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Identification Through Heteroscedasticity in a Multicountry and Multimarket Framework*

The Effects of European Central Banks on European Financial Markets

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Identification through Heteroscedasticity in a Multicountry and Multimarket Framework: The Effects of European Central Banks on European Financial Markets

Abstract
This paper formally proves that Rigobon and Sack (2004)’s approach of identifying monetary policy shocks through heteroscedasticity can be extended to a multimarket and multicountry framework. Applying our multivariate framework allows deriving consistent estimators of monetary policy effects. The advantage of our extended approach is illustrated by applying it to European financial markets. We analyse monetary policy actions of the European Central Bank (ECB), the Bank of England, the Swiss National Bank, and the Swedish Riksbank on major stock indices. First, in line with the Rigobon and Sack (2004) approach, we find an increase in the variance of European stock and money market returns on days when monetary policy committee meetings are held. Second, monetary policy actions have a significant impact on financial markets. Third, we discover that ECB monetary policy moves have spillover effects on the British and Swiss financial markets, but find no evidence of reverse causality.

Keywords: Financial markets, instrumental variable estimation, identification through heteroscedasticity, spillover effects

JEL Classification Numbers: C36, E44, E52, G15

MSC 2000 classification: 62H12, 62P20, 91B64
1. Introduction

A profound knowledge of the monetary transmission mechanism is essential to effective monetary policy. Understanding the underlying processes helps central bankers implement appropriate monetary policy actions. This applies to the transmission of monetary policy shocks both to the real economy as well as to financial markets. Here, we focus on the transmission of such shocks to financial markets. As a consequence of increased globalisation, reduced barriers to international capital flows, and computerised trading, financial markets are increasingly integrated. Thus, when conducting monetary policy, central banks need to take into consideration that their policies may affect more than just one market. In addition, internationally integrated financial markets react not only to domestic, but also to foreign, monetary policy. Thus, an assessment of monetary policy transmission in today’s globalised world requires a multicountry and multimarket approach.

In a seminal paper, Rigobon and Sack (2004) propose to identify monetary policy shocks through an approach based on heteroscedasticity. They note that the channel from monetary policy to financial markets is not unidirected: monetary policy influences financial markets and financial markets simultaneously affect monetary policy. Accordingly, at least two methodological difficulties, endogeneity and omitted variables, arise when estimating the reaction of financial markets to monetary policy. First, there is the problem of endogeneity, causing biased estimators. Because of the simultaneity between policy and market reaction, error terms are correlated with financial market indicators and with monetary policy indicators. Second, there are factors other than monetary policy that influence financial markets. As a consequence, estimators could be biased due to omitted variables. Rigobon and Sack (2004) develop an instrument variable estimator to deal with simultaneity of policy and financial market reactions. However, they only prove its validity for only one monetary policy indicator and one financial market indicator. Given the integrated nature of financial markets mentioned above, theirs is possibly a too-limited test of the approach.

Recent empirical literature contains several approaches for estimating the effects of monetary policy on financial markets, particularly stock markets. There are basically two broad strands of this literature. Thorbecke (1997) applies a vector autoregression (VAR) and an event study to the analysis of the effect of monetary policy on equity returns. He finds evidence that there are monetary policy risk premia in stock returns and that stock markets significantly react to monetary policy shocks. In line with this finding, Patelis (1997) examines short-run effects of monetary policy on stock returns. He
applies VAR and multivariate regressions to study short- and long-run effects and finds a negative association between monetary policy shocks and expected returns. Bjornland and Leitemo (2009) and Bredin et al. (2010) also employ a VAR framework. Bjornland and Leitemo (2009) discover evidence that the S&P 500 reacts to monetary policy and evidence that the federal funds rate reacts to stock market shocks. Bredin et al. (2010) find spillover effects of US monetary policy shocks on German and British excess bond returns.

Some studies of monetary policy effects employ higher frequency data, typically daily. Kuttner (2001) and Bernanke and Kuttner (2005) taking an event-study approach, find evidence that Federal Reserve monetary policy has an impact on US stock and bond markets. However, due to the endogeneity problem touched on above, an event-study analysis provides potentially biased estimates. Rigobon (2003) develops a model based on instrumental variables estimation to correct for such a bias. He utilises heteroscedasticity in the data to derive adequate instruments. Rigobon and Sack (2004) apply the identification through heteroscedasticity approach to the analysis of monetary policy shocks on US major stock indices and bond markets, with findings that support earlier results by Bernanke and Kuttner (2005).

European financial markets have also been studied. Andersson (2007) uses intraday data in an analysis of monetary policy shocks on US and European financial markets. He finds that monetary policy decisions increase stock and bond market volatility in both regions but that the effect is more pronounced in the United States. Bohl et al. (2009), Sondermann et al. (2009), and Kholodilin et al. (2009) apply the identification through heteroscedasticity approach to European markets. Bohl et al. (2009) employ a sample of about 40 monetary policy shock dates, most of which occurred before 2003. The authors analyse the reaction of major stock indices (German, Spanish, Italian, and French) to ECB monetary policy shocks. In a follow-up paper, Sondermann et al. (2009) to also cover the Austrian, Belgian, Finnish, Irish, Dutch, and Portuguese stock market indices. Kholodilin et al. (2009) apply the approach to various sectoral indices. All papers report significant effects of ECB monetary policy on European financial markets.

However, all these studies focus exclusively on the effects of domestic monetary policy on domestic financial markets; that is, they neglect the possible spillover effects of foreign central bank monetary policy on domestic equity indices. Put differently, extant analyses are performed in a bivariate setting and, due to omitted variables, the estimators they rely on are potentially biased. Noting the potential importance of international spillover effects, monticini05 apply two-country event studies to the United Kingdom, the euro
area, and the United States. They find evidence for spillovers of US monetary policy on European stock and bond markets, but not vice versa. British rates, however, react only marginally to FED policy. Hayo et al. (2010), Hayo and Neuenkirch (2011) and Hayo et al. (2011) find evidence of significant reactions by mature and emerging financial markets to US monetary policy action and communication, respectively. Hayo et al. (2011) apply a GARCH model to daily financial data and discover that monetary policy communication by Fed officials affects Canadian financial market, but that Canadian monetary policy communication does not affect US financial markets. In their analysis of US and Australian monetary policy, Craine and Martin (2008) make the first and, at analysing monetary policy spillover effects in the framework of identification through heteroscedasticity. They extend it to a two-country, two-market framework.¹ They find evidence that US monetary policy has an impact on Australian markets but not vice versa. Ehrmann et al. (2010) apply the approach on European and US markets and find significant cross-over effects. However, they do not provide the full proof of the extension of this approach, neither.

This paper makes novel contributions to the literature. First, we formally and rigorously prove that Rigobon and Sack (2004)’s approach can be extended to a multinational and multimarket framework. Thus, our method has the potential to solve the endogeneity and omitted variables problems. We show that only few assumptions are necessary to implement this model in an instrumental variable framework and that the instruments can be generated within the model. In our view, proving that the Rigobon and Sack (2004) approach can be applied to studying monetary policy in globalised financial markets greatly enhances its utility. Second, a multinational and multimarket framework is of particular interest in the study of the impact of monetary policy on European financial markets, which are highly integrated. Thus, in this application we simultaneously analyse the effects of ECB monetary policy on various euro-area markets (multimarket) and spillover effects from other European monetary policies (multinational). Given the high degree of European financial market integration, we expect sizable national spillover effects of monetary policy, particularly from the European Central Bank (ECB), to European financial markets outside the European Monetary Union.

This paper is organised as follows: Section 2 generalises Rigobon and Sack (2004)’s approach to the multimeter and multicountry case and formally derives important characteristics of the estimator. Section 3 applies this approach to European stock

¹Although they extend the basic framework to a multicountry and multimarket framework in their application, they provide a (somewhat sketchy) formal proof only for the two-country, two-market case.
markets, specifically four different central bank actions and eight stock markets. A description of the data is followed by the results of the instrumental variables estimation approach and their interpretation as well as a robustness check. Section 4 concludes.

2. Methodology

Endogeneity and omitted variables are two major obstacles when analysing the effect of monetary policy on financial markets. A country’s short-term interest rate, a widely used indicator of monetary policy stance, could be influenced not only by domestic asset prices but also by asset prices in foreign countries. At the same time, a country’s asset prices could be affected by both domestic and foreign short-term interest rates. Rigobon and Sack (2004)’s framework addresses this issue by applying a simultaneous equation framework estimated by instrumental variables. However, their approach does not address the problem of omitted variables, neither with respect to spillover effects from foreign monetary policy and other financial markets nor with respect to additional factors. To overcome this shortcoming, we extend their framework to a multicountry and multimarket setting and allow a set of macroeconomic variables to enter the model. By including variables that account for business cycle effects, liquidity premia, shocks, and so forth, we avoid the problem of omitted variables. The model is specified as follows:

\[
\Delta i_{kt} = \beta_{1,k} \Delta s_{kt} + \beta_{2,k} \Delta s_{kt} + \ldots + \beta_{n,k} \Delta s_{n,t} + \gamma_{1,k} z_t + \varepsilon_{kt}, \quad (1.1)
\]

\[
\Delta s_{kt} = \alpha_{1,k} \Delta i_{kt} + \alpha_{2,k} \Delta i_{kt} + \ldots + \alpha_{n,k} \Delta i_{n,t} + \gamma_{1,2} z_t + \mu_{kt}, \quad (1.2)
\]

\[\vdots\]

\[
\Delta i_{nt} = \beta_{1,n} \Delta s_{1,t} + \beta_{2,n} \Delta s_{2,t} + \ldots + \beta_{n,n} \Delta s_{n,t} + \gamma_{1,n} z_t + \varepsilon_{nt}, \quad (n.1)
\]

\[
\Delta s_{nt} = \alpha_{1,n} \Delta i_{1,t} + \alpha_{2,n} \Delta i_{2,t} + \ldots + \alpha_{n,n} \Delta i_{n,t} + \gamma_{1,2} z_t + \mu_{nt}, \quad (n.2)
\]

where \(\Delta i_{kt}\) is the difference in the short-term interest rate of country \(k\) at time \(t\), \(\Delta s_{kt}\) is the difference of the price of an asset in country \(k\) at time \(t\), \(\varepsilon_{kt}\) is the monetary shock in country \(k\), and \(\mu_{kt}\) is a financial market shock in country \(k\). The vector \(z_t\) is a set of macroeconomic variables at time \(t\) (e.g. risk premia or oil price shocks).

Equations (1.1) - (n.1) are monetary policy reaction functions. They take into account the influence of financial markets and other macroeconomic variables on monetary policy decisions. However, we are mainly interested in the asset market equations (Equations 1.2) - (n.2), particularly in the estimators \(\alpha_{k,l}, k, l = 1 \ldots n\), which measure the magnitude of the reaction of financial markets to monetary policy actions.
Due to the endogeneity problem, OLS estimates yield biased estimators. However, in a one-market and one-country framework, Rigobon and Sack (2004) show that the two simultaneous equations can be estimated consistently by means of instrumental variable techniques. A convenient feature of their approach is that the instruments are generated within the data. We extended this framework to a multinational and multimarket framework. Given the validity of the underlying assumption, the generalised estimator is consistent. Furthermore, this approach requires fewer assumptions about the variances of the monetary shocks $\varepsilon_i$ than does an event-study approach.

The outline of the proof for the estimator is as follows. We assume that the variance of monetary policy shocks ($\varepsilon_i$) is higher on days when monetary policy is actually undertaken than on other days. Thus, given the institutional framework of monetary policy decisions, we assume that market participants do not expect monetary policy action on days when there is no council meeting. We then separate the sample into two subsamples. The first subsample includes all dates on which monetary policy takes place, henceforth referred to as 'monetary policy dates'. The second subsample includes all other dates, henceforth referred to as 'other dates'. Variances of monetary policy dates should be significantly higher than those of other dates, whereas variances of financial market shocks ($\mu_i$) or variances of the set of macroeconomic variables ($z_i$) should be similar in both samples. For both subsamples, we develop the corresponding covariance matrices. Except for the case of monetary policy shocks, these matrices should be similar. We utilise this similarity by deducting one covariance matrix from the other. Thus, only terms containing the monetary policy shock remain as non-zero entries in the matrix. Then, we obtain the estimator by dividing the relevant entries of the difference in the covariance matrix (shift) by the entries in the first column of the matrix.

As explained above, we split the sample in two subsamples $F_1$ and $F_2$, where $F_1$ includes all monetary policy dates and $F_2$ includes the remaining non-monetary policy dates. This implies:

$$\sigma_{\varepsilon_i}^{F_1} > \sigma_{\varepsilon_i}^{F_2},$$  \hspace{1cm} (1)

$$\sigma_{\mu_i}^{F_1} = \sigma_{\mu_i}^{F_2},$$  \hspace{1cm} (2)

$$\sigma_{z_i}^{F_1} = \sigma_{z_i}^{F_2},$$  \hspace{1cm} (3)

for all countries $i, i = 1 \ldots n$, $t = 1 \ldots T$. Furthermore, we assume that all disturbances
and variables are uncorrelated. To identify the system of equations above, we develop:

\[ u_t = B y_t + \Gamma z_t, \quad \text{(4)} \]

\[ u_t = [\varepsilon_1, \mu_1, \ldots, \varepsilon_n, \mu_n]' \]

\[ B \] is the \( 2n \times 2n \) coefficient matrix, \[ y_t = [\Delta i_1, \Delta s_1, \ldots, \Delta i_n, \Delta s_n]' \]
and \[ \Gamma = [-\gamma_{1,1} - \gamma_{1,2} \ldots - \gamma_{n,1} - \gamma_{n,2}]'. \]

To compute the reduced form, and thus solve this equation, we need the inverse \( B^{-1} \) of \( B \). Showing that such an inverse exists is straightforward, as all the rows of matrix \( B \) are independent, and can be summarised as follows\(^2\):

\[ y_t = B^{-1} u_t - B^{-1} \Gamma z_t \]
\[ = \Lambda z_t + v_t. \quad \text{(5)} \]

\( v_t \) can be characterised as a structural shock, including both monetary and financial market shocks. The principal idea of this estimation technique is to utilise the shift of each subsample \( F_1 \) and \( F_2 \), namely \( E(y_t y_t')_{F_1} \) and \( E(y_t y_t')_{F_2} \), to obtain an estimator with the desired properties.\(^3\)

\[ \Omega = E(y_t y_t'_{F_1}) - E(y_t y_t'_{F_2}) = E(v_t v_t')_{F_1} - E(v_t v_t')_{F_2}. \quad \text{(7)} \]

Given our assumptions (1) - (3), only the covariance matrix of the error term \( v_t \) is relevant. The entries of the shift \( \Omega \) can be used to calculate the estimator. For reasons of simplicity, we focus on the odd rows of the covariance matrix, as only these are used to compute the desired estimators \( \alpha_{k,l} \), \( k, l = 1 \ldots n \).\(^4\) Thus

\[ \Omega = \begin{pmatrix}
  l_1^2 \varepsilon_1 & l_1k_1 \alpha_{1,1} \varepsilon_1 & \ldots & l_1k_n \alpha_{n,1} \varepsilon_1 \\
  \vdots & \vdots & \ddots & \vdots \\
  l_1l_n \varepsilon_n & l_nk_1 \alpha_{1,n} \varepsilon_n & \ldots & l_nk_n \alpha_{n,n} \varepsilon_n
\end{pmatrix}, \quad \text{(8)} \]

\(^2\)A formal analysis of the derived inverse is given in the Appendix.

\(^3\)A mathematical proof as well as a detailed analysis of the covariance matrices is given in the Appendix.

\(^4\)A detailed description of the subtracted covariance matrix is shown in the mathematical Appendix.
where \( \epsilon_j := (\sigma_{\epsilon_j F_1}^2 - \sigma_{\epsilon_j F_2}^2) \). Then the estimator \( \hat{\alpha} \) can be computed as

\[
\hat{\alpha} = \left( \begin{array}{c}
\hat{\alpha}_{1,1} \ldots \hat{\alpha}_{n,1} \\
\vdots & \ddots & \vdots \\
\hat{\alpha}_{1,n} \ldots \hat{\alpha}_{n,n}
\end{array} \right) = \left( \begin{array}{cccc}
1 & \frac{l_{1,1} \alpha_{1,1} \epsilon_1}{l_{1,1}^2} & \cdots & \frac{l_{1,n} \alpha_{1,n} \epsilon_1}{l_{1,n}^2} \\
\vdots & \vdots & \ddots & \vdots \\
1 & \frac{l_{n,1} \alpha_{n,1} \epsilon_n}{l_{1,n}^2} & \cdots & \frac{l_{n,n} \alpha_{n,n} \epsilon_n}{l_{1,n}^2}
\end{array} \right)
\] (9)

We now implement this estimator in an econometric framework. The definition of the sample covariance matrix of a matrix \( X \) is

\[
S = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})(X_i - \bar{X})'.
\] (10)

We calculate the sample covariance matrices of both subsamples and subtract them. Then we divide the entries of the shift of the sample covariance matrices by the entries in the first column of the matrix to obtain our estimator:

\[
\hat{\alpha} = \left( \begin{array}{cc}
(\Delta t_{1,n} \Delta s_{1,n})_{F_1} - (\Delta t_{1,n} \Delta s_{1,n})_{F_2} & (\Delta t_{n,n} \Delta s_{n,n})_{F_1} - (\Delta t_{n,n} \Delta s_{n,n})_{F_2} \\
(\Delta t_{1,n} \Delta s_{1,n})_{F_1} - (\Delta t_{1,n} \Delta s_{1,n})_{F_2} & (\Delta t_{n,n} \Delta s_{n,n})_{F_1} - (\Delta t_{n,n} \Delta s_{n,n})_{F_2}
\end{array} \right),
\] (11)

where \((\Delta t_{ij} \Delta s_{ij})_{F_k}\) denotes the covariance matrix of country \( j \) at time \( t \) in the subsample \( F_k \).

However, this technique is not easily implemented in practice. We therefore follow Rigobon and Sack (2004) and prove that an appropriately constructed instrumental variable estimator is equivalent. Considering that the vector of differences in the interest rate of country \( j \) is ordered chronologically \( \Delta t_j = [\Delta t_{(1),1}, \ldots, \Delta t_{(T),n}] \), we define

\[
\Delta t^*_{(t(k)),j} = \begin{cases} 
\Delta t_{(t(k)),j}, & t(k) \in F_1 \\
-\Delta t_{(t(k)),j}, & t(k) \in F_2
\end{cases}
\] (12)

The instruments can then be defined as

\[
\Delta \omega t_1 = [\Delta t^*_{(1),1}, \ldots, \Delta t^*_{(T),1}]',
\]

\[
\vdots
\]

\[
\Delta \omega t_n = [\Delta t^*_{(1),n}, \ldots, \Delta t^*_{(T),n}]',
\] (14)
and
\[ \omega_i = [\omega_{i1}, \ldots, \omega_{in}]. \]

We show that a set of the same instruments \( \omega_{ij} \) can be used in the extended approach. The instruments \( \omega_{ij} \) are correlated with the regressor but uncorrelated with the error term and the macroeconomic variables. Hence, the estimator for the reaction of financial markets to monetary policy actions can be computed as
\[ \hat{\alpha} = (\Delta_i \omega_i)^{-1}(\Delta_s \omega_i). \] (15)

3. Data and Empirical Results

In this section, we analyse the influence of monetary policy actions of the Bank of England (BoE), the European Central Bank (ECB), the Swiss National Bank (SNB), and the Swedish Riksbank (Riks) on major European stock market indices using daily data from January 1999 to December 2009. To keep the analysis manageable, we focus on the largest EMU countries: Germany, France, Italy, Spain, and the Netherlands. To account for the possibility of spillover effects, we include the United Kingdom, Switzerland, and Sweden in the analysis.\(^6\)

As our indicator for monetary policy, we use one-month money market rates as measured by Euribor, Libor, Swiss Libor, and Stibor. These indicators are computed as daily differences in basis points. The interbank interest rate primarily reacts to monetary policy actions and much less so to other events, which makes it a good indicator for measuring monetary policy shocks. We follow Kleimeier and Sander (2006) in choosing one-month interbank rates because they are less volatile than overnight interest rates but more sensitive than rates with longer maturities. As stock indices, we employ log differences in basis points of daily closing prices for the German DAX, French CAC 40, Spanish IBEX 35, Dutch AEX, Italian FTSE MIB, UK FTSE, Swedish SAX, and Swiss SMI.\(^7\)

The ECB governing council makes decisions on future monetary policy 12 times per year\(^8\) and decisions are announced at 1:45 p.m. on the meeting day. The Monetary

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\(^5\)See the mathematical Appendix for a detailed description.

\(^6\)The United Kingdom and Sweden are members of the European Union but not of the euro area. Sweden is officially committed to introducing the euro, but is not expected to join soon. The UK is not a member of the ERM II and shows little sign of giving up its national monetary independence. Switzerland has not joined the EU but has many bilateral treaties with it. Both Switzerland and the UK play an important role in European capital flows.

\(^7\)Data source: web pages of the respective central banks.

\(^8\)Until November 2001, the ECB had a different meeting schedule.
Policy Committee (MPC) of the Bank of England also meets 12 times per year, and its decisions are published at 11:00 a.m. The Executive Board of the Riksbank holds six scheduled monetary policy meetings each year; decisions are announced at 1:00 p.m. The SNB meets only four times a year. On these four days, money market rates are fixed before the SNB announces its interest rate decision. We expect markets to incorporate the monetary policy decisions after their announcement. Thus, as interbank market rates are already fixed on the decisions day, we employ money market quotes of the subsequent day to capture the change due to the monetary policy move.

We have 167 monetary policy meeting dates for the euro area, 133 for the United Kingdom, 37 for Switzerland, and 39 for Sweden. Following Rigobon and Sack (2004), we employ the days preceding the council meeting dates as non-monetary policy dates so as to avoid unpredictable influences from other economic factors that might accelerate or decelerate the impact of monetary policy actions.

Focussing on the Euribor, Figure 1 illustrates graphically that the identification through heteroscedasticity approach works well in our sample. Standard deviations

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9Since some of these dates coincide, we have a total of 293 monetary policy meeting dates. In the sample period occurring before the financial crisis, we have 144 monetary policy dates for the euro area, 112 monetary policy dates for the United Kingdom, 35 dates for Switzerland, and 32 for Sweden; a total of 251 monetary policy action dates before the crisis. Our system approach requires the same number of observations for each country. Since some of the dates do not contain observations on stock price indices due to country-specific public holidays, some days had to be excluded. Furthermore, we excluded 6/11/2008 and 4/12/2008, as bank rates were reduced by 100 basis points on these occasions, generating huge leverage effects.

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Table 1: Descriptive statistics (in reactions to domestic monetary policy decisions)

<table>
<thead>
<tr>
<th></th>
<th>Before financial crisis</th>
<th>Full dataset</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard deviation</td>
<td>Correlation coefficient with 1-month Euribor</td>
</tr>
<tr>
<td>DAX 30</td>
<td>72.59</td>
<td>67.92</td>
</tr>
<tr>
<td>AEX</td>
<td>65.52</td>
<td>61.29</td>
</tr>
<tr>
<td>CAC 40</td>
<td>66.00</td>
<td>57.43</td>
</tr>
<tr>
<td>IBEX 35</td>
<td>61.27</td>
<td>57.01</td>
</tr>
<tr>
<td>FTSE MIB</td>
<td>57.95</td>
<td>57.55</td>
</tr>
<tr>
<td>FTSE</td>
<td>46.76</td>
<td>49.48</td>
</tr>
<tr>
<td>SMI</td>
<td>93.31</td>
<td>48.86</td>
</tr>
<tr>
<td>SAX</td>
<td>74.41</td>
<td>42.84</td>
</tr>
<tr>
<td>Euribor</td>
<td>5.03</td>
<td>1.69</td>
</tr>
<tr>
<td>Libor GBP</td>
<td>7.44</td>
<td>2.62</td>
</tr>
<tr>
<td>Stibor</td>
<td>3.61</td>
<td>1.57</td>
</tr>
<tr>
<td>Libor CHF</td>
<td>11.18</td>
<td>11.01</td>
</tr>
</tbody>
</table>

$F_1$ denotes monetary policy dates and $F_2$ all other dates.

of major stock indices, and particularly those of short-term interest rates, are substantially higher on monetary policy action dates than on other dates. Descriptive statistics for all series shown in Table 1 suggest the general validity of this identification approach in the euro area. Moreover, the correlation coefficient between the short-term interest rate and major stock indices is negative on monetary policy action dates and positive on other dates. This matches our expectation of a negative relationship between stock market prices and monetary policy shocks. As argued by the present value model (see e.g. Crowder (2006)), monetary policy influences stock markets in two different ways. First, today’s monetary policy influences the expected future cash flow and second, it affects the discount rate of financial market participants. Thus, a hike in the monetary policy rate will lead to a decline in stock prices.\(^\text{10}\)

Finally, we apply the Hausman test for endogeneity, which supports the hypothesis of an endogeneity problem in the data. In fact, the null hypothesis that OLS estimation is consistent must be rejected at the 5% significance level for all countries.\(^\text{11}\)

\(^{10}\)An exception in our dataset is the FTSE, which suggests that we might not find a significant reaction in the regression analysis.

\(^{11}\)Switzerland is an exception here, possibly because there is very little variation in the Swiss Libor rates. An explanation is that there is no domestic Swiss money market rate, only the Libor, which is
The results obtained by application of the generalised identification-through-heteroscedasticity approach support most of our expectations. Table 2 presents the reaction of European stock market indices (DAX 30, AEX, CAC 40, IBEX 35, FTSE MIB, FTSE, SAX, and SMI) to monetary policy actions by four central banks (ECB, BOE, SNB, and the Riksbank).

In the left part of the Table 2, giving estimation results before the financial crisis, we find that most stock markets have negative reactions to ECB monetary policy. The size of the coefficients can be interpreted as the change in daily stock returns following a change in the respective short-term interest rate of 100 basis points. Thus, in the case of an ECB monetary shock of about 25 basis points, stock markets in Europe tend to fall by approximately 1 basis point. Thus, we find that effects of ECB monetary policy within European stock markets are symmetric, as the magnitude of the coefficients is similar in our sample. Moreover, statistically, we cannot reject the hypothesis of equal coefficients across the different stock markets.

Moreover, we find significant monetary policy spillovers from the ECB to Swiss and British stock markets. In line with our expectations based on the descriptive statistics, there are no spillovers from Swedish or Swiss monetary policy actions to the major European stock indices. Furthermore, we observe no reactions by Swiss, Swedish, or British equity markets to domestic monetary policy.

At least in case of the United Kingdom this outcome is somewhat surprising, but may be explained by the fact that, first, average inflation in Great Britain tends to be above the official inflation target and British market participants do not expect the BoE to react to future inflation risks to their full satisfaction. Second, monetary policy might have asymmetric effects and thus specific sectors react more than the broad index. This explanation is supported by previous studies, e.g. Bredin et al. (2007). They find significant effects of British monetary policy on various sectoral subindices, but only weak effects on the FTSE itself. In case of Switzerland, the lack of reaction to domestic monetary policy might be due to the way the SNB conducts monetary policy. It does not employ an interest rate target but a wide range for the Swiss Libor.

Next, we check the robustness of our results in several ways. First, we extend the sample period by including observations from the financial crisis period. As the right

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12Sweden is an exception here. We find no significant reaction of the Swedish SAX to either domestic or foreign monetary policy shocks.

13We control for 11 September 2001 by including a dummy variable. Excluding this impulse dummy does not notably change our results.
Table 2: Instrumental variable estimation results

<table>
<thead>
<tr>
<th></th>
<th>until August 2007</th>
<th>until December 2009</th>
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<tbody>
<tr>
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<tr>
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<td>number of observations: 483</td>
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* 10% significance level, ** 5% significance level, *** 1% significance level

Values in parentheses are standard errors.

Hand columns in Table 2 show, our results are quite robust with regard to a change in the sample period. When including data from the financial crisis period, the coefficient estimates barely change and are not significantly different from those estimated over the pre-crisis period. In general, the significance of the estimated parameters diminishes when estimating the models over the extended sample. During the crisis period, central banks conducted various unorthodox monetary policy operations (e.g. term auctions in cooperation with other central banks) to increase market liquidity. This might help explain the reduction in statistical significance in the case of some of the estimators.

Second, we employ a different set of non-monetary policy dates. Instead of the day preceding council meetings, we utilise observations from one week before. Again, we find that this variation does not significantly change our results. Third, we include dummies in the analysis to control for (i) specific effects of the financial crisis and (ii) the direction of monetary policy, i.e., we test for asymmetries in the strength of stock market reactions depending on the direction of interest rate changes. Again, our results prove to be robust. Fourth, we employ three-month money market rates rather than one-month rates. This variation has only a negligible influence on our results. Fifth, we employ the FTSE
Table 3: Comparison of multi- and bivariate estimation (until August 2007)

<table>
<thead>
<tr>
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<th>Results of multivariate estimation</th>
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</tr>
</tbody>
</table>

* 10% significance level, ** 5% significance level, *** 1% significance level.
Values in parentheses are standard errors.

350 and the FTSE All-Share indices instead of the FTSE 100 but that does not change results either. 14 Finally, we compare our multivariate results to estimations obtained employing the original Rigobon and Sack (2004) approach. Our potentially superior system approach indicates a bias in the bivariate estimations. As shown in Table 3, the magnitude of the estimators in our multivariate estimations is systematically larger, except for Switzerland and Sweden. Thus, in line with the theoretical expectation of the properties of the respective estimators, real effects of monetary policy are underestimated in the original Rigobon and Sack (2004) framework.15

4. Conclusions

In this paper, we extend the approach of identification through heteroscedasticity of Rigobon and Sack (2004) to a multicountry and multimarket model and prove that this

14Results are available on request.
15However, we cannot clearly distinguish between the potential reasons for the observed bias. In addition to the bias inherent in the bivariate approach, it could be due to omitted variables or variations in the number of observations. A detailed analysis of the relative importance of these potential reasons for the superiority of the multivariate approach is left to future research.
can be implemented using instrumental variable estimation. In an empirical application, we examine the influence of monetary policy on domestic and foreign stock markets. We apply our novel system estimator to data on European short-term interest rates, as an indicator of monetary policy, and various stock market indices, as important financial markets. Our analysis shows that the ECB significantly influences euro-area equity markets. An increase in short-term interest rates by 25 basis points decreases stock market prices by about 1 basis point. Moreover, our analysis reveals that there are significant monetary policy spillovers from ECB monetary policy shocks to British and Swiss equity markets. In contrast, domestic monetary policy in the United Kingdom, Sweden, or Switzerland has no influence on other European stock market indices.

When extending the sample period to cover the financial crisis, we find that our estimated coefficients do not change significantly. Thus, our estimations are quite robust. However, our results also show that some stock markets react less strongly to changes in key interest rates. We believe that this is due to the much greater importance of unorthodox monetary policy measures undertaken during the financial crisis. Comparing our multicountry and multimarket estimator to the original approach put forward by Rigobon and Sack (2004) for our dataset, we find that our coefficient estimates are significantly larger in absolute terms. These empirical findings are in line with our expectations based on econometric theory, which suggests that in a situation of integrated financial markets, the original bivariate approach is biased.

Comparing our estimates using the original Rigobon and Sack (2004) approach with previous findings by Bohl et al. (2009), we find that our qualitative findings for stock market effects due to domestic monetary policy are broadly similar, but that our estimated coefficients are smaller. This difference is probably due to the inclusion of all meeting dates of the councils, instead of focusing on pre-selected shock days as done by Bohl et al. (2009). In our view, it is likely that days characterised by particularly large deviations from expectations cause stronger financial market reactions.

Finally, the efficient market hypothesis of Fama (1970) implies that news that reaches financial markets is immediately incorporated into market prices. However, we demonstrate, at least in the short-run, that monetary policy is not neutral. Moreover, we find evidence that there are spillover effects from foreign monetary policies to other countries’ financial markets.

In a world of integrated financial markets, the results of our study have two important implications for the conduct of monetary policy. First, in line with Bohl et al. (2009), we find evidence that the efficient market hypothesis does not hold in the short
run. Monetary policy has a significant impact on financial markets. Second, we detect spillover effects from ECB monetary policy to British and Swiss equity markets. Thus, the ECB, which is legally obliged to focus on the euro area, ought to be aware of the spillover effects its monetary policy actions have on other financial markets. Additionally, other central banks, e.g., the BoE, need to consider the influence of ECB monetary policy on their domestic financial markets when conducting monetary policy.

Finally, the link between monetary policy and financial markets is of particular interest in a period of financial turmoil. We find that the absolute size of the effect of interest rate changes on equity markets does not significantly decline during the financial crisis. However, the reduction in statistical significance of the estimated coefficients reflects the fact that the variance of interest rates during the financial crisis is quite low, thereby no longer affecting stock market returns. Thus, our findings are in accordance with the observation that standard monetary policy, which focuses on interest rates as the main monetary policy instrument, was superseded by frequent use of unorthodox monetary policy measures during the crisis period.

This work is a starting point for fruitful paths of future research. Our generalised multimarket and multicountry approach can be applied to other parts of the world where financial market spillover effects might be particularly important. For instance, studying the influence of the Federal Reserve Bank on Canadian financial markets looks promising. Also, recent research shows that modern monetary policy does not rely only on actions, i.e., interest rate changes, but also on communication of the monetary stance and economic outlook (Ehrmann and Fratzscher (2007); Hayo et al. (2011)). Our analysis of central bank meeting dates could be extended to include days of formal or informal central bank communication and an analysis made of the effect these events have on monetary policy issues.
A. Mathematical Appendix

The full matrix of the reduced form is \( u_t = By_t + \Gamma z_t \) with

\[
\begin{pmatrix}
\varepsilon_{1t} \\
\mu_{1t} \\
\vdots \\
\varepsilon_{nt} \\
\mu_{nt}
\end{pmatrix}; \quad B = \begin{pmatrix}
1 & -\beta_{1,1} & 0 & -\beta_{2,1} & \ldots & 0 & -\beta_{n,1} \\
-\alpha_{1,1} & 1 & -\alpha_{2,1} & 0 & \ldots & -\alpha_{n,1} & 0 \\
0 & -\beta_{1,2} & 1 & -\beta_{2,2} & \ldots & 0 & -\beta_{n,2} \\
-\alpha_{1,2} & 0 & -\alpha_{2,2} & 1 & \ldots & -\alpha_{n,2} & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & -\beta_{1,n} & 0 & -\beta_{2,n} & \ldots & 1 & -\beta_{n,n} \\
-\alpha_{1,n} & 0 & -\alpha_{2,n} & 0 & \ldots & -\alpha_{n,n} & 1
\end{pmatrix}
\]  \tag{16}

\[
\begin{pmatrix}
\Delta i_{1t} \\
\Delta s_{1t} \\
\vdots \\
\Delta i_{nt} \\
\Delta s_{nt}
\end{pmatrix}; \quad \Gamma = \begin{pmatrix}
-\gamma_{1,1} \\
-\gamma_{1,2} \\
\vdots \\
-\gamma_{n,1} \\
-\gamma_{n,2}
\end{pmatrix} \tag{17}
\]

To find the inverse of matrix \( B \) we apply an elementary matrix \( P \) so that

\[
B^* = PBP' = \begin{pmatrix}
1 & 0 & \ldots & 0 & -\beta_{1,1} & -\beta_{2,1} & \ldots & -\beta_{n,1} \\
0 & 1 & \ldots & 0 & -\beta_{1,2} & -\beta_{2,2} & \ldots & -\beta_{n,2} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \ddots \\
0 & 0 & \ldots & 1 & -\beta_{1,n} & -\beta_{2,n} & \ldots & -\beta_{n,n} \\
-\alpha_{1,1} & -\alpha_{2,1} & \ldots & -\alpha_{n,1} & 1 & 0 & \ldots & 0 \\
-\alpha_{1,2} & -\alpha_{2,2} & \ldots & -\alpha_{n,2} & 0 & 1 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \ddots \\
-\alpha_{1,n} & -\alpha_{2,n} & \ldots & -\alpha_{n,n} & 0 & 0 & \ldots & 1
\end{pmatrix}
\]  \tag{18}

\[
= \begin{pmatrix}
I_n & A_1 \\
A_2 & I_n
\end{pmatrix}. \tag{19}
\]

The inverse \( B^{*-1} \) is therefore

\[
B^{*-1} = \begin{pmatrix}
(I_n - BA)^{-1} & -(I_n - BA)^{-1}B \\
-(I_n - AB)^{-1}A & (I_n - AB)^{-1}
\end{pmatrix}. \tag{20}
\]

16
Be \( l_{j,k} = 1 - \sum_{s=1}^{n} \alpha_{j,s} \beta_{i,k} \) and \( k_{j,l} = 1 - \sum_{s=1}^{n} \alpha_{i,j} \beta_{l,s} \)

\[
B^{*-1} = \begin{pmatrix}
\frac{1}{t_{1,1}} & 0 & \cdots & 0 & \frac{\beta_{1,1}}{t_{1,1}} & \frac{\beta_{2,1}}{t_{2,1}} & \cdots & \frac{\beta_{n,1}}{t_{n,1}} \\
\vdots & \ddots & \ddots & \vdots & \ddots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & \frac{1}{t_{n,n}} & \frac{\beta_{1,n}}{t_{1,n}} & \frac{\beta_{2,n}}{t_{2,n}} & \cdots & \frac{\beta_{n,n}}{t_{n,n}} \\
\frac{\alpha_{1,1}}{k_{1,1}} & \frac{\alpha_{2,1}}{k_{2,1}} & \cdots & \frac{\alpha_{n,1}}{k_{n,1}} & 1 & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\frac{\alpha_{1,n}}{k_{1,n}} & \frac{\alpha_{2,n}}{k_{2,n}} & \cdots & \frac{\alpha_{n,n}}{k_{n,n}} & 0 & 0 & \cdots & \frac{1}{k_{n,n}} \\
\end{pmatrix}
\]  \hspace{1cm} (21)

To subtract the covariances some preconsiderations about the structure of these covariance matrices should be made. It is

\[
E(y_t y_t') = E(y_t)E(y_t')
\]  \hspace{1cm} (22)

\[
= (E(\Lambda z_t) + E(v_t))(E(\Lambda z_t) + E(v_t))'
\]  \hspace{1cm} (23)

\[
= (\Lambda \sigma_z + E(v_t))(\Lambda \sigma_z + E(v_t))'
\]  \hspace{1cm} (24)

\[
= \Lambda \sigma_z \Lambda' + \Lambda \sigma_z E(v_t') + E(v_t)\sigma_z \Lambda' + E(v_t v_t')
\]  \hspace{1cm} (25)

\[
= \Lambda \sigma_z \Lambda' + E(v_t v_t').
\]  \hspace{1cm} (26)

Furthermore, we have to compute the residuals

\[
v_t = B^{-1} u_t
\]  \hspace{1cm} (27)

\[
= \begin{pmatrix}
\frac{1}{t_{1,1}} & \frac{\beta_{1,1}}{t_{1,1}} & 0 & \cdots & 0 & \frac{\beta_{2,1}}{t_{2,1}} & \cdots & \frac{\beta_{n,1}}{t_{n,1}} \\
\frac{\alpha_{1,1}}{k_{1,1}} & \frac{\alpha_{2,1}}{k_{2,1}} & 0 & \cdots & \frac{\alpha_{n,1}}{k_{n,1}} & 1 & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\frac{\alpha_{1,n}}{k_{1,n}} & \frac{\alpha_{2,n}}{k_{2,n}} & 0 & \cdots & \frac{\alpha_{n,n}}{k_{n,n}} & 0 & 0 & \cdots & \frac{1}{k_{n,n}} \\
\end{pmatrix}
\begin{pmatrix}
\varepsilon_{1_t} \\
\varepsilon_{2_t} \\
\vdots \\
\varepsilon_{n_t} \\
\end{pmatrix}
\begin{pmatrix}
\mu_{1_t} \\
\mu_{2_t} \\
\vdots \\
\mu_{n_t} \\
\end{pmatrix}
\]  \hspace{1cm} (28)

\[
= \begin{pmatrix}
l_1(\varepsilon_{1_t} + \sum_{i=1}^{n} \beta_{1,i} \mu_{1_t}) \\
k_1(\mu_{1_t} + \sum_{s=1}^{n} \alpha_{1,i} \varepsilon_{1_s}) \\
\vdots \\
l_n(\varepsilon_{n_t} + \sum_{i=1}^{n} \beta_{i,n} \mu_{n_t}) \\
k_n(\mu_{n_t} + \sum_{s=1}^{n} \alpha_{n,i} \varepsilon_{n_s}) \\
\end{pmatrix}
\]  \hspace{1cm} (29)

where \( k_j = \frac{1}{b_{j,j}} \) and \( l_j = \frac{1}{b_{j,j}} \). We are only interested in the covariance matrix \( E(v_t v_t') \).
Be:

\[ x_{t,m}^{n,o} = \beta_{t,m} \sigma_{\mu_t}^2 + \alpha_{n,o} \sigma_{\epsilon_o}^2 \]  

(30)

\[ y_{t}^{\varepsilon} = \sigma_{\varepsilon_t}^2 + \sum_{i=1}^{n} \beta_{i,t} \sigma_{\mu_t}^2 \]  

(31)

\[ y_{t}^{\mu} = \sigma_{\mu_t}^2 + \sum_{i=1}^{n} \beta_{i,t} \sigma_{\epsilon_t}^2 \]  

(32)

The subtraction of the covariance matrices is as follows

\[ E(v tv_t') = \begin{pmatrix}
    l_1^2 \sigma_{\varepsilon_1} & k_1 l_1 x_{1,1}^{1,1} & \cdots & \cdots & l_1 k_n x_{1,n,n} \\
    k_1 l_1 x_{1,1}^{1,1} & k_1^2 \sigma_{\mu_1}^2 & \vdots & \vdots & \vdots \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    \vdots & \vdots & \cdots & l_n^2 \sigma_{\varepsilon_n}^2 & l_n k_n x_{n,n,n} \\
    l_n k_1 x_{1,1}^{n,n} & \cdots & \cdots & l_n k_n x_{n,n,n} & k_n^2 \sigma_{\mu_n}^2
\end{pmatrix}. \]  

(33)

Be:

\[ \tilde{\sigma}_{t} = \left( \sigma_{\varepsilon_1 F_1}^2 - \sigma_{\varepsilon_1 F_2}^2 \right) \]  

(34)

\[ d_{t,k} = \sum_{i=1}^{n} \alpha_{i,t} \alpha_{k,t} \tilde{\sigma}_{t}, \]  

(35)

\[ E(v_t v_t')_{F_1} - E(v_t v_t')_{F_2} = \Omega = \begin{pmatrix}
    \omega_{1,1} & \cdots & \omega_{2n,1} \\
    \vdots & \ddots & \vdots \\
    \omega_{1,2n} & \cdots & \omega_{2n,2n}
\end{pmatrix} \]  

(36)

\[ \begin{pmatrix}
    l_1^2 \tilde{\sigma}_{1} & l_1 k_1 \alpha_{1,1} \tilde{\sigma}_{1} & \cdots & \cdots & l_1 k_n \alpha_{1,n,1} \tilde{\sigma}_{1} \\
    l_1 k_1 \tilde{\sigma}_{1} & k_1^2 d_{1,1} & \cdots & \cdots & l_1 k_1 \alpha_{n,n} \tilde{\sigma}_{1} & l_1 k_n d_{1,n} \\
    \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
    \vdots & \vdots & \cdots & l_n^2 \tilde{\sigma}_{n} & l_n k_n \alpha_{n,n} \tilde{\sigma}_{n} \\
    l_n k_1 \tilde{\sigma}_{n} & l_n k_1 \alpha_{1,n} \tilde{\sigma}_{n} & \cdots & \cdots & l_n k_n \alpha_{n,n} \tilde{\sigma}_{n} & k_n^2 d_{n,n}
\end{pmatrix}. \]  

(37)
Due to the assumptions there is

\[ l_s^2(\sigma_{\varepsilon^2}^2 + \sum_{i=1}^{n} \beta_{1,i}^2 \sigma_{\mu_i}^2)_{F_1} - l_s^2(\sigma_{\varepsilon^2}^2 + \sum_{i=1}^{n} \beta_{1,i}^2 \sigma_{\mu_i}^2)_{F_2} = l_s^2(\sigma_{\varepsilon^2_{F_1}}^2 - \sigma_{\varepsilon^2_{F_2}}^2), \]

(38)

\[ k_s^2(\sigma_{\mu_i}^2 + \sum_{i=1}^{n} \alpha_{s,i}^2 \sigma_{\varepsilon_i}^2)_{F_1} - k_s^2(\sigma_{\mu_i}^2 + \sum_{i=1}^{n} \alpha_{s,i}^2 \sigma_{\varepsilon_i}^2)_{F_2} = k_s^2(\sum_{i=1}^{n} (\alpha_{s,i}^2 \sigma_{\varepsilon_i}^2_{F_1} - \alpha_{s,i}^2 \sigma_{\varepsilon_i}^2_{F_2}), \]

(39)

\[ l_s k_t \beta_{1,t} \sigma_{\mu_i}^2 + \alpha_{s,t} \sigma_{\varepsilon_i}^2_{F_1} - l_s k_t \beta_{1,t} \sigma_{\mu_i}^2 + \alpha_{s,t} \sigma_{\varepsilon_i}^2_{F_2} = l_s k_t (\alpha_{s,t} \sigma_{\varepsilon_i}^2_{F_1} - \alpha_{s,t} \sigma_{\varepsilon_i}^2_{F_2}), \]

(40)

\[ E(y_i y'_i)_{F_1} - E(y_i y'_i)_{F_2} = \Lambda \sigma_{z} \Lambda' + E(v_i v'_i)_{F_1} - \Lambda \sigma_{z} \Lambda' - E(v_i v'_i)_{F_2}, \]

(41)

\[ = E(v_i v'_i)_{F_1} - E(v_i v'_i)_{F_2}. \]

(42)

The estimator is:

\[ \hat{\alpha} = \left( \begin{array}{c}
\hat{\alpha}_{1,1} \\
\vdots \\
\hat{\alpha}_{n,n}
\end{array} \right) = \left( \begin{array}{c}
k_1 \omega_{2,1} \\
\vdots \\
k_n \omega_{2,n}
\end{array} \right) \left( \begin{array}{c}
k_1 \omega_{1,1} \\
\vdots \\
k_n \omega_{1,n}
\end{array} \right) \left( \begin{array}{cccc}
\omega_{1,1} & \cdots & \omega_{1,n} \\
\vdots & \ddots & \vdots \\
\omega_{n,1} & \cdots & \omega_{n,n}
\end{array} \right). \]

(43)

E.g. \( \omega_{1,1} = l_1^2(\sigma_{\varepsilon_{F_1}}^2 - \sigma_{\varepsilon_{F_2}}^2) \), and \( \omega_{2,1} = l_1 k_1 \hat{\alpha}_{1,1} (\sigma_{\varepsilon_{F_1}}^2 - \sigma_{\varepsilon_{F_2}}^2) \) thus \( \hat{\alpha}_{1,1} = k_1 (\sigma_{\varepsilon_{F_1}}^2 - \sigma_{\varepsilon_{F_2}}^2) \). Note that \( \frac{k_1}{l_1} = \frac{1 - \text{rowsum of country } i}{1 - \text{columnsum of country } j} \to 1 \) as both, rowsum and columnsum go either to infinity or to zero (depending on the underlying data). Be

\[ \Delta(\hat{i}, s)_{l,m} = (\Delta \hat{i}_l \Delta s'_i)_{F_1} - (\Delta \hat{i}_l \Delta s'_i)_{F_2}, \]

(44)

\[ \Delta(s, \hat{i})_{l,m} = (\Delta s_l \Delta \hat{i}'_i)_{F_1} - (\Delta s_l \Delta \hat{i}'_i)_{F_2}. \]

(45)

The estimated covariance matrix is

\[ \Omega = \left( \sum_{i=1}^{n} y_i y'_i \right)_{F_1} - \left( \sum_{i=1}^{n} y_i y'_i \right)_{F_2} \]

(46)

\[ = \begin{pmatrix}
\Delta(\hat{i}, s)_{1,1} & \Delta(\hat{i}, s)_{1,1} & \cdots & \Delta(\hat{i}, s)_{n,1} \\
\vdots & \ddots & \ddots & \vdots \\
\Delta(\hat{i}, s)_{1,n} & \Delta(\hat{i}, s)_{1,n} & \cdots & \Delta(\hat{i}, s)_{n,n}
\end{pmatrix}. \]

(47)
To show, that the same instruments are appropriate let $\Delta_i$, $\Delta s$ and $\omega i$ be

$$
\Delta i = \begin{pmatrix}
\Delta i_{F_1} & \Delta i_{F_2} \\
\vdots & \vdots \\
\Delta i_{nF_1} & \Delta i_{nF_2}
\end{pmatrix},
$$

(48)

$$
\Delta s = \begin{pmatrix}
\Delta s_{F_1} & \Delta s_{F_2} \\
\vdots & \vdots \\
\Delta s_{nF_1} & \Delta s_{nF_2}
\end{pmatrix},
$$

(49)

$$
\omega i = \begin{pmatrix}
\Delta i_{F_1} & -\Delta i_{F_2} \\
\vdots & \vdots \\
\Delta i_{nF_1} & -\Delta i_{nF_2}
\end{pmatrix}.
$$

(50)

Due to the assumption of uncorrelated variances $(\Delta i_j \Delta i_j)_{F_1} - (\Delta i_j \Delta i_k)_{F_2}$ equal zero.

Be $\Delta(s,i)_{j,k} = (\Delta s_j \Delta i_k)_{F_1} - (\Delta s_j \Delta i_k)_{F_2}$

$$(\Delta i \omega i')^{-1}(\Delta s \omega i') = \begin{pmatrix}
\Delta(s,i)_{1,1} & \Delta(s,i)_{1,n} \\
\Delta(i,i)_{1,1} & \Delta(i,i)_{1,n} \\
\vdots & \vdots \\
\Delta(s,i)_{n,1} & \Delta(s,i)_{n,n} \\
\Delta(i,i)_{n,1} & \Delta(i,i)_{n,n}
\end{pmatrix}.
$$

(51)
References


