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The Impact of Persistent Shocks and Concave Objective Functions on Collusive Behavior

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THE IMPACT OF PERSISTENT SHOCKS AND CONCAVE OBJECTIVE FUNCTIONS ON COLLUSIVE BEHAVIOR

Johannes Paha*

ABSTRACT

This paper provides a theoretic model for the analysis of cartel formation in an industry that is subject to profit shocks. The competitive or collusive conduct of a firm is determined by a decision maker who maximizes the present value of utility that accrues to him by earning a share of the firm's profit. The paper assumes that, first, factors like progressive taxation, shareholders' preference for smooth profits, or risk aversion may make the utility function of the decision maker rise concavely in the profits of the firm. Second, collusion causes the decision maker a dis-utility by violating legal and, thus, ethical or social norms. This disutility is independent from the level of profits. Concavity has adverse effects on collusion by making the decision maker value the additional utility from, first, establishing a new cartel, second, deviating from an existing cartel and, third, being punished for this deviation higher when the industry is in a bad state with low profits. Under these conditions, a negative profitability shock must be rather persistent to trigger cartel formation. Persistence prevents a newly formed cartel from falling apart quickly as the intense punishment in this state would also persist for a long time.

Keywords: *cartel formation, collusion, concavity, persistence*

JEL Codes: *D43, K21, L13, L41, M20*

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1 INTRODUCTION

Cartel formation is puzzling. To see this consider some evidence about the establishment of explicitly collusive agreements.

1. Cartels are frequently formed following a shock that substantially changes the profitability of doing business in an industry. Often, one observes an intensification of competition leading to a decline in prices and profit margins prior to the establishment of collusion. For example, intense competition was observed prior to the formation of the conspiracies in Methionine, Soda Ash, Vitamins, and Plasterboard (Grout and Sonderegger, 2005).
2. This pattern seems to be fairly independent of the specific type of shock such as demand shocks (Grout and Sonderegger, 2005), entry or expansion of competitors (Tosdal, 1913; Sonnenfeld and Lawrence, 1978), cost shocks, excess capacity, or changes in the bargaining power of buyers (Ashton and Pressey, 2012).

   Different types of shocks can also have adverse effects on profits as long as their net effect contributes to lowering profits. Evidence of this effect is provided by the copper plumbing tubes cartel that was formed in 1988 although market demand rose at least during the time of the conspiracy. However, at the same time the industry had suffered from over-capacity since the late 1980s due to market entry and expansion of competitors, which lead to price erosion and low profitability (EC, 2004).
3. A persistent profitability shock is typically more likely to lead to cartel formation in a state of low profits than a transitory shock.

   For example, when the Petrochemicals cartel was formed in 1980 the participating firms had already seen a period of low demand starting in 1973/74 which resulted in structural overcapacity and caused the firms to operate even below break-even levels (Grout and Sonderegger, 2005). The Graphite Electrodes case provides another example for a cartel that was formed after a persistent demand shock. It was established in 1992 after demand for graphite electrodes used for steelmaking had decreased throughout the 1980s. This was attributable to technical progress in the process of steelmaking paired with a general decline in steel production (Grout and Sonderegger, 2005). Evidence of cartels that were formed in times of volatile demand or following temporary (in expectation) shocks is much harder to find (Grout and Sonderegger, 2005).

Given these stylized facts one would be tempted to ask: Why would rational decision makers establish collusion in response to a negative profit shock? At least two answers can be given to this question. First, one could focus on the structural characteristics of a firm and argue that in some cases a negative profit shock facilitates cartel formation via two channels: The negative profit shock (i) may raise the additional profits to be earned when switching from competitive to collusive conduct and/or (ii) it may make collusion more stable. For example, this can be shown in models where firms are capacity constrained and subject to cyclical demand (Fabra, 2006). In times of high demand, binding capacity constraints relax competition such that the additional profits that can be earned by colluding are small. Moreover, the threat to punish a deviation from the collusive agreement by eternal reversion to competitive conduct would also be weak. In times of low demand, competition becomes more intense. This makes collusion both more attractive relative to competition and more stable.

A second answer to the question, why a rational decision maker would establish collusion in response to a negative profit shock, is given in this paper. It analyzes the incentives of the decision maker, i.e. an employee, manager, or the owner of the firm, who decides about the firm's market conduct. We derive plausible assumptions about the incentives of the decision maker from related literature and solve our model analytically. This yields the finding that the above patterns of cartel formation can even emerge when a negative profit shock does *not* raise the absolute value of the additional profits that are earned by colluding or foregone by deviating from a collusive agreement. Our model suggests that cartels can even be formed under circumstances that existing literature would not have considered suspect to the establishment of collusion. Insofar, we provide a new explanation for cartel formation that complements and extends prior work.

To study cartel formation, our paper proposes a dynamic model where an industry randomly switches with a predefined probability between a publicly observed state with high profits and a state with low profits. The model is fairly general by focusing on profit shocks without assuming a specific type of shock such as a cost or demand shock. The state of the industry is modeled by a two state Markov process which allows us to assess the effects of the shocks' persistence on the formation and stability of collusion. This shock structure appears to be appropriate to depict the evidence on cartel formation.[1] Given this shock structure, we analyze the strategic decisions, first, whether to establish a

1 Our shock structure builds upon and complements prior literature such as Athey and Bagwell (2008) who study the stability of collusion in a model with privately observed cost shocks that also follow a Markov process. Other literature is concerned with the analysis of the stability of collusion when demand is subject to either (i) fluctuations / independent and identically distributed demand shocks (Green and Porter, 1984; Rotemberg and Saloner, 1986; Staiger and Wolak, 1992), (ii) deterministic business cycles (Haltiwanger and Harrington, 1991; Fabra, 2006; Knittel and Lepore, 2010),

collusive agreement and, second, whether to adhere to it in the subsequent periods. These decisions are made by a decision maker with the objective to maximize the discounted stream of utility that accrues to him[2] from earning a stream of profits. The decision maker's utility function is specified to be either linear in profits or concave in profits.

With a linear objective function, the model is consistent with the commonly made assumption of profit maximizing respectively value maximizing firms. A concave utility function has been used by Spagnolo (1999, 2005) who studies the *stability* of cartels and whose work we complement by focusing on the *formation* of explicitly collusive agreements. He argues that concavity may result, on the one hand, when assuming decisions to be made by utility-maximizing managers. Concavity of the managerial utility function in profits can, for example, result from managers' empirically observed risk aversion and a consequential preference for smooth profits. Moreover, the income of a manager may be concave in the profits of the firm when there is an upper bound for profit-related bonuses. On the other hand, firms themselves may have a concave objective function. Spagnolo (1999) argues that convex external costs of finance, convex tax schedules, or investors' preference for assets with smooth returns are reasons that may result in a concave objective function for a firm.[3]

Our model assumes collusion to be costly for the decision maker. First, transaction costs may be required to establish and operate the cartel. Second, expected fines or repayment for damages are other monetary costs of collusion. A third type of costs has been analyzed in disciplines like criminology, sociology, jurisprudence, and psychology (see, for example, Braithwaite, 1989; Granovetter, 2005; Paternoster and Simpson, 1993, 1996; Tyler 2006). These costs comprise the dis-utility of acting against one's own moral standards and the disapproval of illegal conduct by family, friends, or peers. They serve as a *social influence* (Kahan, 1997) or *social norm*[4] on individuals' decisions to commit crimes. In economics, the effect of social interactions on crimes has, for example, been researched

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(iii) non-deterministic business cycles where growth rates follow a Markov process (Bagwell and Staiger, 1997), or (iv) correlated shocks where demand levels follow a Markov process (Kandori, 1991).

2 The managers convicted of participating in illegal cartels are most frequently men. Moreover, criminological literature suggests that crimes are often more likely to be committed by men. Therefore, we refer to the decision maker in the male form.

3 To name other related literature, Asplund (2002) studies how risk aversion affects firms' competitive best response strategies in presence of either cost or demand uncertainty. However, his study is not directly concerned with the case of collusion. Our paper is also related to the literature on the effect of managerial decision making on collusion (see, for example, Olaizola, 2007; Aubert, 2009; Gillet et al., 2001; Han, 2011).

4 For example, see the special issue of the Journal of the European Economic Association (Vol. 11, No. 3, 2013) on *Social Norms: Theory and Evidence from Laboratory and Field*.

empirically by Glaeser et al. (1996). Unlike the first two cost types these opportunity costs are not measured in monetary units but represent the dis-utility associated with certain actions of the decision maker. Our point of such costs being important for cartel formation is supported by the statement of a General Electric division vice president who said about the formation of the 1960 U.S. electrical contractors cartel (Sonnenfeld and Lawrence, 1978): “I think we understood it was against the law ... The moral issue didn't seem to be important at that time [...]. I've seen the situation change, primarily due to overcapacity, to almost a situation where people thought it was a survival measure.”

Given such costs of collusion three cases can emerge: First, when the costs of collusion are high, a cartel will neither be formed in the good nor in the bad state. Second, when the costs of collusion are low, collusion will be established in both states. Third, for intermediate cost levels collusion will only be established in one of the two states of the industry, i.e. when the gain from collusion exceeds its costs.

The second case, where collusion is established in both states, would most likely occur when one considers the first two types of costs only. Transaction costs may be considered to be fairly small relative to the gains of cartels. Fines are also found to be usually lower than the gains from collusion (Connor and Lande, 2005; Harrington, 2010). Hence, these two types of costs are typically too small to effectively deter cartel formation. This is true both when profits are high or low such that these costs must not be believed to be the decisive factor that produces the above patterns of cartel formation.

The third case, where collusion is established in just one state, is most relevant for explaining cartel formation in response to profit shocks. One might be tempted to think that a concave objective function in combination with opportunity costs of collusion suffices to explain cartel formation in response to a negative profit shock. This is because under the assumption of concavity the decision maker values a certain collusion-induced gain in profits the higher the lower the competitive profits are. Hence, a negative profit shock has the ability to raise the additional utility from collusion above the costs of collusion. However, with concave utility a negative profit shock also raises the gain that can be earned by deviating from a collusive agreement. The same is true for the valuation of the loss of profits – as compared to continuing collusion – in the punishment phase following an observed deviation. Therefore, the state of the industry also affects the stability of collusion. As stability is a prerequisite for cartel formation, the existence of a concave utility function and of opportunity costs of collusion alone does not explain cartel formation in response to a negative profit shock. Such circumstances can even be consistent with cartel formation in the good state of the industry.

The persistence of industry conditions determines whether the decision maker's incentive to adhere to collusion is higher in the good or in the bad state. When shocks occur frequently the expected value of the future punishment does not vary much with the current state of the industry and deviations are mainly driven by the short-run gain in utility. Therefore, a decision maker would rather deviate in the bad state. In this situation, a cartel (if any) would be formed in the good state and remain active during the short time only that it takes for another shock to occur, which drives the industry into the bad state where collusion is unstable. When shocks are rather persistent, deviations are more likely to occur in the good state where profits are high and the punishment following the deviation is being perceived as soft. A cartel would be formed in the bad state where collusion is stable. Such a cartel would be characterized by a longer lifetime because the industry remains rather persistently in the bad state. These effects will be shown and explained in greater detail in section 3 of this paper.

This article has implications for future research as well as competition policy. By assuming decision makers who pursue a concave objective function and incur opportunity costs of collusion our model produces outcomes that match observable patterns of cartel formation. Therefore, the model would suggest to research the assumptions of concavity and opportunity costs in greater depth. This is important because prior research – while laying important groundwork by examining the effects of structural factors on collusion, i.e. capacity constraints, different types of cost functions, explicit costs of collusion etc. – has put relatively little attention on concavity and opportunity costs, yet. Future work might be concerned with finding further evidence to support (or reject) the assumptions of concavity and opportunity costs of collusion and to quantify their magnitude. In this context, one might analyze the specific causes of concavity such as, for example, risk averse behavior or convex costs of external finance. Better knowledge about these causes should help to further refine both theoretical models of collusion and policy measures for the deterrence of cartels. Similarly, additional insights into the opportunity costs of collusion can be helpful in making compliance programs even more targeted as these have received increasing attention in competition policy lately.[5]

The structure of the paper is as follows. Section 2 presents the model. Section 3 analyzes the properties of the incentive compatibility constraint for cartel stability and the participation constraint for cartel formation. Section 4 concludes.

5 For example, in 2010 the British Office of Fair Trading published report 1227 on “Drivers of Compliance and Non-compliance with Competition Law” that can be downloaded at: http://www.oft.gov.uk/shared_oft/reports/comp_policy/oft1227.pdf

2 THE MODEL

We consider an infinitely repeated game with two players $i \in \{1,2\}$ who each manages a single firm. The firms are active in the same market.[6] The timing and basic structure of the game are as follows.

1. At the beginning of every period t the state of the industry $s \in \{\bar{s},\underline{s}\}$ is revealed and becomes common knowledge. The state can be either good $\bar{s}$ or bad $\underline{s}$ with every firm i making a higher profit π_i in the good state than in the bad state, i.e. $\pi_i(\bar{s})>\pi_i(\underline{s})$. Our assumptions on the state of the industry and its changes over time are explained in subsection 2.1.
2. After the state s_t has been revealed, every player chooses an action a_i from his strategy set $A_i=\{c,k\}$ which is the same for both players and encompasses two pure strategies. Strategy c implies competitive behavior in the product market. Strategy k stands for collusive behavior in the product market.
3. After all players have chosen a strategy, every firm i earns a profit $\pi_i(s,a_i,a_{-i})$ which is a function of the state of the industry s, the own strategy a_i of firm i, and the strategy a_{-i} of the other player. Our assumptions on firms' profits are explained in subsection 2.2. Earning profit π_i provides the decision making manager with utility $u_i(\pi_i)$. The profit π_i is net of all monetary costs that are associated with choosing either strategy. However, when choosing the collusive strategy k the manager incurs non-monetary opportunity costs κ. Further information on the utility function and the opportunity costs is provided in subsection 2.3.
4. The strategic choices of all firms become common knowledge and the stage game is repeated.

The decision maker chooses strategies c or k with the objective to maximize his present value of utility as is explained in subsection 2.4. Section 3 solves our model and presents the conditions that must be satisfied such that a decision maker switches from competitive to collusive conduct (see subsection 3.2 on cartel formation) or from collusive to competitive conduct (see subsection 3.1 on firms' deviation decision). In particular, it is analyzed how different assumptions on the shape of the utility function affect these patterns in the presence of opportunity costs of collusion and for different levels of shocks' persistence.

6 The basic structure of the model is similar to, for example, Rotemberg and Saloner (1986) or Thoron (1988).

2.1 State of the Industry

In the introductory section 1 we present case evidence which suggests that cartels are frequently formed following a shock that lowers the profitability of doing business in a particular industry. Moreover, a persistent profitability shock is typically more likely to lead to cartel formation in a state of low profits than a transitory shock.

The first feature, i.e. varying levels of profitability, is captured by a variety of models that study the stability of collusion in the presence of changes in demand. The most common models assume independent and identically distributed demand shocks (Green and Porter, 1984; Rotemberg and Saloner, 1986; Staiger and Wolak, 1992), deterministic business cycles (Haltiwanger and Harrington, 1991; Fabra, 2006; Knittel and Lepore, 2010), or non-deterministic business cycles where growth rates follow a Markov process (Bagwell and Staiger, 1997). The second feature, i.e. persistence, is modeled by Athey and Bagwell (2008) and Kandori (1991) who use a Markov-process to model changes of parameters that affect firms' profits. While Athey and Bagwell (2008) use a model with private cost shocks, Kandori (1991) focuses on demand shocks with persistence.

Given the evidence of cartel formation we model the state s of the industry by a two-state Markov-process with persistence probability ρ and, thus, follow Athey and Bagwell (2008) and Kandori (1991). If the industry is in the good state $\bar{s}$ in period t it remains in the good state in period t+1 with probability ρ and switches to the bad state $\underline{s}$ with probability 1-ρ. The same transition probabilities apply if the industry is initially in the bad state $\underline{s}$. Hence, the firms know about the possibility of shocks to occur in future periods but cannot predict the actual timing of shock events. By assuming *some* shock on profits rather than assuming a specific shock, e.g. on demand or costs, our model is somewhat more general than prior literature. This is in line with case evidence which suggests that persistently negative profit shocks facilitate cartel formation independently from the nature of the shock, for example, as a cost or demand shock.

2.2 Profits

The profit $\pi_i(s,a_i,a_{-i})$ of firm i is a function of the state of the industry $s \in \{\underline{s},\bar{s}\}$, the own strategy a_i of firm i, and the strategy a_{-i} of the other firm. By earning profit π_i the manager of firm i receives utility $u(\pi_i)=u(s,a_i,a_{-i})$ as is argued in subsection 2.3 below. To simplify notation we write $\bar{\pi}_i(a_i,a_{-i})$ in the good state $\bar{s}$ and $\underline{\pi}_i(a_i,a_{-i})$ in the bad state $\underline{s}$. When referring to some unspecified state s or its complementary state s' we write $\pi_i(a_i,a_{-i})$ or $\pi_i'(a_i,a_{-i})$.[7] Utility is denoted accordingly as $\underline{u}(a_i,a_{-i})$, $\bar{u}(a_i,a_{-i})$, $u(a_i,a_{-i})$, or $u'(a_i,a_{-i})$. Four situations can arise with regard to the strategies of the firms.

1. Both firms compete (a_i=c, a_{-i}=c). A firm is said to compete when it chooses a value of its strategic variable (e.g. price or quantity) according to its best response function in the product market game. Competitive profits and utility are denoted as $\pi_i(c,c)=\pi_{i,c}$ and $u_{i,c}$.
2. Both firms collude (a_i=k, a_{-i}=k). A firm is said to collude by choosing a value of its strategic variable that was agreed upon by the (explicitly) colluding firms. Collusive profits and utility are denoted as $\pi_i(k,k)=\pi_{i,k}$ and $u_{i,k}$.
3. Firm i behaves according to its best response function (a_i=c) while the other firm colludes (a_{-i}=k). When firm i acts as a deviator from a collusive agreement, profits and utility are denoted as $\pi_i(c,k)=\pi_{i,d}$ and $u_{i,d}$.
4. Firm i acts collusively (a_i=k) while the other firm competes (a_{-i}=c). We use the index s in $\pi_i(k,c)=\pi_{i,s}$ and $u_{i,s}$ as this situation *sucks*.[8] This is because the profit $\pi_{i,s}$ is below collusive profits $\pi_{i,k}$.

We concentrate on the actions of firm i and skip the individual index i where possible. For the below reasons, the model is kept as general as possible and does not assume a specific model of product market competition. We rather assume that the profits can be ranked $0<\pi_c<\pi_k<\pi_d$ both in the good and in the bad state of the industry, i.e. we impose a prisoner's dilemma structure on the game (Harrington and Chang, 2009). The sucker's payoff is below the collusive profit ($\pi_s<\pi_k$) and might even be below the competitive profit ($\pi_s<\pi_c$) but plays a negligible role in our model anyway. The remainder of this subsection explains with recourse to both theory and practice why it is reasonable to use a prisoner's dilemma structure to model industries that are prone to collusion.

In contrast to related literature such as Rotemberg and Saloner (1986), Haltiwanger and Harrington (1991), Staiger and Wolak (1992), Fabra (2006), or Knittel and Lepore (2010) we do not specify a full model of competition and collusion by making assumptions on the structural characteristics of the industry (demand, costs, capacity constraints, strategic variables, shock types etc.) and the features of the cartel (e.g. type of the agreement, mode for splitting profits, monitoring

7 For $s=\underline{s}$ we define $s'=\bar{s}$ while for $s=\bar{s}$ we define $s'=\underline{s}$. It proves convenient to work with state s and the complementary state s'. This allows us to solve our model in section 3 for some state s and then plug in $\underline{s}$ or $\bar{s}$ to analyze how this affects our solutions.

8 The term *sucker's payoff* is fairly common in the literature on prisoner's dilemma games but less often used in the literature on collusion.

measures). This prior literature analyzes how different structural characteristics of an industry affect the value of the profit differences π_d-π_k, π_c-π_k, and π_c-π_d when the industry moves into different states that are typically modeled by different types of cost- or demand-shocks. It is examined how these assumptions affect the stability of collusion by having an effect on the critical discount factor. This literature assumes that the utility function of the decision maker is linear in the profits of the firm and, thus, assumes profit-/value-maximizing behavior. Our model complements this prior work by asking how cartel formation and stability are affected by different shapes of the utility function under the assumption of a manager who maximizes his present value of utility (see subsection 2.3). Therefore, we *assume* a prisoner's dilemma structure of profits rather than *deriving* it from a full industry model. This approach allows us, first, to analyze cartel formation in the most general model we can think of and, second, to assume that the profit differences are the same in the high and in the low state of the industry, i.e. $\underline{\pi}_k-\underline{\pi}_c=\bar{\pi}_k-\bar{\pi}_c$, $\underline{\pi}_d-\underline{\pi}_k=\bar{\pi}_d-\bar{\pi}_k$, and $\underline{\pi}_d-\underline{\pi}_c=\bar{\pi}_d-\bar{\pi}_c$. This prevents that state-dependent profit differences exercise a confounding effect in our efforts to analyze how different shapes of the utility function affect the best responses of firm *i* with regard to the strategies collusion and competition.[9]

Our model also assumes that the profit π is net of all monetary costs associated with either strategy. This assumption can be motivated by a prudent firm that retains the expected present value of such future expenses in the form of equity reserves rather than distributing them in the form of managerial income or dividends. Even if such prudent behavior does not occur the value of the firm may still be the same as in the situation of retained earnings when managers and owners/shareholders consider the expected present value of such payments in their decision making.[10] This assumption is particularly relevant for π_k in the case of collusion where explicit costs are incurred in two forms. On the one hand, this refers to the transaction costs for managing the collusive agreement. On the other hand, this also refers to expected fines or payments for damages. It is well-known that these (expected) costs are typically too low to effectively deter cartel formation (Connor and Lande, 2005; Harrington, 2010) which implies $\pi_c<\pi_k$ in both states of the industry.

9 Note that our model is consistent with the cited ones which are used to analyze the effect of structural factors on collusion. In Appendix B we provide an example where our model is specified as a Cournot duopoly model with firms being subject to fixed cost shocks.

10 The latter argument is similar to the Barro-Ricardo equivalence theorem which states that consumers do not respond to debt-financed tax reductions with higher consumption. This is because they expect to pay higher taxes in the future. Ricardian equivalence may break down when the time horizon of the decision maker is finite. This would go beyond our assumption of an infinite game.

We assume the firms to make positive competitive profits in both states, i.e. $0<\underline{\pi}_c<\overline{\pi}_c$. This rules out situations where an industry is in such a severe crisis that competitive profits become negative. In such a situation, collusion might be viewed as the most desirable strategy compared to other high-risk and/or high-cost strategies of last resort such as product or process innovations or a war of attrition where each firm hopes that its competitors exit the industry first. Under these circumstances, the formation of a cartel would not be surprising news given the evidence on crisis cartels.[11] Assuming positive profits in competition ($0<\pi_c$) implies that our paper is concerned with the more interesting case of cartel formation in industries where a negative profit shock does not endanger the survival of a firm.

The prisoners' dilemma structure of our game assumes the deviation profits to be higher than the collusive profits in both states of the industry, i.e. $\overline{\pi}_k<\overline{\pi}_d$ and $\underline{\pi}_k<\underline{\pi}_d$. A secret deviation occurs when the firm does not adhere to the collusive agreement by, for example, setting a lower than agreed upon price or selling a higher output. Hence, firm *i* behaves according to its best response function ($a_i=c$) while the other firm colludes ($a_{-i}=k$). Deviation profits would not necessarily be above collusive profits when the betrayed cartel member can observe and punish a deviation instantaneously. Therefore, with regard to the above assumptions on the timing of the game a period is defined as the time required to detect a deviation from collusion.[12]

The condition $\pi_k<\pi_d$ would also no longer apply when both firms decided to deviate simultaneously.[13] Such joint deviations are less likely to occur when some asymmetry among firm's production technology, products or customer base exists. Asymmetry implies that in many cases only one firm would have an incentive to deviate from the cartel. Moreover, assume that the firms cannot learn these asymmetries from observing the market as would be the case when their fixed costs differ. Such imperfect knowledge about others' incentives affects the expectations of a firm: Often a firm can only rely on the assumption to be the sole deviator. This is in line with most of the literature on collusion and implies $\pi_k<\pi_d$.

A firm could also adhere to the cartel agreement ($a_i=k$) while the other deviates ($a_{-i}=c$). Such a betrayed firm would make a lower than collusive profit $\pi_s<\pi_k$ which may sometimes be even below the competitive profit $\pi_s<\pi_c$. The literature on cooperation experiments (Rapoport and Chammah, 1965) argues that the rate of cooperation may be the lower the smaller is π_s. π_s is considered in an extension to

11 The issue of crisis cartels has, for example, been discussed in greater depth at the OECD's 10th Global Forum on Competition in February 2011. The background documentation can be found here: http://www.oecd.org/competition/globalforum/GlobalForum-February2011.pdf

12 See Harrington (2006) for an overview on monitoring and enforcement practices in real cartels.

13 See Thoron (1998) on coalition-proof stable cartels.

our model (see Appendix C) where it is shown that the possibility of being betrayed by the other cartel firm makes collusion less profitable in expectation and, thus, becomes less stable. It is also shown that considering π_s leaves our qualitative conclusions unaltered (see section 3). Therefore and along with the majority of literature on collusion, π_s remains unconsidered in the main model for reasons of parameter parsimony.

Firm *i* would also make profit π_s when a coordination failure occurs in the formation stage, i.e. firm *i* already colludes while the other firm does not participate. The prevalence of coordination failure may possibly be reduced by prior talks and negotiations that create focal points for coordinated behavior (Farrell and Rabin, 1996). In this context, cartels are frequently formed and managed in practice while firms come together and talk at the meetings of a trade association (Harrington, 2006). Therefore, our negligence of such coordination failures means that our model has more to say on explicit collusion than on tacit collusion.

2.3 Utility Function and Opportunity Costs

Our model assumes that the market conduct $a_i \in \{c,k\}$ of firm *i* is determined in every period *t* by a manager of the firm.[14] The manager chooses a strategy a_i with the objective to maximize the present value V_t of utility $u(\pi_i)$ net of the opportunity costs of collusion κ, with $\beta \in [0,1]$ denoting the discount factor of the decision maker. This is shown in equation (1) and detailed in subsection 2.4

$$\begin{aligned} V_t &= V\left(s, a_i, a_{-i}\right) \\ V_t &= u\left(s, a_i, a_{-i}\right) - \kappa \cdot \iota_k\left(a_i\right) + \rho \cdot \beta \cdot V\left(s, a_i^*, a_{-i}\right) \\ &\quad + \left(1-\rho\right) \cdot \beta \cdot V\left(s', a_i^*, a_{-i}\right) \end{aligned} \tag{1}$$

Utility $u(\pi_i)$ is a function of profits $\pi_i(s,a_i,a_{-i})$ that depend on the state of the industry *s*, the strategy a_i of firm *i*, and the strategy profile a_{-i} of the other firms. Hence, utility $u(s,a_i,a_{-i})$ can also be written as a function of these parameters. The manager is assumed to incur opportunity costs of collusion κ when choosing the collusive strategy *k*. Thus, the indicator function $\iota_k(a_i)$ takes a value of 1 when the manager colludes and 0 otherwise.

14 Harrington (2006) presents evidence for several European cartels where the most critical decisions on prices and quantities are made by top level managers while the monitoring and implementation of the collusive agreement is made on lower management levels. Based on this evidence Buccirossi and Spagnolo (2008) conclude “that the decision to form a cartel is typically taken at the very top level of the firm hierarchy and is then implemented by issuing instructions to lower level managers that try to hide the collusive arrangement.” These patterns are also supported by evidence from 56 European cartel cases prosecuted between 1990 and 2009: Ashton and Pressey (2012) find that in 64.3% of cartels representatives of the two highest management levels were involved.

$$\iota_k(a_i)=\begin{cases}1 & \text{if } a_i=k\\ 0 & \text{if } a_i=c\end{cases} \tag{2}$$

Below, we explain the economic and mathematical properties of objective function u. Moreover, we elaborate on the reasons for considering opportunity costs κ at all and why we do so by using an additive term.

In period t+1, the industry remains in state s with probability ρ and switches to the complementary state s' with probability 1-ρ. In these cases, the present value of utility of the decision maker is denoted $V(s,a_i^*,a_{-i})$ respectively $V(s',a_i^*,a_{-i})$. We assume that decision maker i behaves optimally in period t+1 by choosing the best response a_i^* to the strategy profile a_{-i} of the other firm. The best responses a_i^* of firm i are derived in section 3 where we also analyze how a_i^* depends on the state of the industry s and its persistence ρ, as well as the values of the discount factor β and the opportunity costs κ.

We analyze two functional forms of the objective function u (Granger and Machina, 2006):

1. Utility may be linear in profits, i.e. $\partial u/\partial\pi>0$ and $\partial^2 u/\partial\pi^2=0$.
2. Utility is allowed to be concave in profits, i.e. $\partial u/\partial\pi>0$ and $\partial^2 u/\partial\pi^2<0$.

We comment on the properties of these functional forms in turn. The condition of utility to rise in the profits of the firm ($\partial u/\partial\pi>0$) is satisfied when the manager is the owner and, thus, residual claimant of the firm. If this is not the case, concavity applies under the conditions, first, that the compensation of a manager who does not (fully) own the firm himself rises in the profit of the firm and, second, that the manager's marginal utility from additional income is positive. In practice, such compensation schemes frequently consist of a fixed element and a profit-related bonus (Murphy, 1999).

We are also interested in the effects of the curvature of the utility function u on the additional utility $u(\pi_1)$-$u(\pi_0)$ that the decision maker may obtain by changing the conduct of the firm and earning a profit π_1 instead of the previously earned profit π_0. To analyze the effects of the curvature of u, we approximate the additional utility by Taylor series (3).

$$u(\pi_1)-u(\pi_0) \quad\approx\quad \frac{\partial u(\pi_0)}{\partial\pi}\cdot(\pi_1-\pi_0)+\frac{\partial^2 u(\pi_0)}{\partial\pi^2}\cdot\frac{(\pi_1-\pi_0)^2}{2} \tag{3}$$

Utility is linear in profits ($\partial^2 u/\partial\pi^2=0$) especially when the manager is the owner / residual claimant of the firm and shows a neutral attitude towards risk. Under the assumption of linearity ($\partial u/\partial\pi>0$ and $\partial^2 u/\partial\pi^2=0$) the condition $u(\pi_1)$-$u(\pi_0)$=($\partial u/\partial\pi$)(π_1-π_0) applies. The behavior of the decision maker is identical to that of a profit- respectively value-maximizing firm.

A concave utility function ($\partial u/\partial\pi>0$, $\partial^2 u/\partial\pi^2<0$) can be observed in a variety of situations both at the firm-level and at the level of managers. Based on an extensive review of prior literature, Spagnolo (1999) argues that concavity may be a result of managers' empirically observed aversion to intertemporal substitution of profits. Concavity of utility in profits can also be a result of concavity in income, which is the defining characteristic of a risk averse manager. The concavity of the utility function may also result from concavity of managerial income in the firm's profits (capped bonuses). Spagnolo (1999) also provides evidence that the objective function of the firm itself might be concave. For example, firms are usually risk averse as can be inferred from their efforts to hedge risks. Reasons for a concave objective function might also be convex costs of external finance, convex tax schedules, or investors' preference for assets with smooth returns.[15]

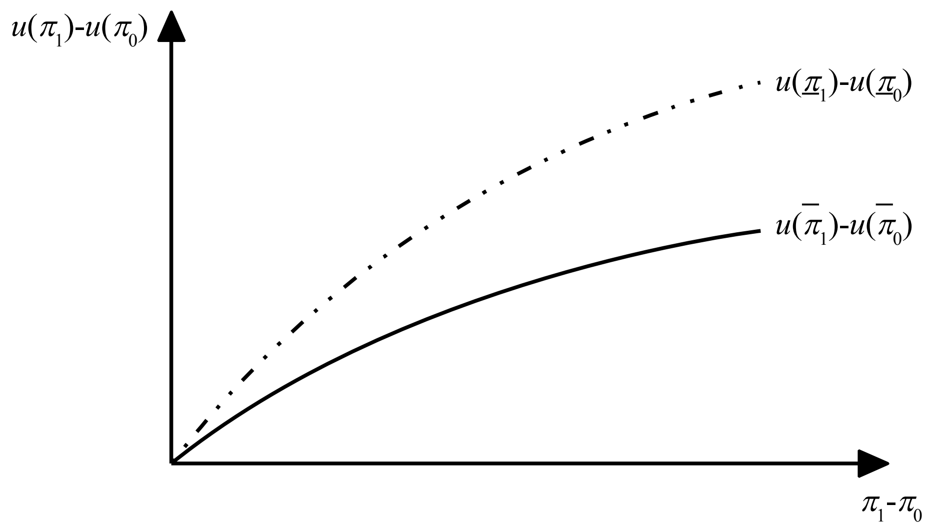

Figure 1: Change in utility

When the objective function is concave in profits ($\partial u/\partial\pi>0$, $\partial^2 u/\partial\pi^2<0$) it is possible to show that the change in utility $u(\pi_1)$-$u(\pi_0)$ is also concave in the additional profits π_1-π_0 (see Appendix A) and takes a value of $u(\pi_1)$-$u(\pi_0)=0$ for π_1-$\pi_0=0$. We concentrate on profit gains, i.e. π_1-$\pi_0\geq 0$, because this article is concerned with the decisions to switch from competitive to collusive behavior or from collusive to deviant conduct where a firm earns positive additional profits. Moreover, assume two sets of profits with identical additional profits, i.e. $\underline{\pi}_1$-$\underline{\pi}_0=\bar{\pi}_1$-$\bar{\pi}_0$ and $\underline{\pi}_0<\bar{\pi}_0$. It can be shown that the additional utility from earning a certain amount of additional profits is the higher the lower is the value of *status quo* profits π_0, i.e. $\partial(u(\pi_1)$-$u(\pi_0))/\partial\pi_0<0$ respectively $u(\underline{\pi}_1)$-$u(\underline{\pi}_0)>u(\bar{\pi}_1)$-$u(\bar{\pi}_0)$. This is also shown in Figure 1. In other words, a manager with a concave utility function values a 1EUR increase in

15 A more detailed treatment of concave objective functions is provided in Spagnolo (1998).

profits the higher the lower profits had been prior to the change.

Our model assumes that a decision maker chooses the market conduct a_i of firm i with the objective to maximize value function (1). Strategy k (cartelization) is against the law in many jurisdictions. Therefore, we model law-breaking as the outcome of a rational process of decision making which is related to the opportunity theory of crime in criminology: "When an individual has internalized a certain goal, and when the legitimate means for achieving that goal are blocked, the individual is under pressure to resort to illegitimate means to achieve the goal" (Braithwaite, 1989).[16]

We argue based on prior literature that the decision to act against the law by colluding may cause the decision maker a pain/dis-utility κ (Mongin and d'Aspremont, 1999). The opportunity cost parameter κ refers to non-monetary costs of collusion at the level of the decision-maker as is explained below. Notice that κ is different from and does not include the explicit costs of collusion. These have already been considered when calculating profits π (see subsection 2.2). The opportunity costs of collusion are modeled as an additive term in value function (1) rather than being included in the utility function u for mainly two reasons.[17] First, u is a function of variables that are measured in monetary terms, i.e. profits π, while κ is a non-monetary measure of dis-utility. Second, the opportunity costs of breaking the law are assumed to be independent of the size of profits π and, thus, shall also be independent of the state s of the industry. In other words, a manager is assumed to experience the same feelings of, for example, shame for breaking the law both in the good state with high profits and in the bad state with low profits. However, when his utility function is concave he values the additional profits from collusion higher in the bad state.

In the following, we discuss some origins of the opportunity costs. We also show why collusion typically causes opportunity *costs*, i.e. a loss of utility, instead of a utility *gain*. One could hypothesize that a manager might gain utility from acting against the law ($\kappa<0$). For example, he might be thrilled (Paternoster and Simpson, 1996) or expect rewards from his social environment by e.g. being considered a daredevil. Alternatively, acting against the law may cause him a dis-utility ($\kappa>0$) by, first, acting against one's own moral standards and, second, anticipating the disapproval by relevant social groups (family, friends, colleagues etc.) once the manager's active role in the illegal conspiracy is discovered. Senior managers have also been reported to be "concerned about their personal reputation within the marketplace, which would be adversely affected if they were to be associated with a business

16 Braithwaite (1989) explicitly names a price fixing agreement among brewing companies as an example for such decision making.

17 Krupka and Weber (2013) use a similar additivity assumption.

that had been the subject of an alleged competition law breach" (OFT, 2010). Netting out these effects, we assume that colluding implies opportunity costs for the manager, i.e. $\kappa>0$. This is for the following reasons.

Concerning the effects of law-breaking on one's own moral standards, one finds that people who believe strongly in the importance of complying with the law are less likely to violate the law.[18] Based on the review of other criminological research as well as own experimental results Paternoster and Simpson (1993, 1996) find that moral inhibitions or feelings of shame are negatively correlated with offending. Similar results are found by Zarkada-Fraser and Skitmore (2000) who interviewed 72 managing employees in the Australian construction industry about their propensity to collude in the form of tendering agreements. Only 11 out of the 72 managing employees in the Australian construction industry revealed a propensity to collude. Sonnenfeld and Lawrence (1978) provide further case evidence.

The anticipation of disapproval by relevant social groups could also be called social censure (Granovetter, 2005) or shaming (Braithwaite, 1989) that may have deterrence effects. These effects also gain importance in the antitrust literature on compliance programs (Sokol, 2012). Research (Paternoster and Simpson, 1996) indicates that the moral climate within the company, i.e. the acceptance of illegal behavior by colleagues and/or superiors, may affect an agent's disposition to commit white collar crimes. Shaming effects can even be multiplied. The wrongdoing of one offender can make his proximate community (e.g. family members or people in his company) feel ashamed who then transmit this dis-utility to the offender (Braithwaite, 1989; Tyler, 2006). Note that the perceived dis-utility of feeling ashamed can differ across cultures (Braithwaite, 1989) and time. Bigoni et al. (2009) provide experimental evidence that cultural differences between Italians and Swedes may affect collusive outcomes.

An alternative explanation for the opportunity costs of collusion would be expected non-monetary costs associated with the prosecution of the conspiracy once it is revealed to a competition authority. For example, managers may incur opportunity costs in the form of time and efforts devoted to competition trials. In jurisdictions where cartelization is a criminal offense opportunity costs may also relate to the expected time being spent in prison.

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18 See Braithwaite (1989) for further literature on this issue.

2.4 Value Functions

The decision maker of firm i is assumed to choose a strategy a_i with the objective to maximize his present value of utility $V(s,a_i,a_{-i})$ (see equation (1)). The value of $V(s,a_i,a_{-i})$ also depends on the strategy profile a_{-i} of the other firm, the state of the industry s, the persistence ρ of industry conditions, the discount factor β of the decision maker, and the shape of his utility function $u(\pi_i)$.

Equation (4) shows the utility $V(s,c,c_{-i})=V_c$ of decision maker i when he and the other firm decide to compete ($a_i=c$, $a_{-i}=c$) in period t.

$$\begin{aligned} V_c &= u_c + \rho\cdot\beta\cdot\left[\left(1-\iota_k(a_i)\right)\cdot V_c + \iota_k(a_i)\cdot V_k\right] \\ &\quad + (1-\rho)\cdot\beta\cdot\left[\left(1-\iota_k'(a_i)\right)\cdot V_c' + \iota_k'(a_i)\cdot V_k'\right] \end{aligned} \tag{4}$$

In period t+1, the state of the industry remains in the same state s as in period t with probability ρ and switches to the complementary state s' with probability 1-ρ. In either situation, the decision maker must decide whether he continues competing ($\iota_k(c)$=0 given $a_{-i}=c$) resulting in a competitive value V_c respectively V_c' or whether he sits together with the other firm and establishes a cartel ($\iota_k(k)$=1 given $a_{-i}=k$) resulting in a collusive value V_k respectively V_k'.[19] A cartel is formed when for both firms the discounted present utility value in case of collusion exceeds that when continuing to compete, i.e. $V_k>V_c$.

To analyze the participation decision of firm i, we define the decision maker's incremental value of collusion $\Omega(s)$ as the difference between the collusive and the competitive present value of utility in state s.

$$\begin{aligned} \Omega &= V_k - V_c \\ &= V(s,k,k) - V(s,c,c) \end{aligned} \tag{5}$$

The incremental value to collude is a measure for the participation constraint of the decision maker. He would participate in a cartel when colluding raises his present value of utility (i.e. Ω>0) and continues to compete otherwise (i.e. $\Omega\leq0$). The participation decision is analyzed in greater detail in subsection 3.2. It is not only a function of s, ρ, β, and the shape of u, it also depends on the expected stability of the cartel as is argued in the following and detailed in subsection 3.1.

19 Recall that our analysis concentrates on explicitly collusive agreements which allows us to disregard coordination failures where firm i colludes while the (majority of the) other firms continues to compete.

Equation (6) denotes the present value of utility when colluding in period t, i.e. $V(s,k,k_{-i})=V_k$.

$$\begin{aligned} V_k \;=\; u_k-\kappa \;&+\; \rho\cdot\beta\cdot\left[(1-\iota_d(a_i))\cdot V_k + \iota_d(a_i)\cdot V_d\right] \\ &+\; (1-\rho)\cdot\beta\cdot\left[(1-\iota_d{}'(a_i))\cdot V_k{}' + \iota_d{}'(a_i)\cdot V_d{}'\right] \end{aligned} \tag{6}$$

When colluding in period t the decision maker incurs opportunity costs κ. In period t+1, the industry remains in state s with probability ρ or switches to the complementary state s' with probability 1-ρ. The indicator function $\iota_d(a_i)$ shows whether the decision maker would deviate ($\iota_d(c)$=1 given a_{-i}=k) in period t+1 or not ($\iota_d(k)$=0 given a_{-i}=k).[20] A decision maker would want to deviate from the collusive agreement if his discounted present value of utility in case of a deviation V_d exceeds that when continuing to collude V_k. This is summarized in condition (7).

$$\iota_d(a_i)=\begin{cases} 1 & \text{if } a_i=c \text{ given } a_{-i}=k \;\Leftrightarrow\; V_d>V_k \\ 0 & \text{if } a_i=k \text{ given } a_{-i}=k \;\Leftrightarrow\; V_d\leq V_k \end{cases} \tag{7}$$

The collusive agreement is stable as long as no decision maker has an incentive to deviate. The deviation decision is a function of s, ρ, β, the shape of u, and the type of punishment that the other firms impose on the deviator i once the deviant conduct has been detected. The choice of a particular punishment strategy affects the size of the present value of utility $V(s,c,k)=V_d$ as is shown in condition (8).

$$V_d \;=\; u_d \;+\; \sum_{\tau=t+1}^{\infty} \beta^{\tau-t}\cdot\left[prob_\tau(s)\cdot u_c+(1-prob_\tau(s))\cdot u_c{}'\right] \tag{8}$$

Condition (8) assumes a grim trigger strategy (Friedman 1971) where the firms punish a deviation from the cartel by eternal reversion to the competitive Nash equilibrium. Hence, from period t+1 onward the decision maker receives utility u_c in state s and u_c' in state s'. $prob_\tau(s)$ denotes the conditional probability that the industry is in state s in period $\tau \in [t+1,\infty[$ when it was in state s in period t. This is the case when the number of state-switches θ is even, i.e. $\theta \in \{0,2,4,...\}$. Since a switch between states occurs with probability 1-ρ we can use a binomial distribution to calculate $prob_\tau(s)$ as is shown in (9).

$$prob_\tau(s)=\sum_{\substack{\theta\leq\tau \\ \theta\in\{0,2,4,...\}}} \binom{\tau-t}{\theta}\cdot(1-\rho)^\theta\cdot\rho^{\tau-t-\theta} \tag{9}$$

The state s that was observed in period t has no predictive power for the state in period τ when τ is in the very distant future, i.e. $\lim_{\tau\to\infty} prob_\tau(s)=0.5$. 1-$prob_\tau(s)$ denotes the complementary probability that

20 Appendix C shows that allowing for situations where the other firm deviates (a_{-i}=c) while firm i colludes (a_{-i}=k) makes collusion less profitable but does not alter our qualitative conclusions. Therefore, this case remains unconsidered in the main part of this article.

the industry is in state s' in period τ.

As an alternative to assuming a grim trigger strategy one might also think of other punishment strategies like temporary price wars (Green and Porter, 1984), most collusive pricing (Rotemberg and Saloner, 1986; Haltiwanger and Harrington, 1991), or optimal punishments (Abreu, 1986). These bear the potential to stabilize collusion for lower discount factors β or higher opportunity costs κ than the ones derived in subsection 3.1 under the assumption of a grim trigger strategy. We assume grim trigger punishments for mainly three reasons. First and least important, assuming a grim trigger strategy is computationally convenient and keeps the model fairly lean and transparent. Second, there is no indication that besides making collusion more stable the assumption of other than grim punishments would alter our qualitative conclusions. Third and foremost, there is no other punishment method that clearly stands out as a better description of reality. This is because evidence from the lab indicates that individuals might rather be playing variants of tit-for-tat strategies (Fudenberg et al., 2012) instead of using the punishment strategies named above.

3 RESULTS

In subsection 3.1, we show for which combinations of the discount factor β and the opportunity costs of collusion κ the decision maker of firm i would have an incentive to deviate from a collusive agreement in no, one, or both states of the industry. We show that it depends on the persistence of profit shocks ρ whether the deviation incentive is higher in the good state $\bar{s}$ or the bad state $\underline{s}$. Given these patterns, subsection 3.2 shows under what conditions – especially on the opportunity costs κ – the decision maker of firm i would want to start colluding in no, one, or both states of the industry and whether the participation incentive is stronger in the good or the bad state. It is shown that under the assumption of a concave utility function the best responses of firm i resemble the patterns of cartel formation presented in the introductory section 1.

3.1 Deviation Decision

Following the discussion in subsection 2.4, the decision maker of firm i has an incentive to deviate from a collusive agreement in state s when his present value of utility in case of the deviation is above the collusive present value of utility, i.e. $V_d > V_k$.

Proposition 1: The higher the opportunity costs κ the more desirable is it for a decision maker to deviate from a collusive agreement. (a) For $\kappa \leq \kappa_{2stab}$ a deviation neither occurs in state s nor in state s'. (b) The collusive agreement is stable in just one state for $\kappa_{2stab} < \kappa \leq \kappa_{0stab}$. (c) The decision maker would want to deviate from the collusive agreement in both states for $\kappa > \kappa_{0stab}$.

Proposition 2: There is a trade-off between the opportunity costs of collusion κ and the discount factor β. Higher values of β raise the critical values κ_{2stab} and κ_{0stab} and, thus, for a given observed value of κ contribute to stabilizing collusion.

Proposition 1 and *Proposition 2* are plausible and in line with prior literature. We prove them before turning to the more interesting effects stemming from the shape of the utility function u and the persistence ρ of profit shocks.

Proof of Proposition 1 (c): When collusion is unstable in both states, the conditions $\iota_d = \iota_d' = 1$ apply. In period $t+1$, the best response of firm i is to deviate irrespective of the state of the industry. The present value of utility in collusion (see equation (6)) can be written as in (10).

$$\begin{aligned} V_k &= u_k - \kappa + \rho \cdot \beta \cdot V_d + (1-\rho) \cdot \beta \cdot V_d' \\ &= u_k - \kappa + \rho \cdot \beta \cdot u_d + (1-\rho) \cdot \beta \cdot u_d' + \sum_{\tau=t+2}^{\infty} \beta^{\tau-t} \cdot \left[prob_\tau(s) \cdot u_c + (1-prob_\tau(s)) \cdot u_c' \right] \end{aligned} \tag{10}$$

We re-write the deviation present value of utility (see equation (8)) as is shown in (11).

$$V_d = u_d + \rho \cdot \beta \cdot u_c + (1-\rho) \cdot \beta \cdot u_c' + \sum_{\tau=t+2}^{\infty} \beta^{\tau-t} \cdot \left[prob_\tau(s) \cdot u_c + (1-prob_\tau(s)) \cdot u_c' \right] \tag{11}$$

The expected present value of (punishment) profits from period $t+2$ onward is the same irrespective of choosing collusion or deviation in the current period t. We use instability condition $V_d - V_k > 0$ to solve for the critical value κ_{0stab} which the true opportunity costs of collusion κ must exceed for collusion to be unstable in both states.

$$\begin{aligned} \kappa &> (u_k - u_d) + \rho \cdot \beta \cdot (u_d - u_c) + (1-\rho) \cdot \beta \cdot (u_d' - u_c') \\ &> \kappa_{0stab} \qquad \blacksquare \end{aligned} \tag{12}$$

Further below, we analyze whether inequality (12) is stricter in the good or in the bad state.

Proof of Proposition 1 (a): We turn to the case where collusion is stable in both states, i.e. the conditions $\iota_d = \iota_d' = 0$ apply. The present value of utility in collusion (see equation (6)) can be written as in (13).

$$\begin{aligned} V_k &= u_k - \kappa + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \cdot \left[prob_\tau(s) \cdot (u_k - \kappa) + (1 - prob_\tau(s)) \cdot (u_k' - \kappa) \right] \\ &= u_k - \frac{\kappa}{1-\beta} + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \cdot \left[prob_\tau(s) \cdot u_k + (1 - prob_\tau(s)) \cdot u_k' \right] \end{aligned} \tag{13}$$

Stability condition V_d-$V_k \leq 0$ can be solved for the critical value κ_{2stab} which the true opportunity costs κ must not exceed when the decision maker shall adhere to the collusive agreement in both states.

$$\begin{aligned} \kappa &\leq (1-\beta) \cdot \left[u_k - u_d + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \cdot \left[prob_\tau(s) \cdot (u_k - u_c) + (1 - prob_\tau(s)) \cdot (u_k' - u_c') \right] \right] \\ &\leq \kappa_{2stab} \end{aligned} \tag{14}$$

■

Proof of Proposition 1 (b): The decision maker would want to adhere to the collusive agreement in just one state s (ι_d=0) and deviate in the complementary state s' (ι_d'=1) when the opportunity costs of collusion are between the above bounds, i.e. $\kappa_{2stab} < \kappa \leq \kappa_{0stab}$. Therefore, the present value of utility (see equation (6)) can be written as in (15) for state s and (16) for state s'.

$$\begin{aligned} V_k &= u_k - \kappa + (1-\rho) \cdot \beta \cdot V_d' + \rho \cdot \beta \cdot V_k \\ &= \left(u_k - \kappa + (1-\rho) \cdot \beta \cdot V_d'\right) / (1 - \rho \cdot \beta) \end{aligned} \tag{15}$$

$$\begin{aligned} V_k' &= u_k' - \kappa + (1-\rho) \cdot \beta \cdot V_k + \rho \cdot \beta \cdot V_d' \\ &= u_k' - \kappa + \frac{(1-\rho) \cdot \beta}{1 - \rho \cdot \beta} \cdot (u_k - \kappa) + \left[\rho \cdot \beta + \frac{[(1-\rho) \cdot \beta]^2}{1 - \rho \cdot \beta} \right] \cdot V_d' \end{aligned} \tag{16}$$

Stability in state s is equivalent to V_d-V_k<0. Solving this condition for κ yields $\kappa > \kappa_{2stab}$. Instability in state s' is equivalent to V_d'-$V_k' \geq 0$. Solving this condition for κ yields $\kappa \leq \kappa_{0stab}$. ■

We turn to *Proposition 2*, i.e. higher values of the discount factor β raise the critical values κ_{0stab} and κ_{2stab} and, thus, contribute to stabilizing collusion. Given $u_c < u_k < u_d$ the condition $\partial\kappa_{0stab}/\partial\beta > 0$ follows immediately from (12). The proof of $\partial\kappa_{2stab}/\partial\beta > 0$ is presented in Appendix A.

In the following, we interpret κ_{0stab} and κ_{2stab} with respect to the shape of utility function u and the state of the industry s. When the utility of the decision maker is linear in the profits of the firm ($\partial u/\partial\pi > 0$, $\partial^2 u/\partial\pi^2 = 0$) the size of κ_{0stab} and κ_{2stab}, i.e. the incentive to deviate, only depends on the size of the profit differences π_d-π_k, π_c-π_k, and π_c-π_d which are functions of the industry's structural characteristics (see subsection 2.2) and the state of the industry. To isolate the effect of the utility function on collusion from the effect of these structural characteristics we assumed $\underline{\pi}_k$-$\underline{\pi}_c$=$\overline{\pi}_k$-$\overline{\pi}_c$ and $\underline{\pi}_d$-$\underline{\pi}_k$=$\overline{\pi}_d$-$\overline{\pi}_k$. Given these conditions and the assumption of a linear utility function our model predicts the deviation incentive to be independent of the state of the industry, i.e. the conditions $\underline{\kappa}_{0\text{stab}}=\overline{\kappa}_{0\text{stab}}$ and $\underline{\kappa}_{2\text{stab}}=\overline{\kappa}_{2\text{stab}}$ apply.

This is different when the utility function is concave in profits ($\partial u/\partial\pi>0$, $\partial^2 u/\partial\pi^2<0$). The discussion in subsection 2.3 shows that even for $\underline{\pi}_d$-$\underline{\pi}_k$=$\overline{\pi}_d$-$\overline{\pi}_k$ concavity causes the short-run utility gain from a deviation to be higher in the bad state than in the good state, i.e. $\underline{u}_d$-$\underline{u}_k$>$\overline{u}_d$-$\overline{u}_k$. This effect is the stronger the higher the curvature of the utility function. However, the effect of concavity alone does not cause the deviation incentive to be always stronger in the bad state $\underline{s}$ of the industry than in the good state $\overline{s}$. This is because the punishment for deviating, i.e. earning competitive instead of collusive profits, is also perceived to be stronger in the bad state, i.e. $\underline{u}_k$-$\underline{u}_c$>$\overline{u}_k$-$\overline{u}_c$. Given these adverse effects, the size of the critical values $\kappa_{0\text{stab}}$ and $\kappa_{2\text{stab}}$ across the states $\underline{s}$ and $\overline{s}$ depends on the decision maker's valuation of future profits (as measured by the discount factor β) and the persistence of states (as measured by ρ).

Given *Proposition 1*, a decision-maker would want to adhere to the collusive agreement in at least one state if the inequality $\kappa\leq max(\underline{\kappa}_{0stab},\overline{\kappa}_{0stab})$ applies. Similarly, the decision-maker would adhere to the collusive agreement in both states under the condition $\kappa\leq min(\underline{\kappa}_{2stab},\overline{\kappa}_{2stab})$. The critical values $\underline{\kappa}_{0stab}$, $\overline{\kappa}_{0stab}$, $\underline{\kappa}_{2stab}$, and $\overline{\kappa}_{2stab}$ depend on the relative size of ρ and β as is shown in the following.

Proposition 3: When the utility function of the decision maker is concave in profits and shocks are short-lived ($\rho<\rho^*$) the incentive to deviate is higher in the bad state, i.e. $\underline{\kappa}_{0stab}<\overline{\kappa}_{0stab}$ and $\underline{\kappa}_{2stab}<\overline{\kappa}_{2stab}$. When shocks are fairly persistent ($\rho>\rho^*$) the incentive to deviate is higher in the good state, i.e. $\underline{\kappa}_{0stab}>\overline{\kappa}_{0stab}$ and $\underline{\kappa}_{2stab}>\overline{\kappa}_{2stab}$.

Proof of Proposition 3: We define ρ^* as the value that separates high and low values of ρ. It is found by equating $\underline{\kappa}_{0stab}$ and $\overline{\kappa}_{0stab}$.

$$\rho^* \equiv 0.5+\frac{(\underline{u}_d-\underline{u}_k)-(\overline{u}_d-\overline{u}_k)}{2\cdot\beta\cdot\left[(\underline{u}_d-\underline{u}_c)-(\overline{u}_d-\overline{u}_c)\right]} \tag{17}$$

It can be shown that $\rho^*\geq 0.5$ applies. The parameter ρ^* can also be found by equating $\underline{\kappa}_{2stab}$ and $\overline{\kappa}_{2stab}$ as is shown in Appendix A. It can be shown that for $\rho<\rho^*$ the inequalities $\underline{\kappa}_{0stab}<\overline{\kappa}_{0stab}$ and $\underline{\kappa}_{2stab}<\overline{\kappa}_{2stab}$ apply. ■

This implies that with $\rho<\rho^*$ a firm's incentive to deviate is higher in the bad state where the short-run gain from a deviation is higher than in the good state, i.e. $(\overline{u}_d-\overline{u}_k)<(\underline{u}_d-\underline{u}_k)$. In other words, the inequalities for instability in just one state ($\kappa>\kappa_{2stab}$) or in both states ($\kappa>\kappa_{0stab}$) are more likely to be satisfied in the bad state. This is because with such short-lived shocks the intensity of future punishments, i.e. gaining utility u_c instead of the higher u_k, varies relatively little with the current state of the industry. The deviation incentive is mainly driven by the short-run utility u_d-u_k. This is different when industry conditions are rather persistent ($\rho>\rho^*$). When the industry is in the bad state in period t it remains in the bad state in the near future with a fairly high probability. Hence, a decision-maker who deviates in the bad state $\underline{s}$ because of the higher short-run gain in utility $\underline{u}_d$-$\underline{u}_k$ also expects a high long-run loss of utility $\underline{u}_c$-$\underline{u}_k$ by being punished in the bad state, too. Given this long-lasting punishment the decision maker would rather refrain from deviating in the bad state $\underline{s}$ but do so in the good state $\overline{s}$.

Given $(\overline{u}_d-\overline{u}_k)<(\underline{u}_d-\underline{u}_k)$ and $(\overline{u}_k-\overline{u}_c)<(\underline{u}_k-\underline{u}_c)$, collusion is only stable in the bad state when the decision maker puts a relatively large weight on the long-run utility loss from being punished for the deviation. This is the case when his discount factor β is high. When β is low the decision maker is rather driven by the short-run gain from deviating than the long-run loss from being punished for the deviation. This is summarized in *Proposition 4*.

Proposition 4: When the utility function of the decision maker is concave in profits, a decision maker with a high discount factor β is more likely to deviate in the good state than a more impatient decision maker with a low realization of β.

Proof of Proposition 4: Given *Proposition 3* a deviation in the good state is the more likely the lower is ρ^*. For some value of ρ a lower value of ρ^* raises the chance that the inequality $\rho<\rho^*$ applies where the decision maker deviates in the good state. It can be seen from (17) that higher values of β indeed lower ρ^*, i.e. $\partial\rho^*/\partial\beta<0$. ■

This subsection shows that effects like risk aversion, progressive taxation, or investors' preference for profit smoothing, which are causal for a concave utility function, have the ability to affect the stability of collusion in the presence of profit shocks. In particular, for collusion to be stable in the bad state these unfavorable conditions must be rather persistent. This is in line with case evidence which suggests that stable cartels are formed after a negative and persistent profit shock has hit an industry. Since stability of collusion is only one prerequisite for the existence of a cartel subsection 3.2 analyzes the decision to participate in a collusive agreement in greater detail.

3.2 Participation Decision

A decision-maker has an incentive to participate in a collusive agreement when his incremental value to collude $\Omega=V_k-V_c$ (see equation (5)) is positive. In the following, we analyze cartel formation in the three cases identified in subsection 3.1, i.e. collusion being stable in no, one, or both states of the industry. It is shown that under the assumption of a concave utility function a decision maker does not necessarily have an incentive to start colluding even if an existing collusive agreement would be stable. To provide an intuition for this effect: In the stage game, the incremental utility from colluding net of the opportunity costs can be negative in the good state and positive in the bad state. Therefore, when being in the good state cartel formation would be delayed until the industry moves into the bad state. However, the firms would not deviate from the cartel once the industry moves back into the good state as this would trigger eternal punishment and prevent the firms from gaining positive additional utility in any future period where the industry is in the bad state. We show that a negative profit shock is particularly likely to induce cartel formation when the negative conditions are fairly persistent.

We start with the case where a decision-maker would want to deviate from a cartel in both states, i.e. condition (12) is violated ($\kappa>max(\underline{\kappa}_{0\text{stab}},\overline{\kappa}_{0\text{stab}})$). This does not necessarily imply that the decision maker has no incentive to participate in a collusive agreement at all. For example, Axelrod and Hamilton (1981) describe strategies where the firms alternately exploit each other by taking turns in playing the collusive or the deviant strategy. Such turn taking is not consistent with the cartel evidence and, thus, is not explored any further in this article.

Consider the case where the cartel is stable in both demand states s and s' with $\iota_d=\iota_d'=0$, i.e. condition (14) is satisfied ($\kappa\leq min(\underline{\kappa}_{2\text{stab}},\overline{\kappa}_{2\text{stab}})$). Using condition (6) for the collusive present value of utility and condition (4) for the competitive present value of utility, the incremental value to collude can be written as in (18).

$$\Omega=\frac{[u_k-\kappa-u_c]+(1-\rho)\cdot\beta\cdot(1-\iota_k')\cdot[V_k'-V_c']}{1+\rho\cdot\beta\cdot(1-\iota_k)} \tag{18}$$

Proposition 5: Assume a decision maker would adhere to a collusive agreement in both states, i.e. $\kappa\leq\kappa_{2\text{stab}}$. (a) When his opportunity costs of collusion are sufficiently small, i.e. $\kappa<\kappa_{2\text{x}2\text{form}}$, the decision maker would want to start participating in the cartel in either state. (b) For somewhat higher opportunity costs of collusion, i.e. $\kappa_{2\text{x}2\text{form}}\leq\kappa<\kappa_{2\text{x}1\text{form}}$, the decision maker would want to start participating in the cartel in just one state.

Proof of Proposition 5 (a): When the decision maker finds it utility-maximizing to start colluding in both states s and s' the condition $\iota_k=\iota_k'=1$ applies and the incremental value to collude from equation (18) simplifies to (19).

$$\Omega \quad = \quad u_k - u_c \quad - \quad \kappa \quad > \quad 0 \quad \forall \quad s \in \{\underline{s}, \bar{s}\} \tag{19}$$

Hence, the decision maker must receive a positive utility from colluding net of the opportunity costs in both states. We use (19) to derive (20).

$$min\left(\underline{u}_k - \underline{u}_c, \bar{u}_k - \bar{u}_c\right) > \kappa \quad \text{with} \quad u_k - u_c \equiv \kappa_{2\text{x}2\text{form}} \tag{20}$$

For $\kappa<\kappa_{2\text{x}2\text{form}}$ a decision maker would want to establish collusion in both states without having an incentive to deviate in either state because of $\kappa\leq\kappa_{2\text{stab}}$. ■

This case where the decision maker would want to establish and adhere to the collusive agreement in both states is of moderate interest as no profit shock is necessary to induce collusion. The industry would be characterized by collusion right from its start. Therefore, the below discussion centers on the more relevant case where the decision maker would adhere to an existing collusive agreement in both states ($\kappa\leq\kappa_{2\text{stab}}$) but would want to start participating in the cartel in just one of them. It is shown that this situation occurs for somewhat higher opportunity costs of collusion, i.e. $\kappa_{2\text{x}2\text{form}}\leq\kappa<\kappa_{2\text{x}1\text{form}}$.

Proof of Proposition 5 (b): To see that for $\kappa_{2\text{x}2\text{form}}\leq\kappa<\kappa_{2\text{x}1\text{form}}$ a stable cartel is formed in just one state consider the following situation. The decision maker would want to start participating in the cartel in the current state s ($\iota_k=1$, $\Omega>0$) but would not do so in the future state s' ($\iota_k'=0$, $\Omega'\leq0$). This situation occurs under two conditions. First, his opportunity costs are too high for collusion to be started in both states, i.e. $\kappa_{2\text{x}2\text{form}}\leq\kappa$. Second, the opportunity costs must not be so high as to deter cartel formation altogether, i.e. $\kappa<\kappa_{2\text{x}1\text{form}}$. We re-write the incremental value to collude from (18) as is shown in (21).[21]

$$\Omega \quad = \quad \left[u_k - u_c - \kappa\right] \quad + \quad \frac{(1-\rho)\cdot\beta}{1+\rho\cdot\beta}\cdot\left[u_k' - u_c' - \kappa\right] \tag{21}$$

Given (21), the inequality $\Omega>0$ enables us to solve for the critical value $\kappa_{2\text{x}1\text{form}}$ that the actual κ must not exceed for the cartel to be formed in state s.[22]

21 Condition (21) is derived from (18) by using $\iota_k=1$, $\iota_k=0$ and the functional symmetry between Ω and Ω', i.e. Ω' looks like Ω in (18) with all s and s' being exchanged.

22 Our example with Cournot-competition (see Appendix B) indicates that the condition $\kappa_{2\text{stab}}<\kappa_{2\text{x}1\text{form}}$ applies: When a decision maker would want to adhere to an existing collusive agreement he would have an incentive to start colluding in at least one state of the industry.

$$\kappa<\kappa_{2x1form}\equiv\frac{(1+\rho\beta)\cdot(u_k-u_c)+(1-\rho)\beta\cdot(u_k{}'-u_c{}')}{1+\beta} \quad \blacksquare \tag{22}$$

In the following, we explore under what conditions and in what state of the industry the decision maker would want to initiate the collusive agreement.

Proposition 6: (a) The case where a cartel is stable in both states ($\kappa\leq\kappa_{2stab}$) but is formed in just one state ($\kappa_{2x2form}\leq\kappa<\kappa_{2x1form}$) requires a concave objective function u. (b) With a concave objective function collusion will be established in the bad state of the industry. (c) This requires the discount factor β of the decision maker and the persistence ρ of industry conditions to be sufficiently high.

Proof of Proposition 6 (a): When the utility function is linear in profits ($\partial u/\partial\pi>0$, $\partial^2u/\partial\pi^2=0$) and under the assumption of identical profit differences, i.e. $\underline{\pi}_k-\underline{\pi}_c=\bar{\pi}_k-\bar{\pi}_c$, $\underline{\pi}_k-\underline{\pi}_d=\bar{\pi}_k-\bar{\pi}_d$, $\underline{\pi}_d-\underline{\pi}_c=\bar{\pi}_d-\bar{\pi}_c$, one obtains two findings. First, the critical values $\kappa_{2x2form}$ and $\kappa_{2x1form}$ are independent from the current state of the industry, i.e. $\underline{\kappa}_{2x2form}=\bar{\kappa}_{2x2form}$ and $\underline{\kappa}_{2x1form}=\bar{\kappa}_{2x1form}$. Second, they are identical, i.e. $\kappa_{2x2form}=\kappa_{2x1form}$. With a linear objective function and given the stability of collusion in both states ($\kappa\leq\kappa_{2stab}$) the decision maker has the same incentive to establish collusion in either state of the industry. It is easy to show that the inequality $\kappa_{2x2form}\neq\kappa_{2x1form}$ only applies for $u_k'-u_c'\neq u_k-u_c$. Under the assumption of state-independent profit differences this requires a non-linear utility function u. With concave utility ($\partial u/\partial\pi>0$, $\partial^2u/\partial\pi^2<0$) condition (20) is a function of the state of the industry, where $\underline{\kappa}_{2x2form}>\bar{\kappa}_{2x2form}$ applies. Hence, the relevant condition for a positive incremental value to collude to exist in just one state is $\bar{\kappa}_{2x2form}\leq\kappa<\kappa_{2x1form}$. It can be shown that the conditions $\bar{\kappa}_{2x2form}<\underline{\kappa}_{2x1form}$ and $\bar{\kappa}_{2x2form}<\bar{\kappa}_{2x1form}$ apply. ■

Proof of Proposition 6 (b): This brings us to the question in what state the decision maker would want to initiate collusion. Above, we state that the incremental value to collude in (21) shall be positive in state s ($\Omega>0$) but negative in state s' ($\Omega'\leq0$). These conditions imply $\Omega>\Omega'$. With concave utility this inequality is only satisfied in the bad state $\underline{s}$ of the industry. This is because in the bad state the additional utility from colluding is higher than in the good state, i.e. $\underline{u}_k-\underline{u}_c>\bar{u}_k-\bar{u}_c$. ■

Not having an incentive to initiate a new cartel in the good state $\bar{s}$ does not mean that the decision maker would also want to deviate from an existing cartel in the good state. This is because the decision maker gains positive additional utility every time the industry moves into the bad state $\underline{s}$. This advantage would be forgone by deviating and, thus, provoking punishment. The incremental value to collude in (21) shows that a cartel is formed in state $\underline{s}$ when the additional utility from doing so, i.e. $\underline{u}_k-\kappa-\underline{u}_c>0$, is large enough to compensate the manager for maintaining the cartel in the unfavorable state $\bar{s}$

with $\overline{u}_k$-κ-$\overline{u}_c$<0.

Proof of Proposition 6 (c): It can be shown that the incremental value to collude in the bad state (see equation (21)) rises in the discount factor β (given ρ>0.5) and in the persistence parameter ρ, i.e. $\partial\underline{\Omega}/\partial\beta>0$ and $\partial\underline{\Omega}/\partial\rho>0$. ■

These conditions are interpreted in the following. Given the above findings, an incentive to establish a stable collusive agreement in only one state of the industry, i.e. the bad state, exists under four conditions. First, the opportunity costs κ of the decision maker must be high enough ($\kappa_{2x2form}\leq\kappa$) such that he refrains from colluding in the good state. However, they must not be so high as to induce deviations ($\kappa\leq\kappa_{2stab}$). Second, the utility function of the decision maker must be concave. Therefore, his additional utility from colluding is higher in the bad state than in the good state. Third, condition $\partial\underline{\Omega}/\partial\rho>0$ implies that cartel formation requires the bad conditions to be sufficiently persistent such that the decision maker earns positive additional utility $\underline{u}_k$-κ-$\underline{u}_c$>0 long enough before moving in the high state again. Fourth, condition $\partial\underline{\Omega}/\partial\beta>0$ implies that the decision maker must be patient enough to sustain the cartel through the high state where collusion net of the opportunity costs gives him a lower utility than in competition, i.e. $\overline{u}_k$-κ-$\overline{u}_c$<0.

Let us turn to the case where the opportunity costs of collusion are so high ($\kappa_{2stab}<\kappa\leq\kappa_{0stab}$) that collusion is stable in just one state.

Proposition 7: (a) When profit shocks are persistent ($\rho>\rho^*$), i.e. collusion is stable in the bad state $\underline{s}$ only ($\overline{\kappa}_{2stab}<\kappa\leq\underline{\kappa}_{0stab}$), the decision maker would always have an incentive to participate in the collusive agreement in the bad state. (b) When profit shocks are transitory ($\rho<\rho^*$), i.e. collusion is stable in the good state $\overline{s}$ only ($\underline{\kappa}_{2stab}<\kappa\leq\overline{\kappa}_{0stab}$), the decision maker does not necessarily have an incentive to participate in the collusive agreement.

Proof of Proposition 7 (a): When profit shocks are persistent ($\rho>\rho^*$) collusion will only be established in the bad state when the firm receives a positive net utility from colluding in the stage game, i.e. $\kappa<\underline{u}_k$-$\underline{u}_c$. It is easy to show that this inequality is always satisfied when $\kappa\leq\underline{\kappa}_{0stab}$ applies, i.e. $\kappa\leq\underline{\kappa}_{0stab}<\underline{u}_k$-$\underline{u}_c$ must be satisfied.

$$\begin{array}{ccc} \underline{\kappa}_{0stab} & < & \underline{u}_k-\underline{u}_c \\ (\underline{u}_k-\underline{u}_d)+\rho\cdot\beta\cdot(\underline{u}_d-\underline{u}_c)+(1-\rho)\cdot\beta\cdot(\overline{u}_d-\overline{u}_c) & < & \underline{u}_k-\underline{u}_c \\ \dfrac{(1-\rho)\cdot\beta}{1-\rho\cdot\beta} & < & \dfrac{\underline{u}_d-\underline{u}_c}{\overline{u}_d-\overline{u}_c} \end{array} \tag{23}$$

The lowest value that the right-hand side of (23) can take under the assumptions of our model is 1.

Plugging this value into (23) and solving for β yields $\beta<1$. ■

Proof of Proposition 7 (b): When profit shocks are transitory ($\rho<\rho^*$) the inequality $\bar{\kappa}_{0\mathrm{stab}}<\bar{u}_k-\bar{u}_c$ can be used to find condition (24).

$$\frac{(1-\rho)\cdot\beta}{1-\rho\cdot\beta} < \frac{\bar{u}_d-\bar{u}_c}{\underline{u}_d-\underline{u}_c} \tag{24}$$

Under the assumptions of our model the right-hand side of (24) takes a maximum value of 1. This would be the case when the utility function is linear. For $0<\rho<1$ and $0<\beta<1$ the left-hand side of (24) takes values smaller than 1. Therefore, a decision maker with an almost linear utility function would want to establish collusion in the good state when profit shocks are transitory ($\rho<\rho^*$). However, the higher the curvature of a concave utility function the smaller is the right-hand side of (24) and the less likely is it that collusion is established under these conditions. ■

4 CONCLUSION

This paper uses a dynamic model to analyze the decision whether to establish and subsequently adhere to collusion under the assumption of the decision-maker pursuing a concave utility function and incurring non-negligible opportunity costs of collusion. These assumptions are well-grounded in both theory and evidence in fields neighboring competition economics but have received little attention in the Industrial Organization literature on collusion, yet. Given our model's suitability in predicting observable patterns of cartel formation one might think about considering these assumptions in competition economics to a greater extent. In this context, more research appears necessary to find evidence of concave objective functions as well as opportunity costs of collusion and explore their causes. An improved understanding of these effects provides a double benefit. First, non-behavioristic causes of concavity, e.g. rising costs of external finance, can be included more directly in structural models of collusion. Second, by identifying recurrent patterns of behavioral effects these effects can be stripped of some aftertaste, i.e. the risk to explain cartel formation by non-verifiable and possibly non-recurring events.

Specific areas of future research are the following. First, it will be interesting to search for more evidence of concave utility functions in practice. Second, it is relevant to identify the causes of concavity as these have implications for the design of competition policy measures that are intended to deter cartels. For example, concavity resulting from progressive taxation might call for changes of the tax system that go beyond the sphere of competition policy. When concavity is mainly a result of managers' risk aversion, cartel formation could possibly be reduced by lowering the share of the profit-dependent bonus in the compensation scheme of managers.

Our paper shows that the existence of opportunity costs of collusion may both deter and destabilize cartels. The underlying effects have been researched more extensively in fields like criminology, sociology and psychology. Our paper interprets these effects as opportunity costs and shows that they can easily be included in economic models. Future research might be directed to exploring possibilities to raise the opportunity costs and, thus, deter cartels. This may include further efforts to make the public aware of the detrimental effects of explicit collusion and deter cartels by a more intense naming and shaming approach. It could also be researched whether in countries like Germany, where competition laws are civil laws, stigmatization of explicitly collusive practices by prosecuting offenses under criminal laws may improve deterrence. As the opportunity costs of collusion also depend on the (dis)approval of business conduct by colleagues, compliance programs might possibly work through their effect on the relations among colleagues. Our model provides a framework to analyze such issues in greater depth.

Our paper focuses on the effects of substantial profitability shocks modeled by a two state Markov process. This shock structure has been identified from cartel cases to be relevant for the establishment of cartels and allows to research the effects of the persistence of industry conditions on collusion. Therefore, our model adds to existing literature on collusion that concentrates on demand or cost shocks which can either be transitory or follow a cyclical pattern.

Future research might be concerned with refining the Markov shock structure to account for additional patterns that are observed in practice. For example, one might think of asymmetric transition probabilities: The French beef cartel was formed in 2001 after demand had declined substantially because of the discovery of the mad cow disease in 2000 (EC 2003). The anticipated probability of this shock to occur was presumably smaller than the probability that demand would recover after some time. In other cases profits are subject to trends and (singular) shifts of those trends: In the cartel for flat glass the level of demand and the growth path of demand for energy efficient glass was shifted upwards by tougher legislation while prices had nonetheless been characterized by a steady decrease (EC 2007: 14, 22). This also raises the question in what ways unexpected shocks and expectable events like changes in legislation differ in their effect on collusion.

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APPENDICES

Appendix A Proofs

Proof 1: $u(\pi_1)$-$u(\pi_0)$ is concave in π_1-π_0

To see that $u(\pi_1)$-$u(\pi_0)$ is concave in π_1-π_0 we calculate the first derivative of the Taylor series (3) with respect to π_1-π_0.

$$\frac{\partial\left(u(\pi_1)-u(\pi_0)\right)}{\partial(\pi_1-\pi_0)} = \frac{\partial u(\pi_0)}{\partial\pi}+\frac{\partial^2 u(\pi_0)}{\partial\pi^2}\cdot(\pi_1-\pi_0) > 0 \tag{25}$$

To see that this first derivative is positive consider that an increase in profits π_1-π_0>0 implies an increase in utility $u(\pi_1)$-$u(\pi_0)$>0. Given this inequality we divide (3) through π_1-π_0 and find condition (26).

$$\frac{\partial u(\pi_0)}{\partial\pi}+\frac{\partial^2 u(\pi_0)}{\partial\pi^2}\cdot\frac{(\pi_1-\pi_0)}{2} > 0 \tag{26}$$

It is easy to show that the value of (25) exceeds that of (26). Hence, the first derivative (25) must be positive. The second derivative of $u(\pi_1)$-$u(\pi_0)$ is negative as can be seen from (27). Therefore, $u(\pi_1)$-$u(\pi_0)$ is concave in π_1-π_0.

$$\frac{\partial^2\left(u(\pi_1)-u(\pi_0)\right)}{\partial\left(\pi_1-\pi_0\right)^2} = \frac{\partial^2 u(\pi_0)}{\partial\pi^2} < 0 \tag{27}$$

Third, for a given profit difference π_1-π_0=Δπ the utility difference $u(\pi_0+\Delta\pi)$-$u(\pi_0)$ rises when the profit π_0 falls. Condition (28) shows that a gain in profits raises utility the less the higher the *status quo* profits π_0. The second line in (28) follows from the first by assuming $\partial^3 u/\partial\pi^3=0$.

$$\begin{aligned}\frac{\partial\left(u(\pi_0+\Delta\pi)-u(\pi_0)\right)}{\partial\pi} &= \left[\frac{\partial^2 u(\pi_0)}{\partial\pi^2}\cdot\Delta\pi-\frac{\partial u(\pi_0)}{\partial\pi}\right]+\left[\frac{\partial^3 u(\pi_0)}{\partial\pi^3}\cdot\frac{\Delta\pi^2}{2}-\frac{\partial^2 u(\pi_0)}{\partial\pi^2}\cdot\Delta\pi\right] \\ &= -\frac{\partial u(\pi_0)}{\partial\pi} < 0 \quad \blacksquare\end{aligned} \tag{28}$$

Proof 2: $\partial\kappa_{2\text{stab}}/\partial\beta>0$

Note that the derivative $\partial\kappa_{2\text{stab}}/\partial\beta$ can be written as in (29).

$$\begin{aligned}\frac{\partial\kappa_{2\text{stab}}}{\partial\beta} &= -\left[u_k-u_d+\sum_{\tau=t+1}^{\infty}\beta^{\tau-t}\cdot\left[prob_\tau(s)\cdot(u_k-u_c)+(1-prob_\tau(s))\cdot(u_k'-u_c')\right]\right]\\ &\quad+(1-\beta)\cdot\sum_{\tau=t+1}^{\infty}(\tau-t)\cdot\beta^{\tau-t-1}\cdot\left[prob_\tau(s)\cdot(u_k-u_c)+(1-prob_\tau(s))\cdot(u_k'-u_c')\right]\\ &= u_d-u_k+\sum_{\tau=t+1}^{\infty}\left[(1-\beta)\cdot(\tau-t)\cdot\beta^{\tau-t-1}-\beta^{\tau-t}\right]\cdot\left[prob_\tau(s)\cdot(u_k-u_c)+(1-prob_\tau(s))\cdot(u_k'-u_c')\right]\end{aligned} \tag{29}$$

Given the assumptions of our model, one finds u_d-u_k>0, u_k-u_c>0, u_k'-u_c'>0 and 0<$prob_\tau(s)$≤1. This leaves to show that the remaining term in brackets is positive.

$$\begin{aligned}(1-\beta)\cdot(\tau-t)\cdot\beta^{\tau-t-1}-\beta^{\tau-t} &> 0\\ (1-\beta)\cdot(\tau-t) &> \beta\\ \tau-t &> \beta\cdot(1-\tau+t)\end{aligned} \tag{30}$$

The inequality $\tau\geq t+1$ applies by definition. Therefore, the left-hand side of (30) is positive while $(1-\tau+t)\leq 0$ on the left-hand side applies. The inequality in (30) is always satisfied which implies $\partial\kappa_{2\text{stab}}/\partial\beta>0$. ■

Proof 3: Deriving ρ^* from $\underline{\kappa}_{2stab}=\overline{\kappa}_{2stab}$

In the following we show that the parameter ρ^* can also be found by equating $\underline{\kappa}_{2stab}$ and $\overline{\kappa}_{2stab}$ (see equation (14)).

$$\begin{aligned}\kappa_{2\text{stab}}(\underline{s}) &= \kappa_{2\text{stab}}(\overline{s})\\ (u_d-u_k)-(\overline{u}_d-\overline{u}_k) &= \sum_{\tau=t+1}^{\infty}\left[\beta\cdot(\rho-(1-\rho))\right]^{\tau-t}\cdot\left[(u_k-u_c)-(\overline{u}_k-\overline{u}_c)\right]\\ (u_d-u_k)-(\overline{u}_d-\overline{u}_k) &= \frac{\beta\cdot(\rho-(1-\rho))}{1-\beta\cdot(\rho-(1-\rho))}\cdot\left[(u_k-u_c)-(\overline{u}_k-\overline{u}_c)\right]\\ \rho^* &= 0.5-\frac{(u_d-u_k)-(\overline{u}_d-\overline{u}_k)}{2\cdot\beta\cdot\left[(\underline{u}_c-\underline{u}_d)-(\overline{u}_c-\overline{u}_d)\right]}\end{aligned} \tag{31}$$

■

Appendix B An Example with Cournot Competition

In the following, the general model from sections 2 and 3 is illustrated on basis of a Cournot duopoly model with linear demand Q=130-p. Two symmetric firms are assumed to produce a homogeneous good at constant marginal costs c=10 that are common knowledge to all firms. The shock to profitability is modeled as a shock to the fixed costs of the firms. Each firm incurs fixed costs $\overline{F}$=0 in the good state and $\underline{F}$=1000 in the bad state.

Given these assumptions, the firms collude by setting the quantity q_k that maximizes joint profits while splitting output and profits evenly.

$$q_i(a_i=k\,,a_{-i}=k) \quad = \quad q_k \quad = \quad 30 \tag{32}$$

Firm i competes (a_i=c_i) by acting according to its best response function (33). This means setting quantity q_c in the Cournot Nash equilibrium where the other firm also sets q_c. Firm i sets quantity q_d when unilaterally deviating from the collusive agreement while the other firm continues to set q_k.

$$\begin{array}{lcccl} q_i(c\,,a_{-i}) & & & = & 60-\dfrac{q_{-i}}{2} \\ q_i(c\,,c) & = & q_c & = & 40 \\ q_i(c\,,k) & = & q_d & = & 45 \end{array} \tag{33}$$

The corresponding profits $\pi(a_i,a_{-i})$ are shown in (34).

$$\begin{array}{lcccl} \pi_i(k\,,k) & = & \pi_k & = & 1800-F \\ \pi_i(c\,,c) & = & \pi_c & = & 1600-F \\ \pi_i(c\,,k) & = & \pi_d & = & 2025-F \\ \pi_i(k\,,c) & = & \pi_s & = & 1350-F \end{array} \tag{34}$$

The ranking of profits corresponds to our assumption of a prisoner's dilemma structure, i.e. $0<\pi_c<\pi_k<\pi_d$ (see subsection 2.2). Utility is modeled by equation (35) with a constant coefficient of relative risk aversion α.

$$u(\pi)=\frac{\pi^{(1-\alpha)}}{1-\alpha} \quad \text{with} \quad 0\leq\alpha<1 \tag{35}$$

Table 1 provides the profits and utility of both firms / decision makers for all strategy combinations in our duopoly model. The values are obtained when setting α=0.3. Under the above assumptions the profit differences are the same in the bad and in the good state, i.e. $\underline{\pi}_d-\underline{\pi}_k=\overline{\pi}_d-\overline{\pi}_k=225$ and $\underline{\pi}_k-\underline{\pi}_c=\overline{\pi}_k-\overline{\pi}_c=200$. However, the utility differences are higher in the bad state than in the good state, i.e. $u(\underline{\pi}_d)-u(\underline{\pi}_k)>u(\overline{\pi}_d)-u(\overline{\pi}_k)$ (29.14>23.32) and $u(\underline{\pi}_k)-u(\underline{\pi}_c)>u(\overline{\pi}_k)-u(\overline{\pi}_c)$ (28.06>21.48).

$\pi_i \mid \pi_{-i}$		**firm -*i***			
		k		*c*	
firm *i*	*k*	1800	1800	1350	2025
	c	2025	1350	1600	1600

Profits in the good state

$\pi_i \mid \pi_{-i}$		**firm -*i***			
		k		*c*	
firm *i*	*k*	800	800	350	1025
	c	1025	350	600	600

Profits in the bad state

$u(\pi_i) \mid u(\pi_{-i})$		**firm -*i***			
		k		*c*	
firm *i*	*k*	271.39	271.39	221.89	294.71
	c	294.71	221.89	249.91	249.91

Utility in the good state

$u(\pi_i) \mid u(\pi_{-i})$		**firm -*i***			
		k		*c*	
firm *i*	*k*	153.84	153.84	86.25	182.98
	c	182.98	86.25	125.78	125.78

Utility in the bad state

Table 1: Profits and Utility

Collusion is modeled as is described in subsection 2.4. In particular, the firms are assumed to play a grim trigger strategy to punish observed deviations from the cartel, i.e. following a deviation they revert to competitive conduct forever. The cartel is stable in both states when $\kappa \leq \kappa_{2stab}$ applies. For $\kappa \leq \kappa_{0stab}$ it is stable in the bad state only. The cartel cannot be stabilized in any of the states for $\kappa > \kappa_{0stab}$ (see *Proposition 1*). In Figure 2 we illustrate the three areas of stability for different values of β and κ under the assumption of ρ=0.9. One can see nicely that the critical values κ_{2stab} and κ_{0stab} rise in the size of the discount factor β. This supports *Proposition 2* that for a given value of κ higher values of β contribute to stabilizing collusion. *Proposition 3* argues that for values $\rho > \rho^*$ the decision maker with a concave objective function finds colluding more difficult in the good state $\bar{s}$. This is the case here because under the above assumptions one finds $\rho^*(\beta=0.6)=0.8911$ and $\rho^*(\beta=0.99)=0.7346$. Therefore, the relevant conditions for cartel stability are $\kappa \leq \bar{\kappa}_{2stab}$ and $\kappa \leq \underline{\kappa}_{0stab}$. To support *Proposition 3* we calculate $\underline{\kappa}_{2stab}=20.7093$, $\bar{\kappa}_{2stab}=19.1133$, $\underline{\kappa}_{0stab}=21.2231$, and $\bar{\kappa}_{0stab}=18.1139$ for β=0.9 and ρ=0.9. This provides evidence for $\bar{\kappa}_{2stab} < \underline{\kappa}_{2stab}$ and $\bar{\kappa}_{0stab} < \underline{\kappa}_{0stab}$ and shows that collusion is more unstable in the bad state $\underline{s}$. To keep Figure 2 as clear as possible we only show $\bar{\kappa}_{2stab}$ and $\underline{\kappa}_{0stab}$. The above values for ρ^*, i.e. $\rho^*(\beta=0.6)=0.8911$ and $\rho^*(\beta=0.99)=0.7346$, show that ρ^* falls in β ($\partial\rho^*/\partial\beta<0$). Hence, for a given ρ a decision maker with a high discount factor β is more likely to deviate in the good state than a more impatient decision maker (see *Proposition 4*).

Figure 2 also supports evidence for *Proposition 5* by showing the area where the decision maker would have an incentive to participate in and adhere to a collusive agreement in both states, i.e. $\kappa \leq \kappa_{2stab} \wedge \kappa < \kappa_{2x2form}$. It also shows that for $\kappa \leq \kappa_{2stab} \wedge \kappa_{2x2form} < \kappa \leq \kappa_{2x1form}$ the decision maker with a concave utility function u would adhere to the cartel in both states but would want to initiate the cartel in the bad state $\underline{s}$ only. In line with *Proposition 6* it can be observed that this requires sufficiently high values of the persistence parameter ρ and the discount factor β. It can also be seen that the condition $\kappa_{2stab} < \kappa_{2x1form}$ applies: When the decision maker would want to adhere to the collusive agreement he would always have an incentive to start colluding in at least one state of the industry.

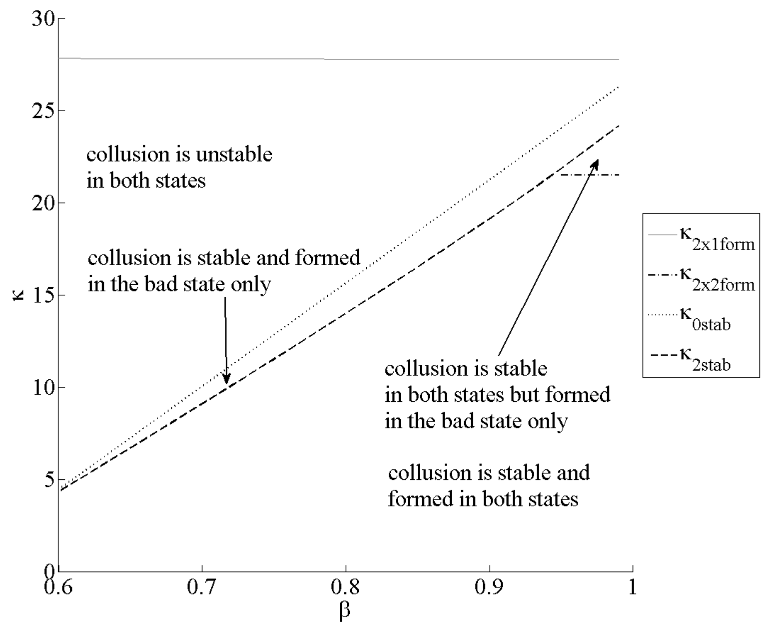

Figure 2: Critical values of κ for ρ=0.9

Appendix C Deviations by the Other Firm

In the general model presented in the main text a deviator from the collusive agreement earns above collusive profits $\pi_d>\pi_k$. Every firm assumes to be the deviator itself. We do not consider situations where a firm reasons that the other firm deviates from the collusive agreement while itself adheres to it. Being betrayed in such a way would result in a below collusive profit $\pi_s<\pi_k$ which may sometimes be even below the competitive profit $\pi_s<\pi_c$ as can be seen in the above example with Cournot competition. This Appendix shows that the effects identified in the main part of the paper remain the same when one allows for deviations by the other firm. The main difference is that the expectation of being possibly betrayed makes collusion less profitable and, thus, less stable.

To see this, assume that a cartel firm expects its co-conspirator to adhere to the collusive agreement with probability $\phi_k \in [0;1]$ and deviate from it with probability 1-ϕ_k. One reason for this uncertainty about the deviation incentive of the other firm can be seen in asymmetric information about the fixed costs of the competitors which cannot be learned from observing price and output. As long as the cartel is stable the firm earns a profit π_k which is dependent on the state of the industry. The decision maker receives utility u_k. In case of a deviation by the other firm the betrayed firm i earns a profit $\pi_s<\pi_k$ and the decision maker receives u_s. Following the deviation in period t the firms enter into a punishment regime from period t+1 on. In our model, punishments are modeled by a grim trigger strategy. Under these assumptions the present value of collusive utility can be written as in (36) which is the analogue to condition (6).

$$\begin{aligned} V_k = \phi_k &\cdot \Big[u_k - \kappa + \rho\cdot\beta\cdot\big[(1-\iota_d(a_i))\cdot V_k + \iota_d(a_i)\cdot V_d\big] \\ &\qquad + (1-\rho)\cdot\beta\cdot\big[(1-\iota_d{}'(a_i))\cdot V_k{}' + \iota_d{}'(a_i)\cdot V_d{}'\big]\Big] \\ (1-\phi_k) &\cdot \Big[u_s - \kappa + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t}\cdot\big[prob_\tau(s)\cdot u_c + (1-prob_\tau(s))\cdot u_c{}'\big]\Big] \end{aligned} \tag{36}$$

Under these assumptions, a cartel is unstable in both states when (37) applies (analogue to (12)) with u_{ks} being defined in (38).

$$\begin{aligned} \kappa &> \big[u_{ks}-u_d\big] + \rho\cdot\beta\cdot\phi_k\cdot\big[u_d-u_c\big] + (1-\rho)\cdot\beta\cdot\phi_k\cdot\big[u_d{}'-u_c{}'\big] \\ &> \kappa_{0\text{stab}} \end{aligned} \tag{37}$$

$$u_{ks} = \big(\phi_k\cdot u_k + (1-\phi_k)\cdot u_s\big) \tag{38}$$

The critical value of $\kappa_{2\text{stab}}$ can be written as in (39) (analogue to (14)).

$$\begin{aligned} \kappa &\le (1-\beta\phi_k)\cdot\Bigg[u_{ks}-u_d + \sum_{\tau=t+1}^{\infty} (\beta\phi_k)^{\tau-t}\cdot\big[prob_\tau(s)\cdot(u_{ks}-u_c) + (1-prob_\tau(s))\cdot(u_{ks}{}'-u_c{}')\big]\Bigg] \\ &\le \kappa_{2\text{stab}} \end{aligned} \tag{39}$$

It can be shown that conditions (12) and (14) are special forms of (37) and (39) with ϕ_k=1. One sees that higher values of ϕ_k raise $\kappa_{0\text{stab}}$. The more likely it is that the other firm adheres to the collusive agreement the more probable is it that firm i itself adheres to the agreement in at least one state. With ϕ_k<1 one sometimes even finds $\kappa_{0\text{stab}}<\kappa_{2\text{stab}}$ so that no stable cartel will be formed at all.