetary policy (medium/long run).

Price markup shocks and to a small extent wage markup shocks are the forces that create unexpected deviations of inflation from its steady state level. Remarkably, technology shocks only account significantly for changes in the growth rate of real wages while they have no significant influence on other variables. This result stands in contrast to the strong responsibility of technology shocks for output variations as typically found in the literature for both the output level (e.g. in Smets and Wouters (2005)) or its growth rate (e.g. in Gertler et al. (2008)) particularly in the long run. Furthermore, monetary policy shocks are irrelevant for forecast errors over all time horizons. At all times they contribute significantly less than the expected average value of 12.5% to unexpected variations in all variables. Merely in the very short run they can account for unexpected interest rate movements. More important for unexpected changes in the nominal interest rate are shocks to the cost of inventory holding. However, in the long term price markup shocks explain more than one third of the forecast error variance of the interest rate. Table 3 reveals that shocks to the price markup and capital investment are the most important shocks in terms of forecast errors.[7]

Table 7 summarizes the variance decomposition of the inventory-sales ratio for different time spans. Within one year capital investment shocks and shocks to government spending explain most of the forecast error variance. But soon their large influence vanishes, government spending shocks even become insignificant in the long term. In the medium and long run price markup shocks as well as shocks to inventory cost contribute more

[7] Not shown in these tables are the forecast error variances of the two specific investment goods that are almost solely caused by the corresponding disturbances. Other shocks do not matter. The same holds for consumption growth where the consumption shock plays the major role.

than 50% to unexpected deviations. Note that we obtain very large confidence intervals for both shocks which means that it is unclear (in statistical sense) whether they are more influential than other structural disturbances. Monetary policy and labor supply shocks are insignificant and can be ignored over all time horizons.

4.4. Assessing the Effect of Inventory Holding

Adding inventories to a standard New Keynesian DSGE model could be a dispensable task given that they do not make a difference compared to a model economy without stockkeeping. Ceteris paribus, a model that allows for inventories has more equilibrium conditions. The problem arises how to incorporate the new variable(s), how to construct the model and lessen possible misspecification. Furthermore, adding the ratio of inventories to sales to the data set can turn out to be more cumbersome than helpful. Kryvtsov and Midrigan (2010) discuss this problem in the context of estimating the model in Smets and Wouters (2007) with inventories. In order to examine whether inventories and the inclusion of the time series inventories-to-sales ratio have an effect on the transmission of shocks we estimate a model without inventories.

The parameter estimates are shown in table 2. For the baseline model, we obtain a significant lower estimate for the investment adjustment cost parameter. With a median value of 1.12 this parameter is quite low with regard to other results in the literature.[8] A high discrepancy is revealed regarding the elasticity of capital utilization cost parameter. With a median value of 7.17 in the baseline model this parameter is estimated to be surprisingly high.

The price and wage adjustment cost parameters are both lower but for wages the difference is significant. The different specifications of the elasticity of current inflation with respect to current marginal cost explain why the price adjustment cost parameter takes a higher value in the inventory model. For wages we suggest that the significant higher estimated value for the inventory model is a result of the discrimination between output and sales and the feasibility of more volatile marginal cost that the model tries to match with the data. Assigned to the indexation parameters are values above the ones estimated for the inventory model, which means that in the model with inventories inflation and wages are more strongly driven by expected future values than past realisations.

[8] As an example, see Gertler et al. (2008) and Justiniano et al. (2010) who use the same formulation of investment adjustment cost. Adolfson et al. (2007) and Sahuc and Smets (2008) even obtain estimates significantly higher than the median estimate of 3.23 for our inventory model.

Overall, we have significantly lower shocks in the non-inventory model, i.e. lower standard deviations (consumption, capital investment and government spending). But the significant higher standard deviation of the price markup shock indicates that inflation can be better explained when we consider inventories. Note that all autocorrelation coefficients change significantly except the one for the technology shock.

The autocorrelations and cross-correlations are plotted in figure 1 (dashed grey lines). Pertaining to the shortcoming regarding the cross-correlations between consumption and investment discussed before we see that the model without inventories even does worse, albeit the models' cross-correlations are not significantly different. Without inventories, the estimated model seems to mimic the autocorrelation of several variables slightly better. But again, the results do not significantly change when we banish inventories.

Taking all together, the inclusion of inventories leads to a better fit in terms of correlations. The greatest improvement is obtained for those involving output growth. In particular the cross-correlations with lagged output growth are much better in comparison to the non-inventory model: for most of the variables the moments differ significantly. In almost all cases the obtained results of the model with inventories better match their empirical counterparts. The same also holds partly for lagged changes in total investment.

Figures 2 to 8 show the impulse responses of the non-inventory model (dashed grey lines). Most striking is the changing response of inflation for all shocks. These results can be attributed to changes in marginal cost only for half of the shocks. The price markup and capital investment shock lead to insignificant deviations in marginal cost's response while for a wage markup shock a lower reaction of marginal cost in the non-inventory model goes along with higher inflation (in comparison to the inventory model). Note that the inventories-to-sales ratio affects production and therefore marginal cost, both driving the price setting decision. Overall, the impulse responses of the model without inventories are similar only for a technology shock. For the remaining shocks we see significant deviations.

Are the observed differences in impulse responses caused by the models' equilibrium conditions or by the parameter estimates? To answer this question we use the parameter draws of the model with inventories and simulate the non-inventory model. Results for selected shocks and variables are presented in figure 9. The results indicate that the ratio of inventories to sales as additional time series used for estimation is the main source for the discrepancy: Only for shocks to capital investment we see partly significant differences between the two models. Different parameter estimates rather than the (partly) different equilibrium conditions create a wedge between the two models and their empirical implications.

On the one hand, the ratio of inventories to sales contains additional information compared to other time series frequently used for DSGE model estimation. On the other hand, the standard model enhanced with inventories ceteris paribus cannot explain the behaviour of the observed inventories-to-sales ratio since the inclusion of this time series leads to different estimation results. Admittedly, these changes are small in most instances but overall the estimation results change with regard to impulse responses, correlations and variance decompositions. However, misspecification could have led to biased estimation results in both models. Since DSGE models are stylized models they will never coincide with the true data generating process. As a result, misspecification will always be present and will affect parameter estimates as well as statistical inference. Moreover, the choice of observable variables in the estimation, i.e. the use of the ratio of inventories to sales as additional observation in the inventory model, influences the parameter estimates and, as a matter of course, the empirical results.

The analysis of the variance decompositions shown in table 8 reveals that capital investment specific shocks are more important in the estimated non-inventory model relative to the model with inventories. Except for the real wage growth series capital investment shocks explain a significantly larger fraction of the endogenous variables' forecast errors over all time horizons. Apart from inflation this is in line with findings of Justiniano et al. (2010) who attribute the forecast error variance of these variables mainly to the capital investment shock. Nevertheless, the estimation results for the model with inventories differ significantly. Furthermore, note that technology shocks explain about 1/4 while government spending shocks explain only circa 10% of the variations in output growth.

In the short and medium run government spending shocks are less important in the model without inventories. Furthermore, price markup shocks are the main reason for forecast errors for inflation in the artificial inventory economy while their importance shrinks significantly in a world without inventories. Therefore, capital investment shocks and shocks to labor supply are more relevant for unexpected deviations of inflation from its steady state. Without inventories, the wage markup shock is important with respect to variations in hours worked and real wage growth. As a result, this shock is responsible for more than 1/3 of the forecast error variance of inflation. When we accout for inventories, inflation is driven by shocks to labor supply only by at most 20% (depending on the time horizon).

5. Conclusion

We presented a New Keynesian DSGE model with inventories. As in Bils and Kahn (2000), firms face an increase in demand when they enlarge their stock of available goods. As a result, output and sales can temporarily differ and firms have to deal with a trade-off between cost of production and cost of storing goods. The model parameters are estimated using a Bayesian approach. To the seven macroeconomic variables commonly employed in DSGE model estimation we add the ratio of nonfarm inventories to final sales. Our estimation results differ from estimates of models without inventories. This deviation is attributed to the inclusion of the additional time series rather than to the model's characteristics.

Our model with inventories can compete with other models previously presented in the literature and the model does a good job in terms of autocorrelations and cross-correlations with regard to the endogenous variables and in particular the inventories-to-sales ratio. A decomposition of the forecast error variances shows that technology shocks mainly affect changes in the growth rate of real wages but have no significant influence on other variables. This is at odds with the strong effect of technology shocks on variations in output that are found in the literature (particularly in the long run). Furthermore, we find that monetary policy shocks are irrelevant for forecast errors over all time horizons. This indicates that studies focussing on monetary policy shocks only such as Jung and Yun (2005) and Kryvtsov and Midrigan (2010) examine factors of business cycles that are empirically of minor relevance.

While studies that do not incorporate inventories often identify capital investment shocks, labor market shocks and partly technology shocks as main driving force for forecast errors, our inventory model does not. With inventories, government spending shocks contribute most to sales growth variance and explain a significant fraction of the unexpected variance in output growth. This may indicate that the importance of technology shocks and capital investment shocks is overestimated when the inventory-sales time series is not considered and more attention should be paid to government spending shocks or variations in net exports.

Due to a change in parameter estimates most impulse responses change significantly in terms of magnitude and persistence when we add inventories to an otherwise standard New Keynesian model. Only for technology shocks the reactions are quite the same. For inflation we see significantly different responses to all shocks which can only in half of the cases assigned to marginal cost behaviour. Regarding the ratio of inventories to sales, we confirm the results in Jung and Yun (2005) while we cannot verify the findings in Kryvtsov and Midrigan (2010) who extended the model in Smets and Wouters (2007) with inventories and inventory adjustment cost. Overall, the inventories-to-sales ratio falls in response to demand shocks (shocks to capital investment, consumption, and government spending) and technology shocks. Monetary policy shocks and price markup shocks cause a rise in the ratio of inventories to sales. For wage markup shocks we obtain that the ratio ascends for about two years and then shrinks compared to its steady state level. For our model, the stylized fact of procyclical inventories and a countercyclical inventories-sales ratio is only observed for technology shocks and, in the short run, for wage markup shocks.

Our estimated model with inventories reveals significant differences with regard to impulse responses and variance decompositions. The main reason are the changing parameter estimates owing to the additional time series inventories to sales, although it must be kept in mind that the results depend on the specification of inventories in a theoretical economy. On the basis of our results further research regarding inventories and structural changes over time as well as monetary policy analysis should be pursued to examine whether the results of previous research still hold when inventory holding is incorporated.

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A. Appendix

Table 1. Estimation Results for the Model with Inventories

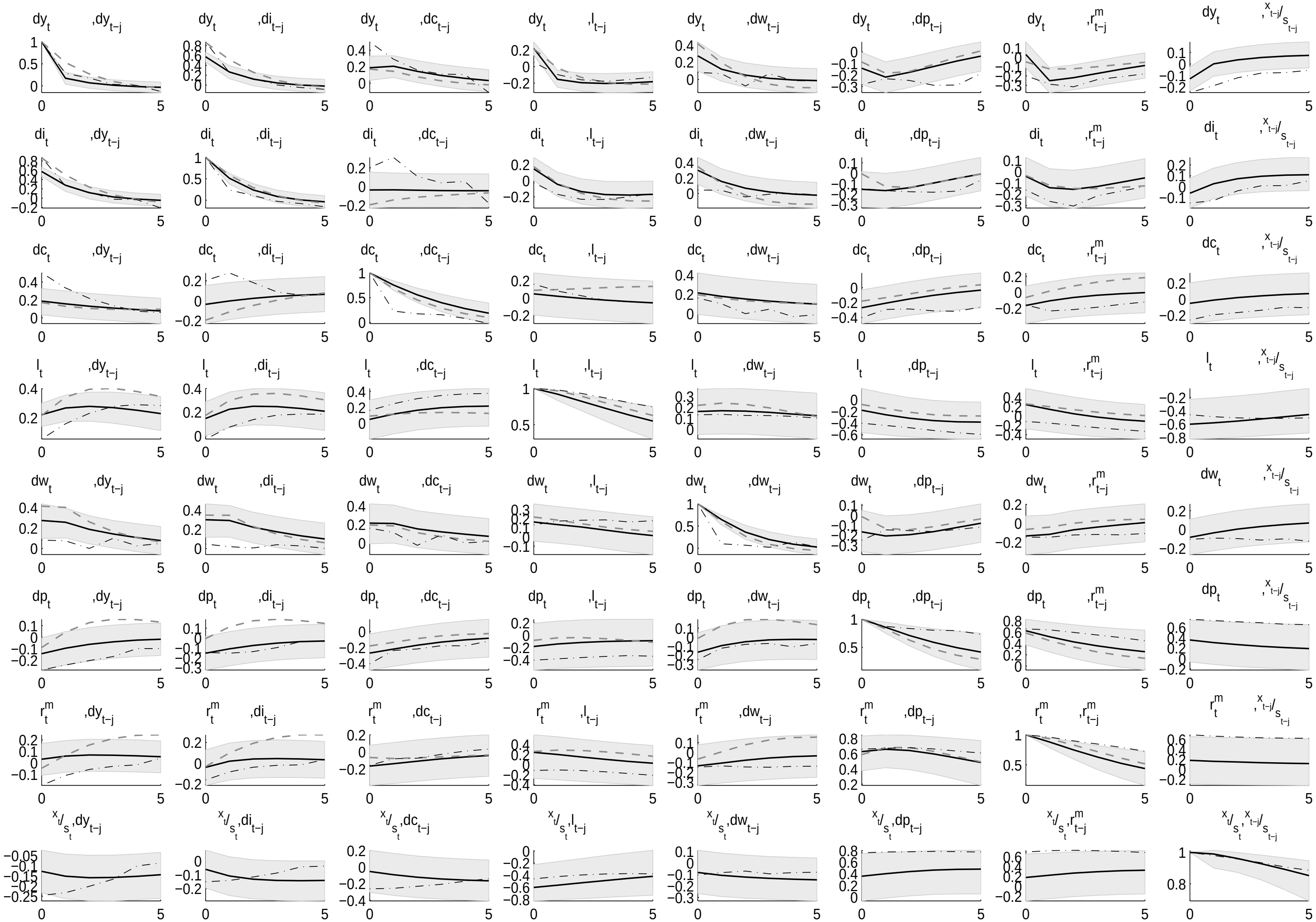
		Prior					Posterior			
		Dis.	Mean	SD	Mode	SD (Hes.)	5%	Med.	Mean	95%
$100\left(\frac{1}{\beta}-1\right)$	Discount factor (quarterly)	G	0.25	0.1	0.20	0.07	0.11	0.21	0.22	0.35
$100(v-1)$	StSt technology growth (quarterly)	N	0.4	0.1	0.36	0.07	0.26	0.36	0.36	0.47
$100(\pi-1)$	StSt inflation (quarterly)	G	0.62	0.1	0.63	0.11	0.51	0.67	0.68	0.87
l_{stst}	StSt hours worked	N	0	2	0.51	0.95	-1.03	0.57	0.57	2.18
b	Consumption habit	B	0.6	0.1	0.85	0.04	0.78	0.84	0.84	0.90
σ_l	Inverse Frisch elasticity	G	2	0.75	1.07	0.43	0.49	1.09	1.14	1.98
θ	Elasticity demand of avail. goods	N	0.6	0.2	0.31	0.04	0.27	0.33	0.33	0.41
ψ	Ratio storage areas to inventories	B	0.4	0.2	0.99	0.08	0.81	0.98	0.97	1.10
S''	Investment adjustment cost	N	4	1.5	2.92	0.81	2.05	3.23	3.31	4.90
σ_a	Elasticity capital adjustment cost	G	4	1.5	1.20	0.64	0.53	1.17	1.31	2.53
κ_p	Price adjustment cost	G	50	20	63.43	21.40	38.13	70.09	72.98	119.17
κ_w	Wage adjustment cost	G	50	20	70.57	25.02	48.54	84.66	87.92	138.43
γ_p	Price indexation	B	0.5	0.15	0.08	0.05	0.05	0.10	0.11	0.19
γ_w	Wage indexation	B	0.5	0.15	0.16	0.04	0.09	0.16	0.16	0.24
ρ_m	Interest rate smoothing	B	0.75	0.1	0.79	0.02	0.75	0.79	0.78	0.82
φ_π	Response to inflation	N	1.5	0.2	1.71	0.11	1.54	1.72	1.72	1.92
φ_y	Response to output	N	0.13	0.05	0.07	0.01	0.05	0.07	0.07	0.10
$\varphi_{\Delta\pi}$	Response to inflation difference	N	0.3	0.1	0.24	0.06	0.16	0.24	0.25	0.33
$\varphi_{\Delta y}$	Response to output difference	N	0.06	0.05	0.13	0.02	0.10	0.13	0.13	0.16
ρ_v	technology	B	0.5	0.2	0.22	0.06	0.11	0.21	0.21	0.31
ρ_p	price markup	B	0.5	0.2	0.98	0.04	0.84	0.94	0.93	0.98
ρ_w	wage markup	B	0.5	0.2	0.83	0.07	0.64	0.79	0.78	0.89
ρ_k	capital investment	B	0.5	0.2	0.57	0.07	0.45	0.57	0.57	0.69
ρ_h	storage area investment	B	0.5	0.2	0.63	0.05	0.55	0.64	0.64	0.73
ρ_c	consumption	B	0.5	0.2	0.90	0.03	0.83	0.91	0.90	0.95
ρ_g	government spending	B	0.5	0.2	0.92	0.02	0.89	0.92	0.92	0.95
σ_v	technology	I	0.1	1	1.19	0.10	1.02	1.17	1.18	1.36
σ_p	price markup	I	0.1	1	0.05	0.01	0.04	0.06	0.06	0.09
σ_w	wage markup	I	0.1	1	0.18	0.02	0.16	0.19	0.19	0.23
σ_k	capital investment	I	0.1	1	0.22	0.06	0.16	0.24	0.25	0.38
σ_h	storage area investment	I	0.1	1	0.26	0.02	0.23	0.26	0.26	0.30
σ_c	consumption	I	0.1	1	0.59	0.06	0.53	0.61	0.62	0.73
σ_g	government spending	I	0.1	1	1.25	0.07	1.15	1.26	1.26	1.38
σ_m	interest rate	I	0.1	1	0.20	0.01	0.19	0.21	0.21	0.23

The first three columns show the prior distributions of the estimated parameters. Column 6 reports the estimated posterior mode and column 7 the associated standard errors (taken from the Hessian). The last group contains the posterior distributions obtained by the Random-Walk Metropolis-Hastings algorithm.

Table 2. Estimation Results for the Model without Inventories

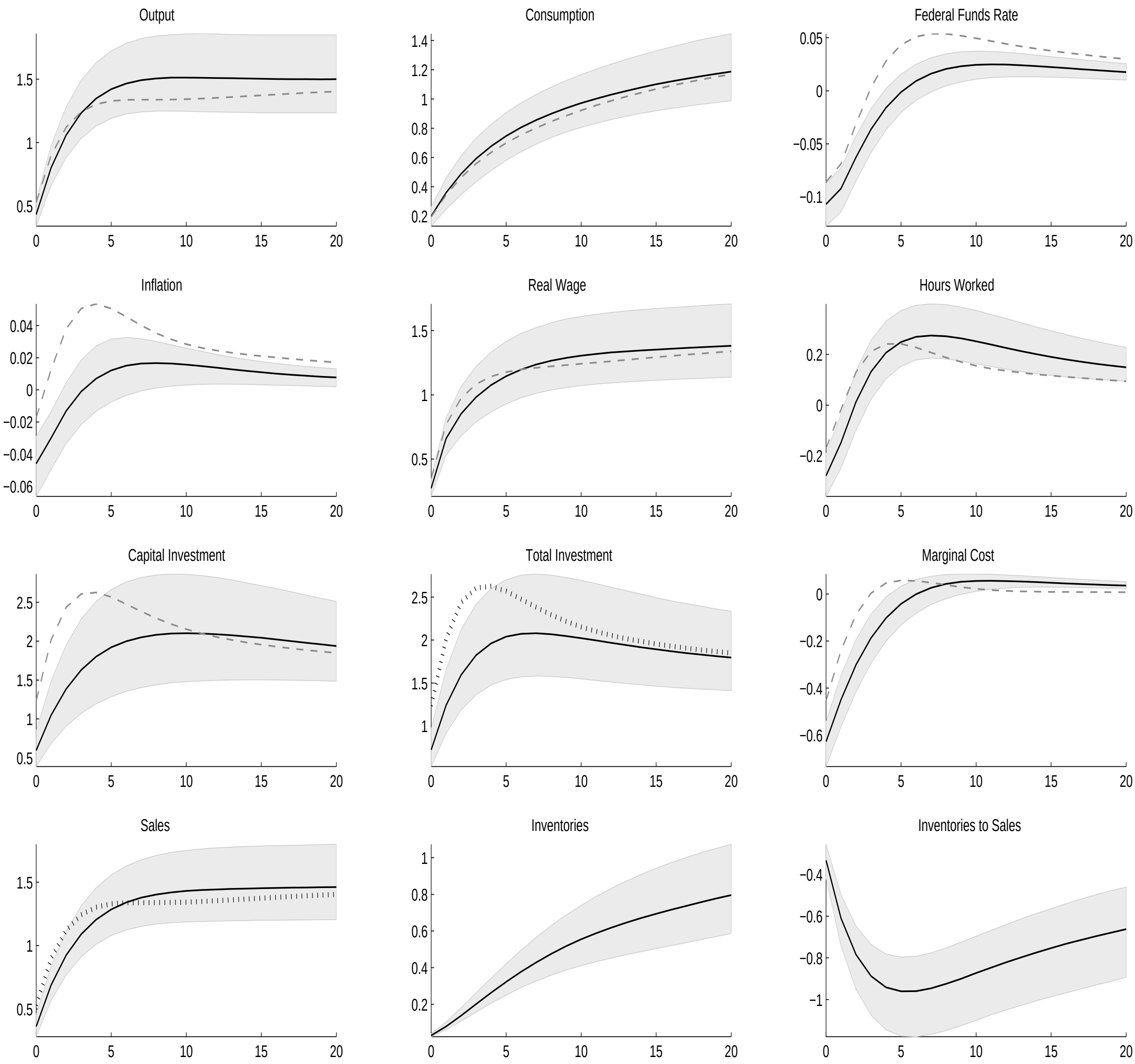
		Prior					Posterior			
		Dis.	Mean	SD	Mode	SD (Hes.)	5%	Med.	Mean	95%
$100\left(\frac{1}{\beta}-1\right)$	Discount factor (quarterly)	G	0.25	0.1	0.25	0.08	0.14	0.26	0.27	0.41
$100(v-1)$	StSt technology growth (quarterly)	N	0.4	0.1	0.33	0.07	0.20	0.32	0.32	0.44
$100(\pi-1)$	StSt inflation (quarterly)	G	0.62	0.1	0.68	0.11	0.53	0.69	0.70	0.87
l_{stst}	StSt hours worked	N	0	2	2.60	1.34	0.16	2.23	2.24	4.38
b	Consumption habit	B	0.6	0.1	0.83	0.04	0.76	0.83	0.83	0.89
σ_l	Inverse Frisch elasticity	G	2	0.75	1.42	0.34	1.01	1.53	1.56	2.21
μ^p	StSt price markup	N	0.15	0.05	0.24	0.04	0.18	0.24	0.24	0.30
S''	Investment adjustment cost	N	4	1.5	0.88	0.49	0.64	1.12	1.21	2.07
σ_a	Elasticity capital adjustment cost	G	4	1.5	7.13	1.68	4.84	7.17	7.35	10.45
κ_p	Price adjustment cost	G	50	20	52.79	18.14	34.69	56.61	59.63	96.21
κ_w	Wage adjustment cost	G	50	20	29.70	9.47	23.01	37.23	39.76	65.56
γ_p	Price indexation	B	0.5	0.15	0.21	0.09	0.12	0.24	0.25	0.39
γ_w	Wage indexation	B	0.5	0.15	0.19	0.05	0.12	0.19	0.19	0.27
ρ_m	Interest rate smoothing	B	0.75	0.1	0.76	0.02	0.73	0.77	0.76	0.80
φ_π	Response to inflation	N	1.5	0.2	1.86	0.12	1.70	1.87	1.87	2.07
φ_y	Response to output	N	0.13	0.05	0.09	0.01	0.06	0.09	0.09	0.11
$\varphi_{\Delta\pi}$	Response to inflation difference	N	0.3	0.1	0.26	0.06	0.17	0.26	0.26	0.35
$\varphi_{\Delta y}$	Response to output difference	N	0.06	0.05	0.16	0.02	0.12	0.15	0.15	0.19
ρ_v	technology	B	0.5	0.2	0.30	0.05	0.21	0.30	0.30	0.39
ρ_p	price markup	B	0.5	0.2	0.84	0.05	0.73	0.83	0.83	0.90
ρ_w	wage markup	B	0.5	0.2	0.95	0.02	0.89	0.94	0.94	0.97
ρ_k	capital investment	B	0.5	0.2	0.89	0.04	0.79	0.87	0.87	0.93
ρ_c	consumption	B	0.5	0.2	0.75	0.07	0.63	0.77	0.76	0.86
ρ_g	government spending	B	0.5	0.2	0.997	0.004	0.987	0.995	0.994	0.999
σ_v	technology	I	0.1	1	1.05	0.07	0.96	1.06	1.07	1.18
σ_p	price markup	I	0.1	1	0.13	0.02	0.10	0.13	0.13	0.16
σ_w	wage markup	I	0.1	1	0.24	0.03	0.18	0.23	0.23	0.30
σ_k	capital investment	I	0.1	1	0.09	0.02	0.08	0.10	0.10	0.13
σ_c	consumption	I	0.1	1	0.51	0.04	0.47	0.52	0.53	0.59
σ_g	government spending	I	0.1	1	0.48	0.03	0.45	0.49	0.49	0.54
σ_m	interest rate	I	0.1	1	0.22	0.01	0.20	0.22	0.22	0.24

The first three columns show the prior distributions of the estimated parameters. Column 6 reports the estimated posterior mode and column 7 the associated standard errors (taken from the Hessian). The last group contains the posterior distributions obtained by the Random-Walk Metropolis-Hastings algorithm.



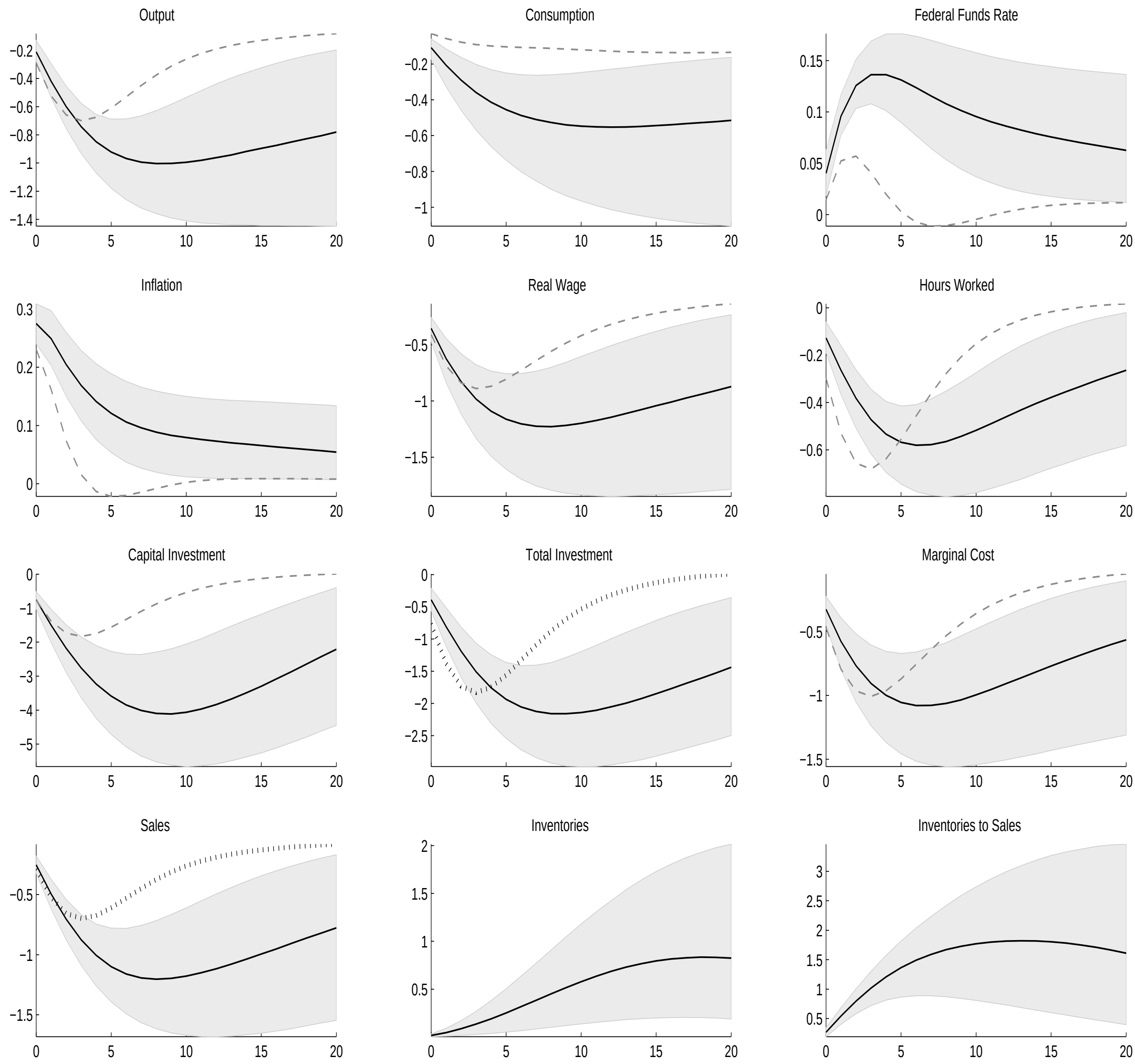
The black and grey lines show the median and the 5% & 95% percentile autocorrelations of the estimated model with inventories, respectively. The dashed grey line reveals the median autocorrelations of the estimated model without inventories. The dash-dotted black line stands for autocorrelations in the data.

Figure 2. Response to a Technology Shock



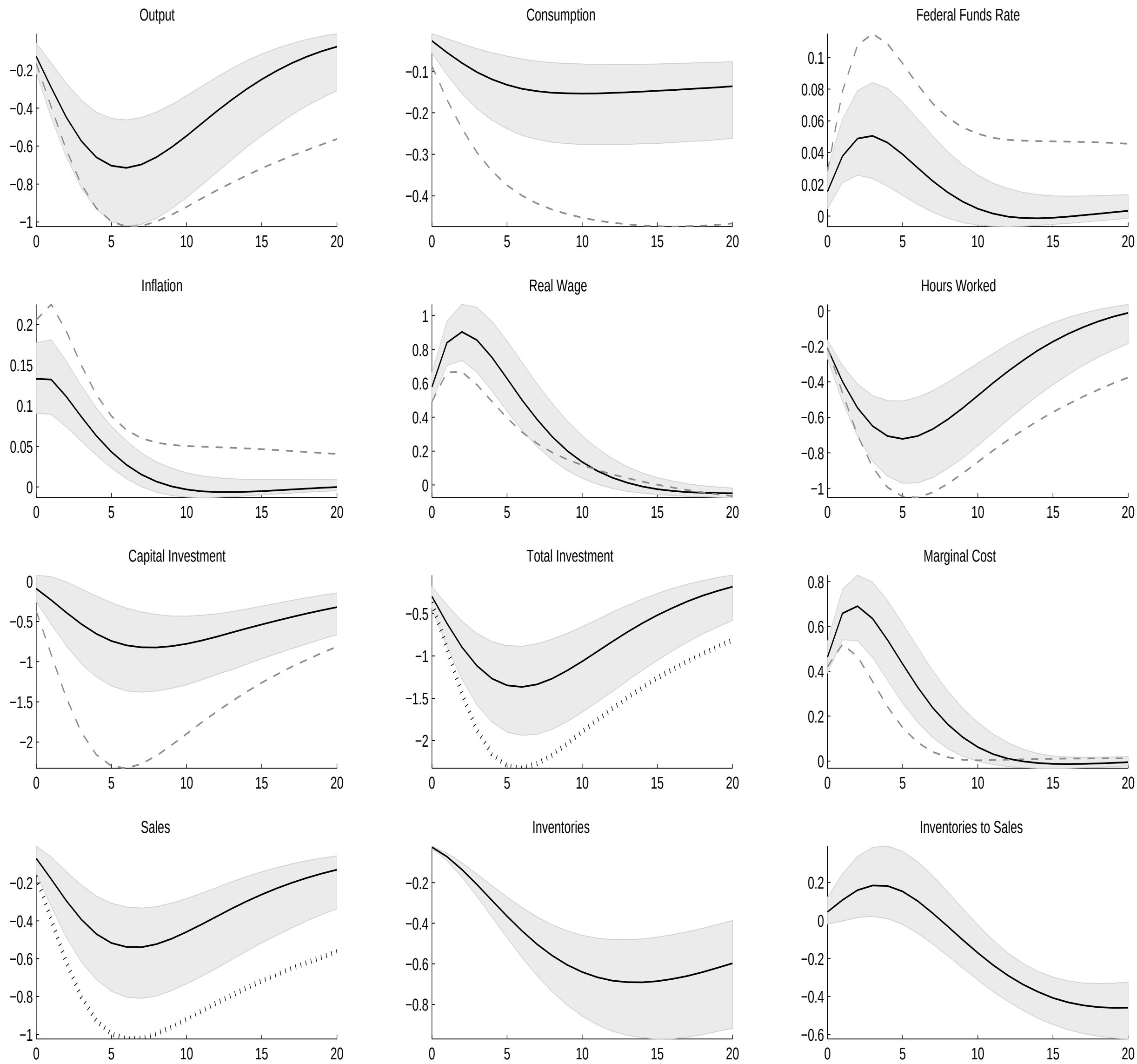
Medians, 5% and 95% percentile responses (solid lines). Dashed lines are median responses for the estimated model without inventories. Dotted lines show median response of the non-inventory model for output (sales) and investment (total investment).

Figure 3. Response to a Price Markup Shock



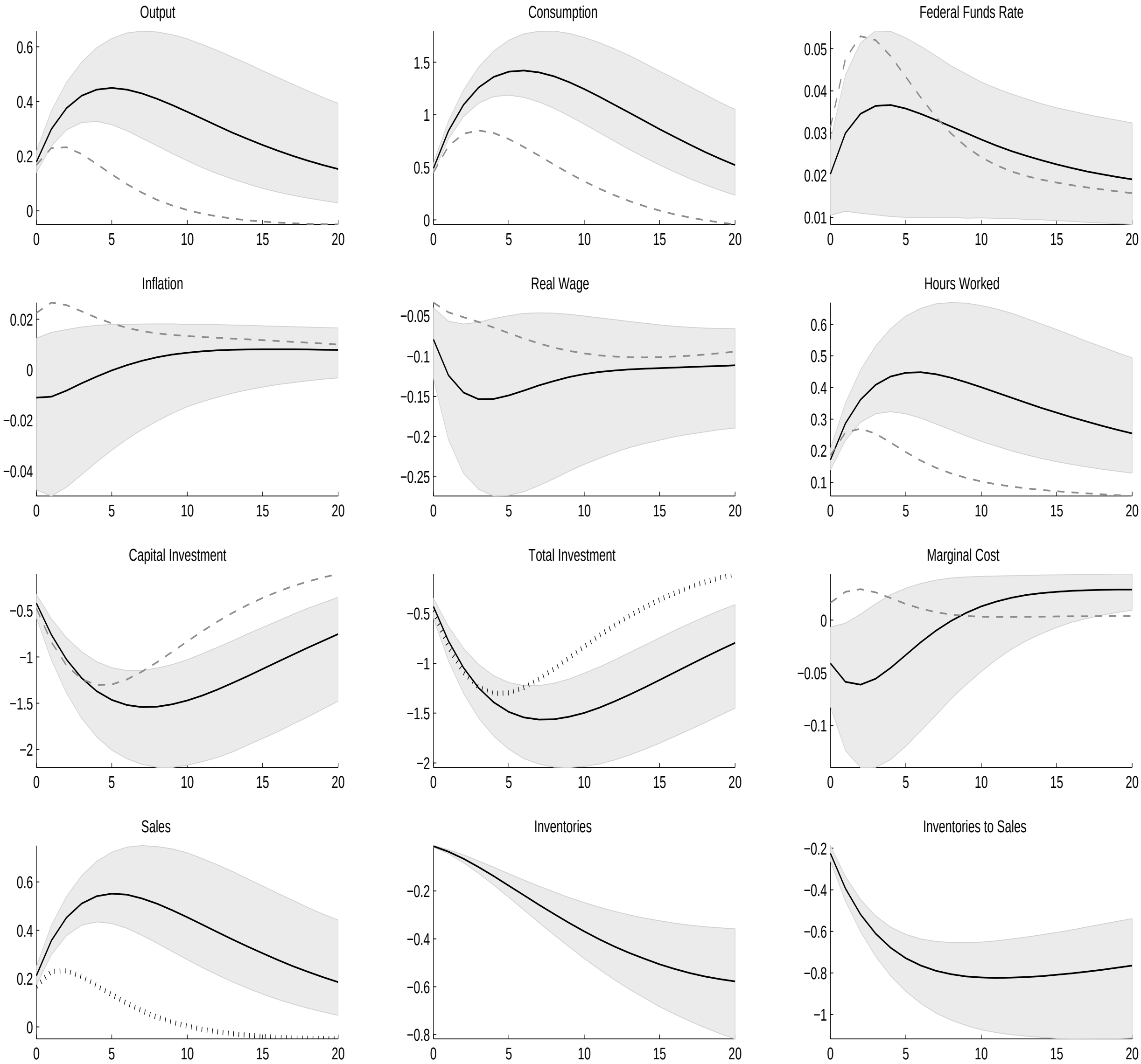
Medians, 5% and 95% percentile responses (solid lines). Dashed lines are median responses for the estimated model without inventories. Dotted lines show median response of the non-inventory model for output (sales) and investment (total investment).

Figure 4. Response to a Wage Markup Shock



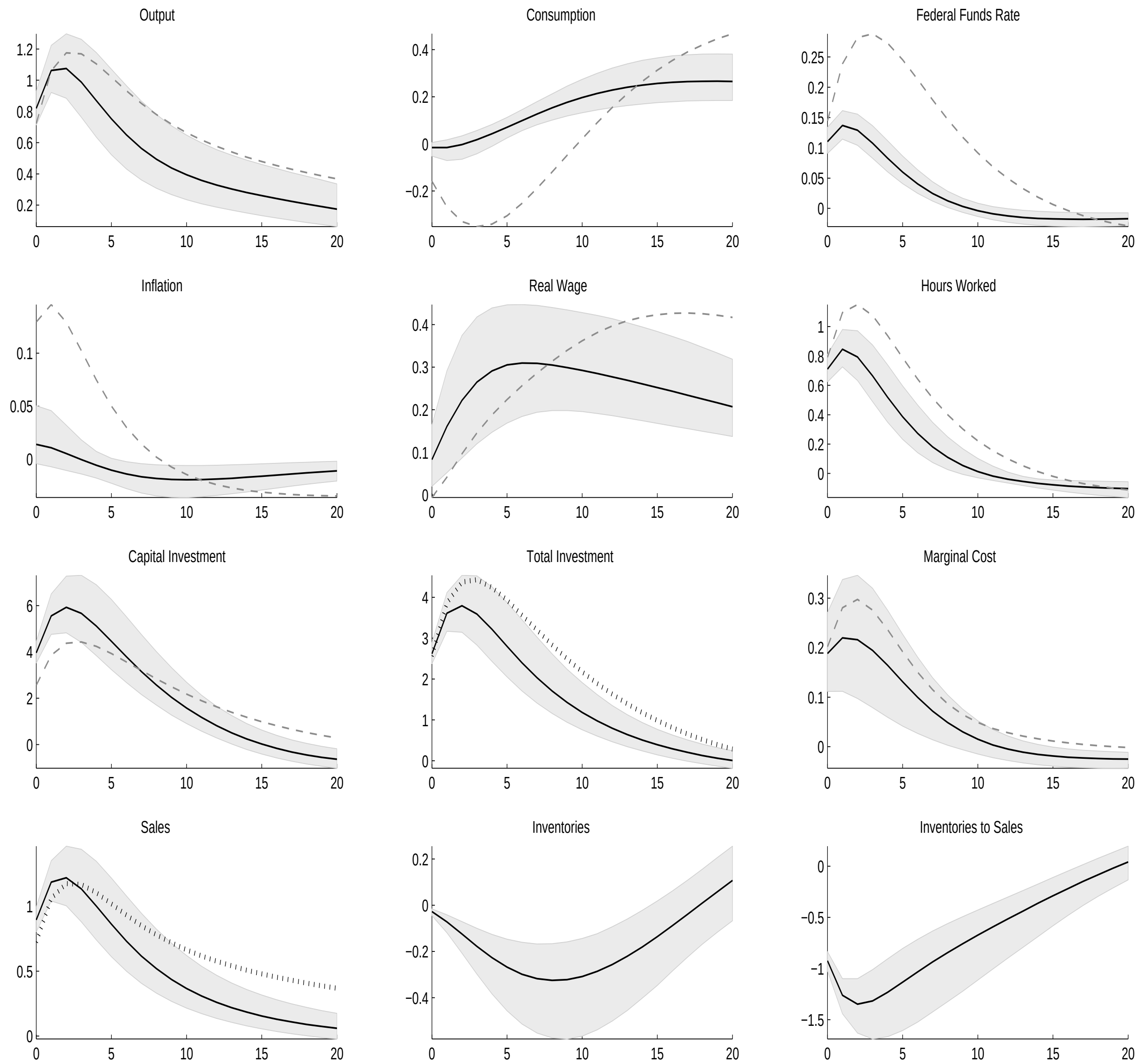
Medians, 5% and 95% percentile responses (solid lines). Dashed lines are median responses for the estimated model without inventories. Dotted lines show median response of the non-inventory model for output (sales) and investment (total investment).

Figure 5. Response to a Consumption Shock



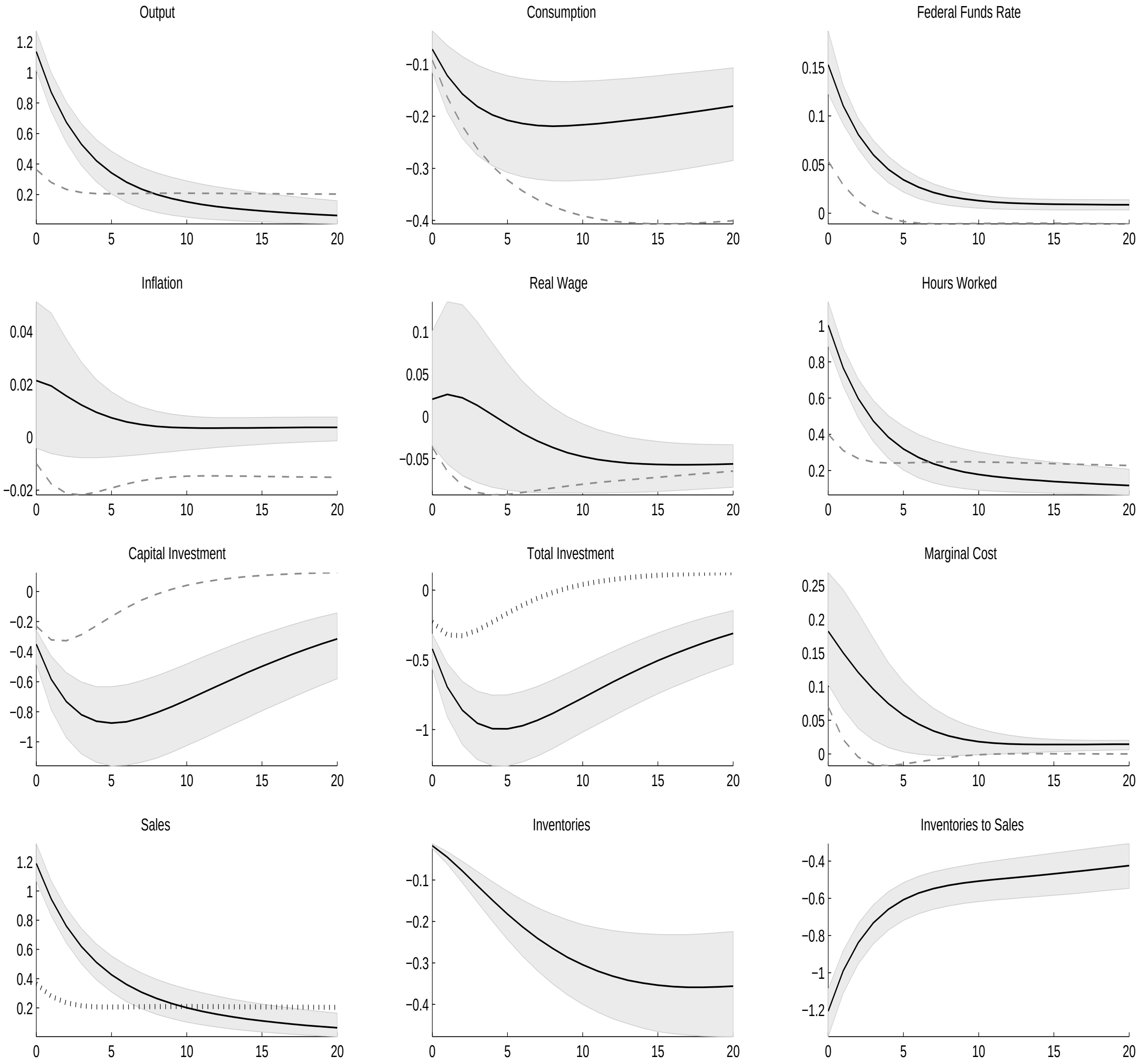
Medians, 5% and 95% percentile responses (solid lines). Dashed lines are median responses for the estimated model without inventories. Dotted lines show median response of the non-inventory model for output (sales) and investment (total investment).

Figure 6. Response to a Capital Investment Shock



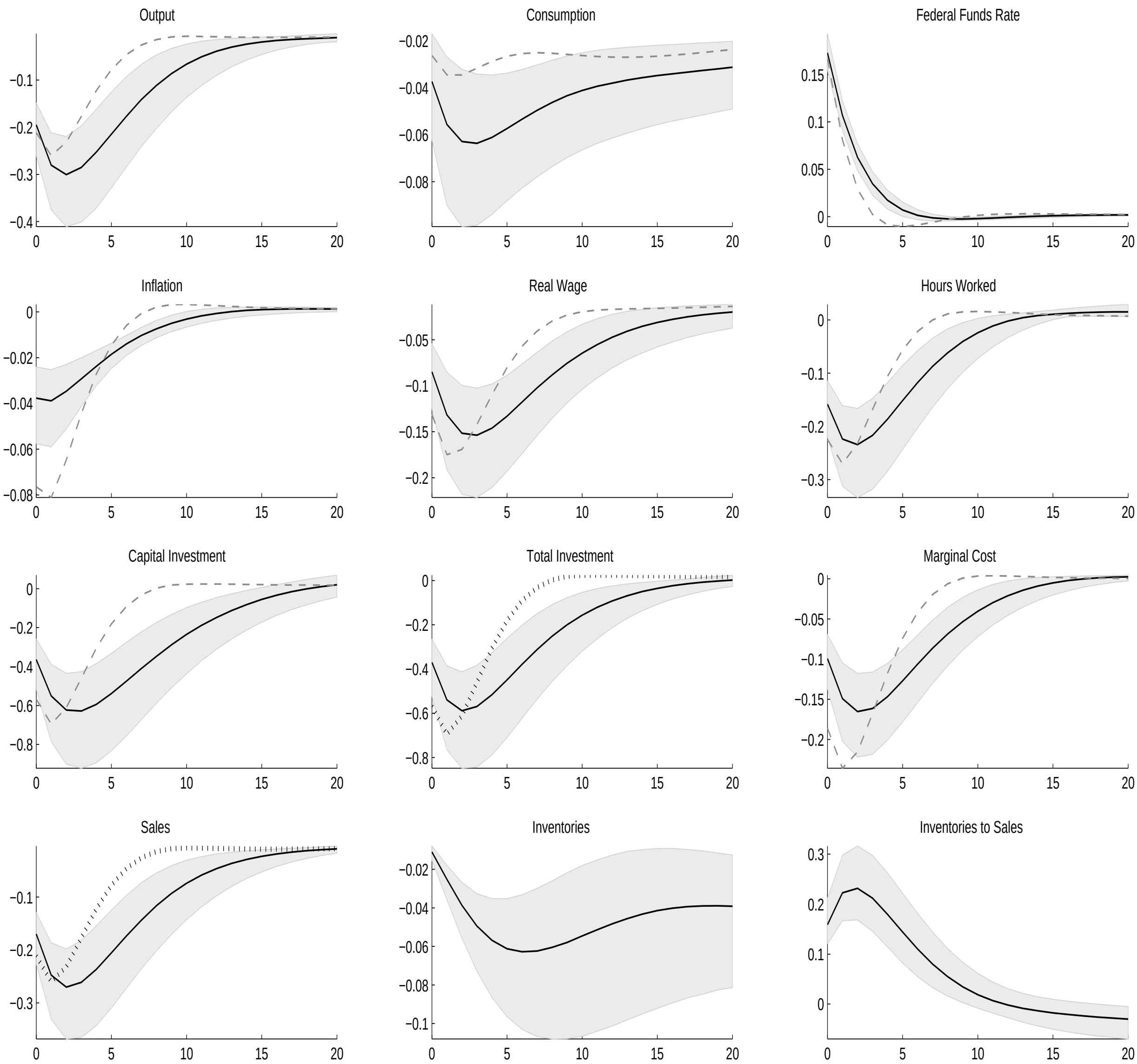
Medians, 5% and 95% percentile responses (solid lines). Dashed lines are median responses for the estimated model without inventories. Dotted lines show median response of the non-inventory model for output (sales) and investment (total investment).

Figure 7. Response to a Government Spending Shock



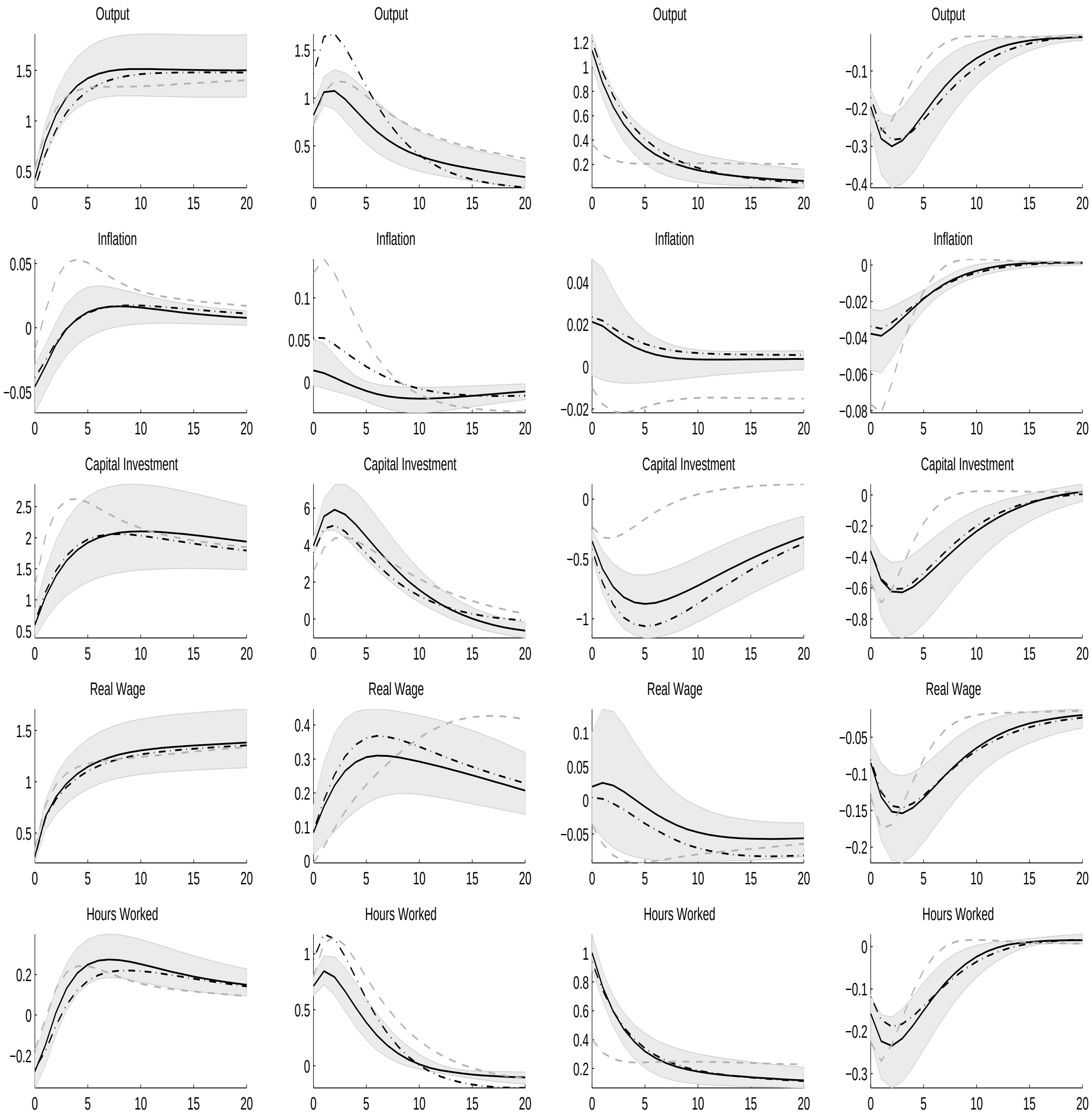
Medians, 5% and 95% percentile responses (solid lines). Dashed lines are median responses for the estimated model without inventories. Dotted lines show median response of the non-inventory model for output (sales) and investment (total investment).

Figure 8. Response to a Monetary Policy Shock



Medians, 5% and 95% percentile responses (solid lines). Dashed lines are median responses for the estimated model without inventories. Dotted lines show median response of the non-inventory model for output (sales) and investment (total investment).

Figure 9. Comparing Impulse Responses



Medians, 5% and 95% percentile responses of the model with inventories (solid lines). Dashed lines are median responses for the model without inventories. Simulated responses of the non-inventory model using parameter draws from the estimation of the inventory model are shown by the dash-dot line. Columns show (from left to right) responses to shocks to: technology, capital investment, government spending, monetary policy.

Table 3. Variance Decomposition: All Horizons (Model with Inventories)

Variable	Shock η_t^c *cons.*	η_t^k *cap.*	η_t^w *labor*	η_t^v *tech.*	η_t^p *prices*	η_t^g *gov.*	η_t^m *mon.*	η_t^h *inv. cost*
Output Growth	1.3	16.4	2.6	9.0	3.8	29.4	1.1	35.5
	[0.8, 2.0]	[12.8, 20.3]	[1.1, 5.0]	[6.3, 12.8]	[2.5, 5.7]	[24.1, 35.1]	[0.6, 1.9]	[30.0, 41.9]
Total Invest-	4.1	67.7	3.5	7.6	6.4	2.5	1.5	5.6
ment Growth	[2.6, 6.4]	[58.4, 76.5]	[1.5, 6.5]	[4.5, 12.3]	[3.6, 10.2]	[1.4, 4.1]	[0.8, 2.8]	[3.0, 8.9]
Real Wage	0.8	1.9	42.2	25.3	26.5	0.1	0.9	0.7
Growth	[0.2, 2.3]	[0.7, 4.3]	[27.0, 56.5]	[18.6, 33.8]	[15.6, 39.6]	[0.0, 1.2]	[0.4, 1.9]	[0.1, 3.3]
Hours Worked	13.3	12.3	16.6	4.7	18.8	11.7	1.0	15.2
	[5.8, 33.9]	[7.7, 18.5]	[7.3, 34.1]	[2.6, 7.8]	[7.3, 37.2]	[7.8, 16.8]	[0.5, 2.1]	[9.7, 22.5]
Inflation	0.8	1.1	11.8	1.4	76.0	0.4	1.2	5.5
	[0.1, 4.2]	[0.1, 4.6]	[4.2, 24.6]	[0.4, 3.4]	[50.5, 91.7]	[0.0, 2.0]	[0.4, 2.9]	[1.7, 14.5]
Interest Rate	3.2	11.6	1.8	5.2	38.3	7.7	6.6	21.3
	[0.8, 7.9]	[4.5, 19.8]	[0.4, 5.6]	[2.3, 8.5]	[14.9, 73.6]	[3.3, 12.3]	[2.9, 10.9]	[8.4, 38.1]
Sales Growth	2.6	28.3	1.8	9.9	7.5	44.1	1.2	3.7
	[1.8, 3.8]	[23.7, 33.8]	[0.7, 4.0]	[6.9, 14.0]	[4.5, 11.0]	[37.5, 50.8]	[0.7, 2.1]	[2.0, 6.0]
Ratio Inventories	11.7	10.0	2.2	9.9	37.3	5.5	0.2	16.6
to Sales	[3.4, 28.3]	[3.1, 20.0]	[0.9, 4.8]	[3.6, 16.7]	[8.5, 78.9]	[2.0, 10.0]	[0.0, 0.4]	[4.9, 37.2]

Medians and 5th/95th percentiles. Percentage values.

Table 4. Variance Decomposition: $t = 0$ (Model with Inventories)

Variable	**Shock**							
	η_t^c *cons.*	η_t^k *cap.*	η_t^w *labor*	η_t^v *tech.*	η_t^p *prices*	η_t^g *gov.*	η_t^m *mon.*	η_t^h *inv. cost*
Output Growth	0.8	17.4	0.4	4.9	1.2	33.6	1.0	40.0
	[0.5, 1.2]	[13.7, 21.7]	[0.1, 1.3]	[2.9, 7.9]	[0.4, 2.1]	[28.0, 39.5]	[0.6, 1.8]	[34.0, 46.9]
Total Invest-	2.1	78.9	1.0	6.0	1.8	2.0	1.6	5.8
ment Growth	[1.4, 3.5]	[70.2, 85.8]	[0.4, 2.2]	[3.3, 10.4]	[0.5, 3.6]	[1.1, 3.6]	[0.8, 3.0]	[2.9, 9.9]
Real Wage	1.1	1.2	59.4	12.8	22.0	0.1	1.2	0.6
Growth	[0.3, 2.8]	[0.1, 4.2]	[40.2, 74.2]	[8.0, 19.9]	[12.4, 34.9]	[0.0, 1.6]	[0.6, 2.5]	[0.0, 3.9]
Hours Worked	1.0	17.4	1.5	2.6	0.6	34.7	0.9	40.8
	[0.6, 1.5]	[13.9, 21.4]	[0.9, 2.6]	[1.3, 4.5]	[0.1, 1.3]	[29.2, 40.3]	[0.5, 1.6]	[34.3, 48.2]
Inflation	0.2	0.2	16.8	2.0	72.5	0.4	1.3	5.7
	[0.0, 2.1]	[0.0, 2.2]	[8.5, 26.8]	[0.8, 4.0]	[53.4, 85.0]	[0.0, 2.3]	[0.6, 2.9]	[2.2, 12.7]
Interest Rate	0.4	10.8	0.2	10.0	1.4	20.7	26.6	28.8
	[0.1, 0.7]	[8.4, 13.6]	[0.0, 0.8]	[7.3, 13.5]	[0.3, 3.5]	[16.0, 25.6]	[19.2, 35.8]	[22.3, 36.7]
Sales Growth	1.7	30.7	0.2	5.0	2.4	54.2	1.1	4.0
	[1.1, 2.4]	[25.7, 36.2]	[0.0, 0.9]	[2.9, 8.2]	[1.2, 4.1]	[47.9, 60.1]	[0.6, 2.0]	[1.9, 6.6]
Ratio Inventories	2.0	33.0	0.1	4.3	2.8	56.3	1.0	0.1
to Sales	[1.3, 2.7]	[27.7, 38.8]	[0.0, 0.6]	[2.4, 7.0]	[1.4, 4.7]	[49.8, 62.4]	[0.5, 1.8]	[0.0, 1.0]

Medians and 5th/95th percentiles. Percentage values.

Table 5. Variance Decomposition: $t = 4$ (Model with Inventories)

Variable	**Shock**							
	η_t^c	η_t^k	η_t^w	η_t^v	η_t^p	η_t^g	η_t^m	η_t^h
	cons.	*cap.*	*labor*	*tech.*	*prices*	*gov.*	*mon.*	*inv. cost*
Output Growth	1.2	16.2	1.9	9.3	3.3	30.7	1.0	35.5
	[0.8, 1.9]	[12.7, 20.2]	[0.8, 4.1]	[6.5, 13.3]	[2.1, 5.0]	[25.3, 36.4]	[0.6, 1.8]	[30.0, 42.0]
Total Invest-	3.8	69.1	2.9	8.4	5.5	2.5	1.5	5.4
ment Growth	[2.4, 5.9]	[59.5, 77.8]	[1.3, 5.6]	[5.0, 13.6]	[3.0, 8.8]	[1.4, 4.1]	[0.8, 2.8]	[2.8, 8.8]
Real Wage	0.9	1.8	40.5	27.3	26.2	0.1	0.9	0.6
Growth	[0.2, 2.5]	[0.5, 4.4]	[25.6, 55.4]	[20.1, 36.2]	[15.2, 39.6]	[0.0, 1.1]	[0.4, 1.8]	[0.0, 2.9]
Hours Worked	5.0	21.6	12.0	1.5	6.3	19.5	1.8	30.4
	[3.3, 8.0]	[16.5, 28.1]	[6.4, 20.7]	[1.1, 2.2]	[3.4, 10.0]	[15.0, 24.8]	[1.0, 3.4]	[23.4, 39.4]
Inflation	0.2	0.2	18.0	1.1	69.9	0.4	1.7	7.1
	[0.0, 3.2]	[0.0, 1.9]	[8.3, 31.1]	[0.4, 2.6]	[47.7, 84.8]	[0.0, 2.2]	[0.7, 3.6]	[2.8, 15.9]
Interest Rate	1.3	16.6	2.2	6.4	16.2	12.0	11.8	31.7
	[0.2, 2.8]	[12.1, 22.6]	[0.5, 5.9]	[4.1, 9.8]	[10.2, 24.3]	[8.8, 15.8]	[8.3, 16.2]	[21.8, 43.1]
Sales Growth	2.4	27.7	1.4	10.1	6.5	46.4	1.1	3.6
	[1.7, 3.5]	[23.1, 33.0]	[0.5, 3.3]	[7.0, 14.4]	[3.8, 9.6]	[39.7, 53.0]	[0.6, 2.0]	[1.9, 5.9]
Ratio Inventories	5.8	33.5	0.5	12.2	15.4	18.2	0.9	11.5
to Sales	[4.1, 8.2]	[24.9, 43.3]	[0.0, 2.0]	[8.8, 17.0]	[8.5, 23.4]	[13.3, 24.5]	[0.5, 1.8]	[6.9, 19.5]

Medians and 5th/95th percentiles. Percentage values.

Table 6. Variance Decomposition: $t = 10$ (Model with Inventories)

Variable				**Shock**				
	η_t^c *cons.*	η_t^k *cap.*	η_t^w *labor*	η_t^v *tech.*	η_t^p *prices*	η_t^g *gov.*	η_t^m *mon.*	η_t^h *inv. cost*
Output Growth	1.2	16.4	2.1	9.2	3.5	29.8	1.1	35.9
	[0.8, 1.9]	[12.9, 20.4]	[0.9, 4.3]	[6.4, 13.0]	[2.2, 5.2]	[24.5, 35.5]	[0.6, 1.9]	[30.4, 42.3]
Total Invest-	3.6	69.2	3.0	7.9	5.6	2.4	1.5	5.8
ment Growth	[2.3, 5.7]	[59.9, 77.7]	[1.3, 5.7]	[4.7, 12.7]	[3.2, 9.0]	[1.3, 4.0]	[0.8, 2.9]	[3.1, 9.2]
Real Wage	0.8	1.8	42.9	26.0	25.1	0.1	0.9	0.7
Growth	[0.2, 2.3]	[0.6, 4.2]	[27.4, 57.4]	[19.2, 34.6]	[14.4, 38.6]	[0.0, 1.2]	[0.4, 1.9]	[0.0, 3.4]
Hours Worked	8.9	15.0	20.1	3.1	13.9	14.0	1.4	20.1
	[4.9, 17.5]	[10.6, 21.2]	[9.8, 36.1]	[2.0, 5.3]	[7.9, 21.4]	[10.3, 18.8]	[0.7, 2.9]	[14.4, 28.1]
Inflation	0.3	0.6	15.5	1.3	71.5	0.4	1.6	7.1
	[0.1, 3.5]	[0.1, 2.8]	[6.6, 28.6]	[0.5, 3.0]	[48.4, 86.7]	[0.0, 2.1]	[0.7, 3.4]	[2.8, 16.1]
Interest Rate	2.2	13.8	2.3	5.3	27.1	9.7	8.9	28.2
	[0.3, 4.6]	[8.8, 20.7]	[0.5, 6.9]	[3.5, 7.9]	[14.3, 42.9]	[6.6, 13.5]	[6.2, 12.4]	[17.9, 41.9]
Sales Growth	2.4	28.5	1.5	10.1	6.7	45.0	1.2	3.8
	[1.7, 3.4]	[23.9, 33.9]	[0.6, 3.4]	[7.0, 14.3]	[3.9, 10.1]	[38.4, 51.6]	[0.7, 2.1]	[2.0, 6.1]
Ratio Inventories	7.3	18.4	0.3	11.5	27.5	8.5	0.4	23.2
to Sales	[4.5, 12.2]	[11.0, 29.8]	[0.1, 1.1]	[7.7, 16.7]	[11.2, 44.6]	[5.8, 12.7]	[0.2, 0.8]	[13.7, 38.4]

Medians and 5th/95th percentiles. Percentage values.

Table 7. Variance Decomposition: Ratio Inventories to Sales (Model with Inventories)

Shock	**Time Horizon**					
	On impact	1 year	2.5 years	5 years	10 years	25 years
η_t^c (consumption)	2.0	5.8	7.3	8.5	10.5	11.7
	[1.3, 2.7]	[4.1, 8.2]	[4.5, 12.2]	[4.2, 16.7]	[3.7, 23.7]	[3.5, 28.1]
η_t^k (capital)	33.0	33.5	18.4	10.2	9.2	10.0
	[27.7, 38.8]	[24.9, 43.3]	[11.0, 29.8]	[4.9, 20.0]	[3.3, 18.8]	[3.2, 20.1]
η_t^w (wages)	0.1	0.5	0.3	1.3	2.1	2.2
	[0.0, 0.6]	[0.0, 2.0]	[0.1, 1.1]	[0.8, 2.4]	[1.0, 4.3]	[1.0, 4.8]
η_t^v (technology)	4.3	12.2	11.5	10.0	10.0	9.9
	[2.4, 7.0]	[8.8, 17.0]	[7.7, 16.7]	[5.6, 16.0]	[4.2, 16.9]	[3.6, 16.7]
η_t^p (prices)	2.8	15.4	27.5	37.5	39.1	37.2
	[1.4, 4.7]	[8.5, 23.4]	[11.2, 44.6]	[10.6, 64.3]	[8.8, 75.5]	[8.5, 78.5]
η_t^g (government)	56.3	18.2	8.5	5.9	5.5	5.5
	[49.8, 62.4]	[13.3, 24.5]	[5.8, 12.7]	[3.4, 9.8]	[2.3, 9.8]	[2.0, 10.0]
η_t^m (monetary)	1.0	0.9	0.4	0.2	0.2	0.2
	[0.5, 1.8]	[0.5, 1.8]	[0.2, 0.8]	[0.1, 0.5]	[0.0, 0.4]	[0.0, 0.4]
η_t^h (inv. cost)	0.1	11.5	23.2	22.8	18.2	16.7
	[0.0, 1.0]	[6.9, 19.5]	[13.7, 38.4]	[10.5, 43.9]	[6.2, 40.1]	[5.0, 37.2]

Medians and 5th/95th percentiles. Percentage values.

Table 8. Variance Decomposition (Model without Inventories)

Variable	**Shock**						
	η_t^c *cons.*	η_t^k *cap.*	η_t^w *labor*	η_t^v *tech.*	η_t^p *prices*	η_t^g *gov.*	η_t^m *mon.*
All Horizons							
Output Growth	2.1	37.9	11.5	26.3	10.5	7.7	3.1
Consumption Growth	55.5	15.4	4.9	18.6	0.5	3.6	0.1
Investment Growth	3.5	60.9	7.8	15.0	8.8	0.5	2.6
Real Wage Growth	0.2	2.1	30.1	34.2	30.0	0.2	2.2
Hours Worked	1.6	23.4	40.5	2.0	8.0	18.8	0.7
Inflation	1.2	20.0	42.7	4.3	15.4	7.1	3.4
Interest Rate	2.8	62.5	15.8	5.9	1.4	3.5	3.9
t = 0							
Output Growth	2.5	46.3	2.4	24.8	7.5	11.8	3.9
Consumption Growth	71.2	8.8	2.6	12.9	0.3	3.0	0.2
Investment Growth	2.4	69.4	1.4	16.1	6.1	0.6	3.4
Real Wage Growth	0.2	0.2	43.4	21.8	30.1	0.2	3.1
Hours Worked	3.3	60.1	4.0	2.8	8.5	15.4	4.9
Inflation	0.4	14.0	35.0	0.3	44.1	0.1	4.8
Interest Rate	1.6	33.7	1.3	12.0	0.4	4.6	45.4
t = 4							
Output Growth	2.0	37.9	10.5	28.1	9.3	8.2	3.1
Consumption Growth	60.8	8.9	5.2	18.9	0.5	4.2	0.2
Investment Growth	3.2	61.1	7.1	16.6	7.9	0.5	2.8
Real Wage Growth	0.1	1.3	29.8	37.2	28.3	0.3	2.2
Hours Worked	2.7	49.0	23.8	1.5	15.8	4.2	2.1
Inflation	0.8	19.9	45.9	2.1	23.8	0.5	5.4
Interest Rate	2.5	72.7	10.1	3.3	1.9	0.9	8.1
t = 10							
Output Growth	2.1	38.0	10.6	26.8	10.4	7.8	3.2
Consumption Growth	58.3	11.8	4.9	19.0	0.5	4.0	0.1
Investment Growth	3.2	60.9	7.1	15.7	9.0	0.5	2.8
Real Wage Growth	0.2	1.8	30.3	34.8	29.6	0.2	2.2
Hours Worked	2.1	34.5	42.2	2.0	12.5	4.1	1.1
Inflation	1.1	18.9	47.0	4.2	21.3	0.8	4.8
Interest Rate	2.7	74.0	11.1	4.5	1.3	0.7	5.2

Medians and 5th/95th percentiles. Percentage values.