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Cartel Formation With Endogenous Capacity and Demand Uncertainty

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CARTEL FORMATION WITH ENDOGENOUS CAPACITY AND DEMAND UNCERTAINTY

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ABSTRACT

This article analyzes the strategic decisions of firms whether to establish and adhere to a cartel when they can also shape competition by investing into production capacity while being subject to unexpected demand shocks with persistence. The model shows that a negative demand shock can facilitate cartel formation despite lowering collusive profits. This is because lower demand reduces capacity utilization and makes competition more intense especially when capacities are durable and when demand falls much within a short time. The model also shows that firms with a low discount rate strive for a dominant position in the market which results in asymmetric capacity distributions. These obstruct collusive strategies. This is interesting because a low discount rate is usually considered a facilitating factor for collusion.

Keywords: *Asymmetric firms, capacity investments, cartel formation, demand shocks, excess capacity*

JEL Codes: *D21, D43, L11, L13, L41*

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1 INTRODUCTION

This paper shows under what circumstances a demand shock raises firms' inclination to collude and what role capacity constraints and the intensity of competition play in this context. The relevance of this analysis is shown by an own evaluation of 41 cartel cases prosecuted by the European Commission between 2001 and 2010. The Commission suspected that changes in demand were causal for cartel formation in 25 of the 41 decisions, with excess capacities being named in 13 cases. An increase in the intensity of competition prior to collusion is reported in 28 cases.[1]

Analyzing the cartels prosecuted by the European Commission shows that demand shocks can take a variety of forms. Demand can move up (e.g. the conspiracy in sodium chlorate) or down (e.g. the French beef cartel). Behavior may be influenced by changes in demand prior to cartel formation (e.g. the two aforementioned cartels) or by changes in demand expected to occur after the establishment of the conspiracy (e.g. the expected decline in demand for carbonless paper or the expected increase in demand for animal feed phosphates). Demand shocks can be quite persistent (e.g. graphite electrodes) or supposedly temporary (e.g. fine art auction houses) while occurring abrupt (e.g. French beef) or slowly (e.g. professional videotapes). To allow for demand patterns that match this case evidence we model demand by a Markov-process (Kandori 1991, Besanko et al. 2010.1, 2010.2) where demand moves up or down across D discrete states while the persistence of demand conditions can be varied by the researcher. This demand model appears to be more in line with the case evidence for cartel *formation* than other demand specifications that are frequently used when analyzing cartel *stability*, i.e. deterministic demand cycles (Haltiwanger and Harrington 1991), i.i.d. demand shocks (Rotemberg and Saloner 1986), or a Markov-growth model (Bagwell and Staiger 1997).

The existence of excess capacities, which are observed in 13 out of the 41 analyzed decisions, can be the result of a decline in demand (e.g. graphite electrodes, needles) or may have reasons in the history of a firm and/or industry like expectations that did not fulfill later. For example, in the hydrogen peroxide case the firms had built capacities in anticipation of an increase in demand that did not come true. This example underlines the property of capacities being the result of endogenous investment decisions of the firms which are modeled explicitly in this article. In doing so, our model complements literature on the stability of collusion that – to date – has assumed capacities to be exogenously given and symmetric (Fabra 2006: 72), to be chosen prior to a repeated product market game (Knittel and Lepore 2010: 133), or to be chosen at the beginning of every period while depreciating completely at

1 I am grateful for this thorough evaluation that was conducted by Daniel Herold for his MSc-thesis.

the end of a period (Staiger and Wolak 1992: 206).

To be more specific about the model, we set up an infinitely repeated game where two capacity-constrained one-product firms supply a near-homogeneous product and are subject to Markov-type demand shocks. At the beginning of every period, the firms learn the current level of demand and capacities as the capacity stock from previous periods is subject to stochastic depreciation events. The firms make two decisions. First, every firm decides whether to compete or collude and sets a price for its good accordingly. Second, the firms decide how much to invest into their production capacity. These decisions are made by every firm with the objective to maximize its value, i.e. the present value of future operative profits net of investment expenditures. The model for competition has initially been described in the online appendix to Besanko and Doraszelski (2004) and has been applied, for example, by Chen (2009) and Besanko et al. (2010.1, 2010.2). The model for collusion is based on Fershtman and Pakes (2000). The colluding firms are assumed to determine prices as the Nash (1950, 1953) solution to a bargaining game. Collusion is stabilized by price wars in combination with a grim trigger strategy (Friedman 1971, Rotemberg and Saloner 1986).

Analyzing firms' decisions concerning competitive or collusive market conduct and investment is non-trivial as these decisions are interdependent and affected by stochastic events. To see this, consider that the profits depend on the state of demand and firms' production capacities. The optimal investment decision depends on future profits and, thus, relies on both the expected evolution of demand and the expected evolution of capacities. As demand shocks are assumed to be exogenous and to occur stochastically, firms can predict from the current level of demand with what probability demand will be in some specific state in a future period. Capacities also evolve to some extent exogenously as depreciation events and investment success are modeled to be stochastic following known probability distributions. However, the probability distribution of investment success also depends endogenously on firms' optimal investment strategies, i.e. the higher the invested amount the higher is the probability of investment being successful.

Our model allows us to calculate the profit function, the policy function, and the value function of a representative firm both in competition and collusion. The profit function shows the profits of the firm for all combinations of the firms' capacities in all demand states. Based on these profits, we use a variant of the Pakes and McGuire (1994) algorithm to calculate the optimal investments of the firm for all combinations of capacity and demand, i.e. the policy function. This enables us to analyze the determinants of firms' investment decision and the resulting distribution of capacities both in competition and collusion. Our calculation also yields a collusive and a competitive value function which show a firm's present value of future profits (net of investment expenditures) for all combinations of capacity and demand. We calculate the incremental value of collusion for every combination of capacities and demand by subtracting the competitive firm value of a firm from its collusive counterpart. We interpret that a cartel is more likely to be formed for higher values of the incremental value to collude.

The contribution of our paper is twofold. First, the paper is among the first to explicitly explore cartel *formation* and, thus, complement the established literature on cartel *stability*. Our model supports the observation that cartel formation is facilitated by events that raise the intensity of competition and, thus, raises the incremental value to collude. Second, the paper studies the relationship between firms' investment decision into capacity and their decision to form and maintain an explicit collusive agreement. It finds that a low discount rate, which is commonly considered to be a facilitating factor for collusion, may make the firms invest into quite asymmetric capacities, which effectively impedes collusion. To see this, we give an overview of the main effects that can be observed in the model.

Demand effect: Profits are a positive function of demand. A decline in demand lowers collusive as well as competitive profits. The same is true for the collusive and competitive firm value. Hence, the incremental value to collude falls along with demand if the demand effect is not offset by the competition effect.

Competition effect: More intense competition, as measured by a low price-cost margin, lowers both the competitive profit and the firm value and, thus, raises the incremental value of collusion. For example, intense competition was observed prior to the formation of the conspiracies in Methionine, Soda Ash, Vitamins, and Plasterboard (Grout and Sonderegger 2005). Competition is intense when production capacities are high relative to demand. Hence, the existence of excess capacities contributes to the intense competition that is observed in 28 of the 41 cartel cases that we refer to above. Such a situation can be brought about by different types of events.

(i) A negative demand shock may make competition more intense. This is especially true when prior to the shock the industry had been characterized by binding capacity constraints which had given the firms some market power. Lower demand makes the capacity constraints less binding which results in more intense competition. This is especially true when the decline in demand is pronounced and occurs quickly.

(ii) When capacities are fairly durable they cannot be adjusted quickly to situations of lowered demand. In this case it takes the firms some time to reduce production capacities by means of depletion and, thus, mitigate the higher intensity of competition. The longer the firms remain subject to intense competition the more strongly does their competitive firm value decrease. This effect is less pronounced when capacity depreciates at a higher rate.

(iii) The collusion-enhancing effect of falling demand vanishes when the firms had produced at capacities that more than suffice to serve demand even before the drop in demand. In this case, the negative demand shock only somewhat raises the intensity of competition and the above demand effect becomes relatively more important, i.e. the incremental value to collude may even fall along with demand. However, high capacities are generally associated with a high level of competition irrespective of the state of demand. This causes the level of the incremental value to collude to be high as well. For example, the citric acid cartel was formed in 1991 after one firm had significantly expanded its capacity in 1985 and 1988 which contributed to a decline in prices of about 45% between 1985 and 1990.[2]

Symmetry effect: A symmetric capacity distribution is found to facilitate cartel formation. With asymmetric capacities, the larger firm enjoys scale- respectively cost-advantages over the small firm in competition and has a lower incentive to establish a collusive agreement. This finding is consistent with the observation that cartels are most frequently observed among firms with similar market shares. When the capacities are fairly skewed one also finds that collusion cannot necessarily be stabilized by price wars in combination with a grim trigger strategy (Friedman 1971, Rotemberg and Saloner 1986, Compte et al. 2002).

The latter effect, i.e. collusion cannot necessarily be stabilized when the capacities of the firms are fairly asymmetric, shows that the scope of collusion depends on the investments which the firms choose in competition and the resulting capacity distributions. The model shows that the investment decision of the firms is mainly affected by the values of firms' discount rate and the depreciation probability of their capacities. We identify three different types of competitive equilibria that have different properties with regard to cartel formation.

Type A – Asymmetric Competition: When capacity depreciates quickly and firms strongly discount future profits they invest into somewhat asymmetric and relatively high capacities. Asymmetry implies that especially the large firm has a small incentive to collude. High capacities are associated with a high intensity of competition irrespective of the state of demand. Thus, a negative demand shock often aggravates cartel formation because of a strong demand effect.

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2 DG Comp Case No COMP/E-1/36 604 – Citric Acid

Type S – Symmetric Competition: When capacity depreciates slowly and the firms strongly discount future profits they invest into rather symmetric and not overly high capacities. As the durable capacities cannot be lowered quickly in response to a drop in demand intense competition would persist for a relatively long time. Therefore, a negative demand shock often facilitates cartel formation because of a strong competition effect.

Type P – Preemption Races: When capacity depreciates quickly and firms discount future profits at a low rate they engage in preemption races in order to attain a dominant position in the market. These result in fairly skewed capacity distributions where collusion is not necessarily stable in our model. Given firms' inability to stabilize a collusive agreement such a cartel will not be formed.

Asymmetric capacities and, thus, instability of collusion is observed when the firms discount future profits at a low rate. This is interesting because the literature typically argues that a low discount rate contributes to stabilizing rather than destabilizing a collusive agreement. Our finding is novel and illustrates the importance of analyzing firms' decision to collude in the context of their entire strategy set that, for example, includes investment strategies as well as market conduct strategies like collusion or competition. Analyzing the interactions between different types of strategies shows that a low discount rate does not always contribute to stabilizing collusion as it may also cause firms to invest into asymmetric capacities which cause collusion to be unstable.

The paper is structured as follows. In section 2, we present the industry model and introduce the specification of fluctuating demand with persistence. Moreover, we specify our assumptions on competitive and collusive conduct. In section 3, we evaluate the model for different parameter combinations and analyze its properties for the three types of competitive equilibria. Moreover, we provide robustness checks. Section 4 concludes. Technical background information and further robustness checks are provided in Appendix A and Appendix B.

2 THE MODEL

This section presents our dynamic duopoly model with near-homogeneous products, demand shocks with persistence, price competition, and endogenous capacity investments. Subsection 2.1 explains our assumptions on demand. Subsection 2.2 elaborates on firms' production costs as a function of their capacity. Subsection 2.3 assumes that the competing firms individually set prices in order to maximize profits. Subsection 2.4 states our assumptions on collusion, i.e. collusive prices are jointly set as the Nash solution to a bargaining game. Collusion is stabilized by price wars in combination with a grim trigger strategy. In competition and collusion, firms are assumed to invest into production capacity with the objective of maximizing the present value of expected operative profits net of investment expenditures. Results are presented in section 3.

2.1 Structure and Evolution of Demand

The representative consumer has utility function (1) as initially proposed by Bowley (1924).

$$u=q_0+a\,q_1+a\,q_2-\frac{b}{2}q_1^2-\frac{b}{2}q_2^2-\gamma\,q_1 q_2\ , \tag{1}$$

where $q_0 \geq 0$ is the consumption of a numeraire good and $q_j \geq 0$ denotes the consumed quantities of the goods produced by the two one-product firms in our duopoly model where firms are being indexed by $j \in \{1,2\}$ and $-j=3-j$. Let $\theta=\gamma/b$, where $\theta \in [0;1[$ measures the degree of product differentiation, ranging from 0 for independent goods to 1 for homogeneous goods.

We maximize utility function (1) with respect to the quantities q_0, q_1, and q_2 subject to the budget constraint $y=q_0+\Sigma q_j p_j$, with y denoting the representative consumer's income and the price of the numeraire being set to $p_0=1$. This yields the linear inverse demand function in (2).

$$p_j(\boldsymbol{q})=a-bq_j-\theta\,b\,q_{-j} \quad \forall\, j\in\{1,2\} \tag{2}$$

$\boldsymbol{q}$ denotes a (2×1)-vector whose elements represent the outputs of the two firms. The representative consumer's demand function for good j looks as shown in (3).

$$q_j(\boldsymbol{p})=\frac{a(1-\theta)-[1+(J-2)\theta]\,p_j+\theta\,p_{-j}}{b\cdot[1+(J-2)\theta-(J-1)\theta^2]} \quad \forall\, j\in\{1,2\} \tag{3}$$

$\boldsymbol{p}$ denotes a (2×1)-vector whose elements represent the prices of the two firms.

Given that persistent demand shocks are thought to be a possible cause of cartel formation we assume Markov-process demand fluctuations (Kandori 1991, Besanko et al. 2010.1 and 2010.2) that allow for such shocks. We introduce an exogenous market size parameter Γ in our model that can be interpreted as the total mass of customers. Total demand for good j ($\tilde{q}_j$) is found by multiplying the demand of one representative customer (3) by Γ.

$$\tilde{q}_j(\boldsymbol{p}) = \Gamma \cdot q_j(\boldsymbol{p}) \quad \forall\, j \in [1;J] \tag{4}$$

Without loss of generality we normalize the median value of Γ to 1.0. Varying market size Γ rotates the demand curve while keeping its intercept with the price-axis fixed. This implies that a drop in demand reduces the number of customers but leaves the distribution of consumers' tastes unchanged.[3]

Conducive to solving the model by the numerical Pakes and McGuire (1994) algorithm, the number of demand states is assumed to be finite. There are D demand states and the demand state in period t, d_t, lies in $\{1,2,...,D\}$.[4] Every demand state translates into a market size, i.e. $\Gamma=\Gamma(d)$ where Γ is increasing in d so that a higher value for d corresponds to a bigger market. We propose a specific functional form for $\Gamma(d)$ in section 3. In Table 1 we define the transition probabilities $prob(d_{t+1}|d_t)$ that govern the transition from state d_t in period t into d_{t+1} in period t+1. Demand moves up or down by no more than one state per period, which occurs with probability (1-ρ)/2, or it remains at its previous level with probability ρ. Hence, ρ can be interpreted as a measure of persistence of demand shocks. In the border demand state d_t=1 (d_t=D) the probability of moving down (up) is zero. This raises the probability of remaining in the old state by (1-ρ)/2. The stationary distribution over demand states is uniform, i.e. over time every demand state is visited with probability 1/D. This specification of demand parallels the one used by Besanko et al. (2010.1).[5]

3 The literature on the stability of cartels typically uses one of four types of demand movements. These are i.i.d. demand shocks (Rotemberg and Saloner 1986), deterministic demand cycles (Haltiwanger and Harrington 1991), non-deterministic demand movements in a Markov-growth model (Bagwell and Staiger 1997, Hamilton 1989), and Markov-process demand fluctuations (Kandori 1991, Besanko et al. 2010.1 and 2010.2).

4 In principle, the number of demand states can take any value. However, computation time requires D not to be excessively large. In section 3, we assume D=9.

5 Besanko et al. (2010.1) measure the persistence of demand only somewhat differently. They define a parameter $\rho_B \in$ [0,0.5] as a measure of demand uncertainty where higher values indicate more volatile demand. In our model, $\rho \in$ [0,1] measures demand persistence where more volatile demand is indicated by lower values of ρ. The two measures relate to each other as follows: ρ_B=(1-ρ)/2.

	$d_t=1$	$1<d_t<D$	$d_t=D$
$prob(d_{t+1}=d_t-1)$	0	$(1-\rho)/2$	$(1-\rho)/2$
$prob(d_{t+1}=d_t)$	$(1+\rho)/2$	ρ	$(1+\rho)/2$
$prob(d_{t+1}=d_t+1)$	$(1-\rho)/2$	$(1-\rho)/2$	0

Table 1: Transition probabilities of demand

2.2 Costs and Capacity

The cost function of the firms is modeled as proposed in the online appendix to Besanko and Doraszelski (2004) and applied by Chen (2009) and Besanko et al. (2010.1). We assume capacity-constrained firms whose capacity $\bar{q}$ takes one of M positive values. In period t, the capacity state of the industry is $\boldsymbol{s}_t=(s_{1t}, s_{2t}) \in \{1, 2, \ldots, M\}$, where s_{jt} denotes firm j's capacity level and firm j's capacity is $\bar{q}_{jt}$. For example, s_{jt} denotes the firm's number of plants with $\bar{q}_{jt}$ being the total output that can be produced by these plants. In section 3, we specify a function $\bar{q}_{jt}=5\cdot s_{jt}$ that relates s_{jt} and $\bar{q}_{jt}$ through a plant size (here: 5) which can be chosen by the researcher.[6] In the following, the time subscript t is sometimes dropped to make the notation more concise.

Given that firm j holds $\bar{q}_j$ units of capacity, the total cost of producing $\tilde{q}_j$ units of output is

$$C_j(\tilde{q}_j|\bar{q}_j)=\frac{1}{1+\eta}\left(\frac{\tilde{q}_j}{\bar{q}_j}\right)^{\eta}\tilde{q}_j\text{, with }\eta>0 \quad . \tag{5}$$

Equation (5) poses a "soft" capacity constraint because for $\tilde{q}_j\leq \bar{q}_j$ marginal costs c_j are relatively small.

$$c_j(\tilde{q}_j|\bar{q}_j)=\left(\frac{\tilde{q}_j}{\bar{q}_j}\right)^{\eta} \tag{6}$$

For $\eta>1$ and $\tilde{q}_j> \bar{q}_j$ marginal costs rise quite steeply, i.e. firms face substantial diseconomies of scale once they produce at or above their capacity. Soft capacity constraints can be considered a reasonable approximation of reality because the assumption of diseconomies of scale can be motivated by the existence of, for example, overtime allowances, higher maintenance costs, or a higher number of rejects. These additional costs make it unprofitable for a firm to produce a quantity that is way above the planned capacity $\bar{q}_j$. Note that the higher the parameter η the closer are we to hard capacity constraints which is about the case for $\eta\geq 10$. Figure 1 presents a graph of marginal costs for $\eta \in \{2.5, 10, 40\}$.

6 Section 3 and Appendix B show that our qualitative conclusions are not sensitive to this choice of parameter values.

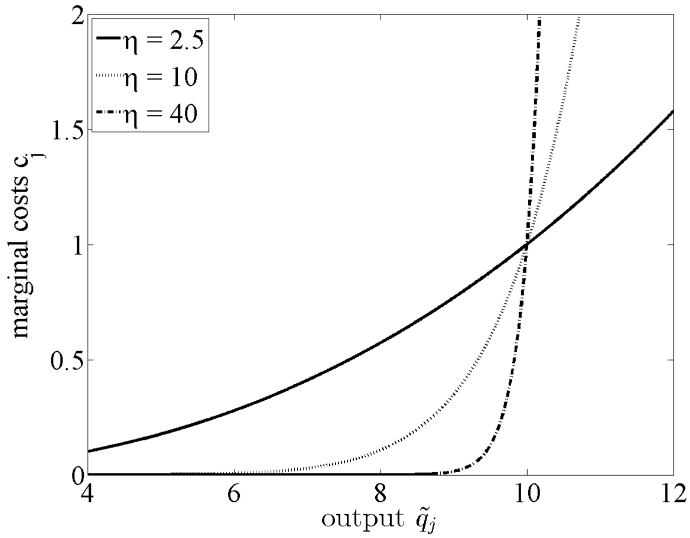

Figure 1: Marginal costs with $\overline{q}_j=10$

From a modeling point of view, the use of soft capacity constraints is convenient because it allows us to concentrate on Nash equilibria in pure strategies in the product market game. To see this consider that in case of hard capacity constraints demand can exceed firms' production capacity. This would require to specify a rationing rule (see, for example, Besanko and Doraszelski 2004: 27) which may result in a profit function that is not quasi-concave such that pure-strategy equilibria often fail to exist. This is different with soft capacity constraints where it is not necessary to ration consumers. This results in profit functions that are continuous and quasi-concave. Hence, a unique pure strategy equilibrium exists (Maggi 1996: 242).

We allow the firms in our model to invest into capacity. In every period, firm j invests an amount $x_j \geq 0$ to raise and/or maintain its production capacity. The higher x_j the higher is the probability $(\alpha x_j)/(1+\alpha x_j)$ that the investment is successful, where $\alpha>0$ indexes how likely investments are successful. The probability $(\alpha x_j)/(1+\alpha x_j)$ rises concavely in both α and x_j. This modeling structure is consistent with a time-to-build model (Besanko and Doraszelski (2004: 28), Kydland and Prescott (1982)). By assumption, a firm may only move up by one step per period, i.e. it may transit from state s_j to s_j+1 but not to s_j+2. A firm's stock of capital may also depreciate which occurs with probability δ. Again, a firm may only move down by one step per period, i.e. it can transit from state s_j to s_j-1 but not to s_j-2.

In writing down the transition probabilities of firm j we distinguish three cases. First, the capacity of a firm that is in the lowest state s_{jt}=1 may only move up or stay at the previous level but cannot move down any further. Second, a firm that is in the highest state $s_{jt}=M$ may only transit to state

$s_{jt+1}=M$-1 or remain in the highest state $s_{jt+1}=M$ but cannot attain a higher capacity. Third, a firm that is in some intermediate state $1<s_{jt}<M$ may either move up or down by one state or remain in the same state. The resulting transition probabilities $prob(s_{jt}|s_{jt+1},x_j)$ are provided by Table 2.

	$s_j=1$	$1<s_j<M$	$s_j=M$
$prob(s_j-1\|s_j,x_j)$	0	$\delta/(1+\alpha x_j)$	$\delta/(1+\alpha x_j)$
$prob(s_j\|s_j,x_j)$	$1/(1+\alpha x_j)$	$(1+\delta\alpha x_j-\delta)/(1+\alpha x_j)$	$(1+\alpha x_j-\delta)/(1+\alpha x_j)$
$prob(s_j+1\|s_j,x_j)$	$(\alpha x_j)/(1+\alpha x_j)$	$((1-\delta)\alpha x_j)/(1+\alpha x_j)$	0

Table 2: Transition probabilities of capacity

2.3 Competitive Conduct

After demand has realized the firms set prices simultaneously and then produce to satisfy demand (4). Equation (7) defines the profits of firm j.

$$\pi_j = p_j \tilde{q}_j(p_j, p_{-j}) - C_j\left(\tilde{q}_j(p_j, p_{-j}) \| \bar{q}_j\right) \tag{7}$$

Individual profit maximization requires solving the system of non-linear first order conditions (8) for the vector of profit-maximizing prices $\boldsymbol{p}_c$.

$$\begin{aligned} \frac{\partial \pi_j}{\partial p_j} &= \tilde{q}_j + \frac{\partial \tilde{q}_j}{\partial p_j} \cdot (p_j - c_j) \overset{!}{=} 0 \\ &\quad \tilde{q}_j - \frac{\Gamma(1+(J-2)\theta)}{b\cdot\left[1+(J-2)\theta-(J-1)\theta^2\right]} \cdot \left(p_j - \left(\frac{\tilde{q}_j}{\bar{q}_j}\right)^{\eta}\right) = 0 \end{aligned} \tag{8}$$

Following Chen (2009), we solve for the values in $\boldsymbol{p}_c$ numerically and calculate the competitive equilibrium profit $\pi_{c,j}$ and quantity $q_{c,j}$ of firm j according to equations (7) and (4). The properties of the profit function across the capacity space have been explored in the online appendix to Besanko and Doraszelski (2004), by Chen (2009), and by Besanko et al. (2010.2). Graphs of the profit function and a discussion of its properties are provided in section 3.1.

Let V_j denote the expected present value of firm j, and let x_j denote the amount firm j invests in the current period t given the industry is in capacity state $\boldsymbol{s}$ and demand is in state d. We focus on symmetric Markov perfect equilibria. This allows us to concentrate on any firm j, knowing that the policy function and the value function of firm j and that of the other firm $-j$ are the same if the capacities of their respective competitors and the state of demand are the same. Equation (9) shows the Bellman equation for this problem.

$$V_j(\boldsymbol{s},d)=\max_{x_j\geq 0}\left(\pi_{c,j}(\boldsymbol{s},d)-x_j+\frac{1}{1+r}\sum_{s_j'=1}^{M} W_j(s_j')\cdot prob(s_j'|s_j,x_j)\right) \quad (9)$$

In this equation, $0<r<1$ is the discount rate and s_j, respectively s_j', are firm j's capacity states in periods t and t+1. We assume that the discount rate is the same for all firms so that r is not indexed by j.

W_j is the expected present value of the firm in period t+1 over all possible future states of the capacity of firm j's competitor s_{-j}' and over all possible future states of demand d'.

$$W_j(s_j')=\sum_{d'=1}^{D}\sum_{s_{-j}'} V_j(s_{-j}',s_j',d')\cdot prob(s_{-j}'|s_{-j},x_{-j}(\boldsymbol{s},d))\cdot prob(d'|d) \quad (10)$$

The first *prob*-term shows the probability that the competitor of firm j moves from capacity state s_{-j} into state s_{-j}' given that the firm behaves according to its policy function $x_{-j}(\boldsymbol{s},d)$. The second *prob*-term shows the probability that the industry moves from demand state d into state d'.

To obtain firm j's policy function we determine the first-order condition (11) of value function (9).

$$-1+\frac{1}{1+r}\sum_{s_j'=1}^{M} W_j(s_j')\cdot\frac{\partial\, prob(s_j'|s_j,x_j)}{\partial x_j}=0 \quad (11)$$

Note that for $1<s_j<M$ the sum in equation (11) consists of three summands because $s_j' \in \{s_j-1, s_j, s_j+1\}$. For $s_j=1$ and $s_j=M$ the number of summands reduces to two. The partial derivatives of the transition probabilities can be calculated from their functional forms provided in Table 2. We plug these terms in equation (11) and define Δ as in (12).

$$\Delta=\begin{cases}\dfrac{\alpha}{1+r}\left[W_j(s_j+1)-W_j(s_j)\right] & \text{if} \quad s_j=1\\ \dfrac{\alpha}{1+r}\left[(1-\delta)\cdot\left(W_j(s_j+1)-W_j(s_j)\right)+\delta\cdot\left(W_j(s_j)-W_j(s_j-1)\right)\right] & \text{if} \quad 1<s_j<M\\ \dfrac{\alpha}{1+r}\left[\delta\cdot\left(W_j(s_j)-W_j(s_j-1)\right)\right] & \text{if} \quad s_j=M\end{cases} \quad (12)$$

We determine the optimal investment strategy $x_j(\boldsymbol{s},d)$ of firm j by solving first-order condition (11) for $x_j(\boldsymbol{s},d)$.

$$x_j(\boldsymbol{s},d)=\begin{cases}0 & \text{if} \quad \Delta<0\\ \max(0,(-1+\sqrt{\Delta})/\alpha) & \text{if} \quad \Delta\geq 0\end{cases} \quad (13)$$

2.4 Collusive Conduct

Our collusive model is based on Fershtman and Pakes (2000). The two firms are assumed to collude in prices but do not coordinate their investment policies (semi-collusion). Such semi-collusion has been reported, e.g., for the Norwegian cement cartel (Steen and Sørgard 1999) or the conspiracies in nitrogenous fertilizer, synthetic fibers, plastics and aluminum (Davidson and Deneckere 1990). We define $V_{k,j}(\boldsymbol{s},d)$ as the collusive present expected value of firm j when it chooses price $p_{k,j}$, sells quantity $q_{k,j}$, makes profits $\pi_{k,j}$, and invests $x_{k,j}$. Moreover, let $V_{dev,j}(\boldsymbol{s},d)$ define firm j's expected present value if it deviates from the collusive agreement. In this case it chooses price $p_{dev,j}$, sells quantity $q_{dev,j}$, makes profits $\pi_{dev,j}$, and chooses investment $x_{dev,j}$. Along with the related literature[7] we assume collusive conduct to be enforced by a grim trigger strategy (Friedman 1971), i.e. after an observed deviation all cartel firms revert to the competitive equilibrium forever. The firms in our model are assumed to be completely informed about the capacities and the strategies of the other players. As a consequence, every firm j anticipates if any other firm has an incentive to deviate at the collusive price and as a precaution lowers its own price to the competitive level in order to render defections unprofitable (Green and Porter 1984). We comment on each of these parameters and assumptions in turn.

Equation (14) shows the collusive present value of profits.

$$V_{k,j}(\boldsymbol{s},d)=\max_{x_{k,j}\geq 0}\left(\pi_{k,j}(\boldsymbol{s},d)-x_{k,j}+\frac{1}{1+r}\sum_{s_j'=1}^{M}W_{k,j}(s_j')\cdot prob(s_j'|s_j,x_{k,j})\right) \tag{14}$$

The collusive expected present value of firm j in period t+1 ($W_{k,j}$) is defined in equation (15), which is similar to the competitive future firm value as defined in equation (10).

$$W_{k,j}(s_j')=\sum_{d'=1}^{D}\sum_{s_{-j}'}V_{k,j}(s_{-j}',s_j',d')\cdot prob(s_{-j}'|s_{-j},x_{-j}(\boldsymbol{s},d))\cdot prob(d'|d) \tag{15}$$

To determine the collusive profit $\pi_{k,j}$ we specify a collusive price vector $\boldsymbol{p}_k$. We assume that the firms determine prices as the solution to a bargaining game and comment on this assumption below. Along with Fershtman and Pakes (2000: 213) we use the Nash (1950, 1953) solution to this game and assume that the firms choose prices $\boldsymbol{p}_{NBS}$ that maximize condition (16).

7 See, for example, Rotemberg and Saloner (1986), Davidson and Deneckere (1991), Haltiwanger and Harrington (1991), Staiger and Wolak (1992), Bagwell and Staiger (1997), Fershtman and Pakes (2000: 213), Fabra (2006), and Knittel and Lepore (2010).

$$\max_{p_j \forall j} \pi_{NBS} = \prod_{j=1}^{J} \left(\pi_{NBS,j} - \pi_{c,j} \right) \tag{16}$$

As in the case of the competitive model, we solve for the profit-maximizing vector of prices $\boldsymbol{p}_{NBS}$ numerically and calculate the equilibrium profit $\pi_{NBS,j}$ and quantity $q_{NBS,j}$ for every firm j.

Our set of assumptions aims at a balance between the computational tractability of the model and its economic meaningfulness. When firms are assumed to be symmetric it is natural to focus on collusive agreements where the firms maximize joint profits and distribute the gains equally. When firms are asymmetric there is less agreement on collusive rules. Using the Nash (1950, 1953) bargaining solution provides a convenient way for dealing with some important topics arising in this context. It allows us to avoid ambiguity by concentrating on a scheme without side payments or output quotas where profits are distributed according to the *earnings follows output* principle (Bain 1948: 618), i.e. “each firm receives revenue only from the output it produces and sells itself.” These desirable features emerge because the Nash bargaining solution ensures, first, that every colluding firm receives at least its competitive profit and, second, that firms with a higher threat point gain more from collusion.[8]

Firm j's incentive to deviate from an unconstrained semi-collusive equilibrium at Nash bargaining prices is determined by its deviation present value of profits as shown in equation (17).

$$V_{dev,j}(\boldsymbol{s},d) = \max_{x_{dev,j} \geq 0} \left(\pi_{dev,j}(\boldsymbol{s},d) - x_{dev,j} + \frac{1}{1+r} \sum_{s_j'=1}^{M} W_j(s_j') \cdot prob(s_j' | s_j, x_{d,j}) \right) \tag{17}$$

Let us define the deviation profit $\pi_{dev,j}(\boldsymbol{s},d,p_{-j},p_{dev,j})$ more formally as a function of firm j's deviation price $p_{dev,j}$ and the price of the other firm p_{-j}. Following our above reasoning we assume that the other firm sets the Nash bargaining price $p_{-j,NBS}$. Given this price we solve numerically for the profit maximizing price of the deviator $p_{dev,j}$ using the competitive first-order condition (8). The assumption of a grim trigger strategy implies that W_j is the competitive expected present value of firm j in period t+1 (see equation (10)), which is fully defined by our solution of the game for competing firms (see section 2). The optimal investment $x_{dev,j}$ is determined according to conditions (11) and (12).

When at least one of the cartel firms would have an incentive to deviate from Nash bargaining prices $\boldsymbol{p}_{NBS}$ we assume that the firms return to competitive prices $\boldsymbol{p}_c$. This implies collusive profits $\pi_{k,j}$ as

8 Note that the assumption of Nash bargaining does not drive the results that are reported in section 3 to a great extent. We find very similar results when assuming the firms to set prices in order to maximize joint profits instead of employing the Nash bargaining solution.

shown in (18).

$$\pi_{k,j}(\boldsymbol{s},d)=\begin{cases}\pi_{NBS,j}(\boldsymbol{s},d) & \text{if} \quad \left\{V_{dev,j}(\boldsymbol{s},d)\le \max\limits_{x_{k,j}\ge 0}\left(\pi_{NBS,j}(\boldsymbol{s},d)-x_{k,j}+\frac{1}{1+r}\sum_{s_j'=1}^{M} W_{k,j}(s_j')\cdot prob(s_j'|s_j,x_{k,j})\right)\right\}\forall j \\ \pi_{c,j}(\boldsymbol{s},d) & \text{otherwise}\end{cases} \tag{18}$$

The assumption of a reversion to competitive prices $\boldsymbol{p}_c$ is also attributed to the uniqueness of this set of prices. In some cases, deviations could also be prevented if the cartel firms chose a set of prices below Nash bargaining prices but above competitive prices. Determining such a pricing vector is difficult for at least two reasons.

First, there is a continuum of constrained semi-collusive equilibria at supra-competitive prices and the firms have to coordinate on one of them. This poses a selection problem not only to the researcher but also to the firms. This selection problem is particularly difficult in industries with asymmetric firms and differentiated products. Second, this set of prices would be dependent on the investment strategy of the firms. To see this, consider that the deviation incentive of the firms does not only depend on the collusive price vector but also on firms' investments in the collusive and the deviant case (see equations (17) and (18)). This deviation incentive would have a feedback effect on the collusive price vector and consequently on firms' profits, values and investments which, again, affect the deviation incentive. The result would be a highly complex optimization problem where one would have to determine endogenously in the optimization process not only the competitive, collusive and deviant value functions as well as policy functions for all firms but also their pricing strategies. Assuming collusion at Nash bargaining prices and reversions to competitive prices provides us with unique pricing strategies that can be calculated independently from the policy functions and value functions of the firms. This greatly simplifies the task of determining firms' optimal investment policies. The properties of the collusive stage game are explored in section 3.1.

We use the above model to analyze the additional value that accrues to the firms in period t from forming a stable cartel. In equation (19), we define firm j's incremental value from collusion Ω_j as the difference between its expected collusive present firm value $V_{k,j}(\boldsymbol{s},d)$ and its expected competitive present firm value $V_j(\boldsymbol{s},d)$.

$$\Omega_j(\boldsymbol{s},d)=V_{k,j}(\boldsymbol{s},d)-V_j(\boldsymbol{s},d) \tag{19}$$

The incremental value to collude $\Omega_j(\boldsymbol{s},d)$ depends on the state of demand d and the production capacities available to the firms $\boldsymbol{s}$ in the current period t. Therefore, $\Omega_j(\boldsymbol{s},d)$ depends on the future profits and investments of firm j given its expectations about the development of demand and the capacity investments of the other firm(s). Given these complex interactions, section 3 studies the effects of demand shocks on the incremental value to collude of a firm j.

3 RESULTS

This section analyzes the properties of our model and derives conclusions for the stability and the formation of collusive agreements. Subsection 3.1 explores the properties of the competitive and the collusive stage game equilibria with regard to changes in demand and capacities. It shows that for a negative demand shock to raise the additional profits from collusion the production capacities of the firms must not be too high. Therefore, subsection 3.2 identifies the determinants of firms' investment policy and the resulting capacity distribution. These mainly depend on the useful life of capacities (measured by the depreciation probability δ) and the discount rate r of the firms. We find three types of competitive capacity distributions whose properties are explored in subsections 3.3 to 3.5. In particular, we analyze how changes in capacity or demand affect the incremental value to collude Ω in these equilibria. Subsection 3.6 shows that our results are robust to changes in the persistence of demand ρ and the hardness of capacity constraints η and that they extend to values of r and δ other than the ones used in subsections 3.3 to 3.5.

3.1 Effects of Demand and Capacity on Profits

Our analysis starts with calculating prices, outputs and profits of the two firms in the stage game of our duopoly model. We set the model parameters at values a=4, b=0.1, α=0.0625, θ=0.9, M=6, J=2, and D=9. Demand states d translate into a market size Γ according to function Γ=0.75+0.05·d, i.e. $\Gamma \in$ {0.80, 0.85, ..., 1.2}. Given this parametrization, demand for firm j's product varies between $\tilde{\tilde{q}}_j(\mathbf{0})$=16.84 in demand state d=1 and $\tilde{\tilde{q}}_j(\mathbf{0})$=25.26 in demand state d=9 when both firms charge a price of zero. We assume that firm j's capacity $\bar{q}_j$ is linked to state s_j by the function $\bar{q}_j=5\cdot s_j$ such that maximum capacity exceeds maximum demand, $\bar{q}_j(s_j=6)>\tilde{\tilde{q}}_j(\mathbf{0},d=9)$. The stage game equilibria are calculated for each of the $M\cdot M\cdot D$=324 combinations of capacities and demand.

Figure 2 shows the collusive and competitive prices and profits of firm 1 for all combinations of capacities s_1 and s_2 in demand states d=1, 5, and 8. We illustrate how the collusive and competitive stage-game equilibria (i.e. costs, output, price, and profits) depend, first, on demand and, second, on the distribution of capacities. Third, we analyze how demand and capacities affect the additional profits gained by switching from competition to collusion $\pi_{k,j}$-$\pi_{c,j}$. This provides some first evidence for the demand effect and the competition effect that are outlined in the introductory section 1.

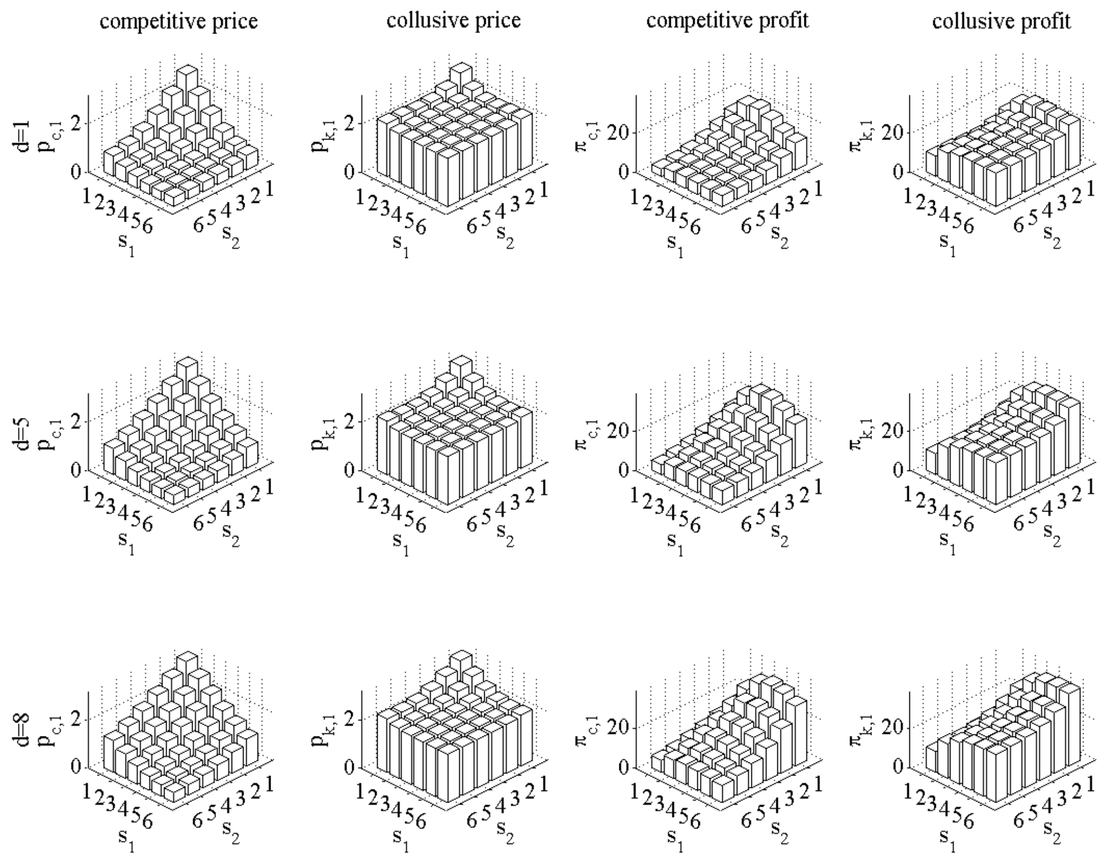

Figure 2: Price and profits in the stage game

We start by analyzing the effect of demand on costs, output, price, and profits. *Demand effect on costs*: For a specific capacity s_j and given our assumption of soft capacity constraints (see equation (6)) the marginal costs c_j of a firm j fall along with its output, i.e. they are smaller in times of lower demand. *Demand effect on output*: With fixed marginal costs, a drop in demand d would lower both the competitive and the collusive equilibrium output of the firms. In our model this effect is partially offset because lower demand reduces marginal costs. However, our numerical evaluations indicate that despite the reduction of marginal costs the competitive and collusive equilibrium outputs fall when demand is reduced. *Demand effect on price* (see Figure 2): As lower demand implies lower output and, thus, lower marginal costs, competitive equilibrium prices fall when demand falls. This is different in the collusive equilibrium where the firms supply a lower than competitive output in all demand states and, thus, produce at marginal costs close to zero for a wide range of capacity combinations $\boldsymbol{s}$. Hence, the collusive price remains fairly constant when demand changes. The collusive price only falls as a response to a drop in demand when the firms produce at low capacities and, thus, are subject to more pronounced diseconomies of scale. *Demand effect on profits* (see Figure 2): Putting the above effects together, one finds that in our model a drop in demand lowers both competitive and collusive profits. This is what we termed *the* demand effect in the introduction.

We continue our analysis by exploring the effect of changes in the own capacity s_1 of firm 1 and of changes in the capacity s_2 of its rival on the costs, output, price, and profits of firm 1. *Capacity effect on costs*: Condition (6) implies that an increase in its own capacity s_1 lowers the marginal costs c_1 of firm 1. There is no direct effect of changes in the capacity s_2 of firm 2 on the marginal costs of firm 1. *Capacity effect on output*: A higher own capacity s_1 raises the equilibrium output of firm 1 by lowering marginal costs c_1. A higher capacity s_2 of the rival firm 2 lowers the rival's marginal costs c_2 and its price p_2. This results in a business stealing effect and lowers the equilibrium output of firm 1. As business stealing is more of an issue in competition the above effects of changes in s_1 and s_2 are more pronounced in the competitive than in the collusive equilibrium. *Capacity effect on price* (see Figure 2): The competitive price of firm 1 falls in its own capacity s_1 because of the lower marginal costs c_1. The price also falls in the capacity s_2 of firm 2 because of the more intense competition and the business stealing effect. In collusion, these price effects of changes in s_1 and s_2 can mainly be seen when the firms produce at low capacities where the firms are subject to pronounced diseconomies of scale.

Capacity effect on profits (see Figure 2): Both the competitive and collusive profits of firm 1 fall when its rival 2 expands its capacity s_2. In competition, this is the consequence of more intense competition resulting in both a lower equilibrium price and output. In collusion, a higher capacity s_2 puts firm 1 at a bargaining disadvantage and enables firm 2 to claim a larger share of aggregate profits. Analogously, when firm 1 expands its own capacity s_1 it typically earns a higher collusive profit.[9] However, the competitive profit of firm 1 is inversely u-shaped in its own capacity s_1. This is a consequence of the assumption of soft capacity constraints. To see this consider a situation where the capacity s_1 of firm 1 has been small initially and is expanded to a medium level. Firm 1 mainly benefits from lower costs and a higher output (see above) which results in a higher profit. Now, consider that capacity s_1 is further expanded from intermediate to high levels. The cost and output effect are not that pronounced any more. However, stealing business from firm 1's rival means lowering the output and marginal costs of firm 2 making it a fiercer competitor as can be seen from the lower prices. As a consequence, the competitive profit of firm 1 decreases when it expands its own capacity s_1 from

9 It can be shown that the assumption of Nash-bargaining is not critical for this result. One attains the same conclusion when assuming the firms to maximize joint profits and distribute them according to the earnings follows output principle.

medium to high levels. This effect has only a minor impact on collusive profits where the firms often keep prices constant when capacity changes.

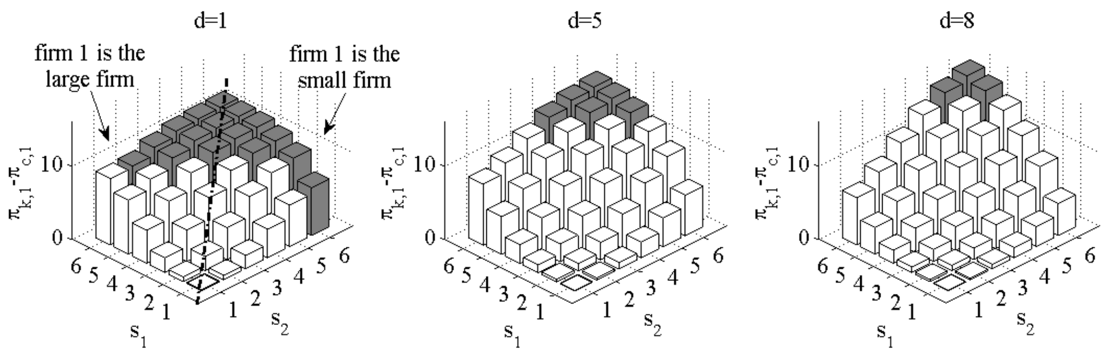

Figure 3: Profit differences in the stage game

We turn to the gain in stage-game profits generated by switching from competition to collusion Figure 3 displays the differences between the collusive and the competitive profits of firm 1, i.e. $\pi_{k,1}-\pi_{c,1}$, for all combinations of s_1 and s_2 in demand states d=1, 5, and 8.[10] The gray bars indicate the capacity combinations where $\pi_{k,1}-\pi_{c,1}$ decreases when demand falls from d=9 into d=8, from d=6 into d=5, or from d=2 into d=1. One observes two effects of changes in demand that depend on the combination of capacities s_1 and s_2 chosen by the firms.

1. When the firms produce at high capacities the short-run gain from collusion $\pi_{k,1}-\pi_{c,1}$ falls when demand moves into a lower state (gray bars). This is mainly a consequence of the *demand effect* that makes collusive profits fall along with demand.

2. When the firms produce at medium or low capacities the short-run gain from collusion $\pi_{k,1}-\pi_{c,1}$ rises when demand moves into a lower state (white bars). This is mainly a consequence of the above effect that production at low capacities means production subject to pronounced diseconomies of scale. A lower demand results in lower output and, thus, lower marginal costs. This results in more intense competition and competitive profits fall more strongly in demand than collusive profits (*competition effect*). This makes collusion relatively more desirable in comparison to competition as can be seen by a higher gain from collusion in the stage game.

These findings raise the question what is deemed to be the border between a high or a low capacity. It can be shown that capacities may be considered high when aggregate capacity suffices to satisfy demand even when the goods are given away for free. Hence, the distinction between high and

10 Note that all graphs in Figure 3 are rotated by 180° as compared to Figure 2. This ensures a better visibility of the plots.

low capacities is a function of the state of demand. *Demand state* d=1: Every firm could sell $\tilde{q}_j(\mathbf{0})$=16.84 units of its product at a price of 0. The total duopoly-quantity of 33.68 could be produced with about 6.7 units of capacity. Figure 3 shows that a drop of demand from state d=2 into state d=1 raises the profit difference π_k-π_c in all but one capacity states $\boldsymbol{s}$ where the sum of capacities s_1+s_2 takes a value of 7 or lower. *Demand state* d=5: Every firm could sell $\tilde{q}_j(\mathbf{0})$=21.05 units of its product at price 0. The total output of 42.11 units could be produced with about 8.4 units of capacity. In the model, a drop of demand from state d=6 into state d=5 raises the profit difference π_k-π_c in all capacity states $\boldsymbol{s}$ with s_1+s_2≤9. *Demand state* d=8: The maximum aggregate output of the two firms is 48.42 and could be produced with about 9.7 units of capacity. When demand drops from d=9 into d=8 the profit difference π_k-π_c rises in all capacity states $\boldsymbol{s}$ where s_1+s_2≤10 applies. Therefore, in all three cases a negative demand shock raises π_k-π_c when the firms produce at binding capacity constraints.

To summarize, a negative demand shock contributes to cartel formation in industries where it makes competition more intense (*competition effect*). Such a situation occurs when prior to the shock binding capacity constraints had given the firms some market power but become less binding thereafter which results in more intense competition. When prior to the shock an industry had been characterized by production capacities that more than suffice to serve demand cartel formation becomes even less likely. This is a consequence of the smaller market size and the lower competitive and collusive profits (*demand effect*).

3.2 Determinants of the Capacity Distribution: δ and r

Subsection 3.1 shows for what combinations of capacity a negative demand shock raises the short-run gain from collusion $\pi_{k,j}$-$\pi_{c,j}$ of firm j. This raises three questions:

1. What distribution of capacities should one expect in the competitive equilibrium?
2. How does this capacity distribution change when the firms switch to collusive conduct and how does this change in capacities affect the incremental value to collude, i.e. the present value of additional future profits.
3. What parameters shape these capacity distributions?

The first and the second question are answered in subsections 3.3 to 3.5 where we analyze the economic properties of the different equilibria found in our model. This subsection 3.2 answers the third question. It shows that we find three types of competitive equilibria which are mainly shaped by the values of the depreciation probability δ and the discount rate r. Subsection 3.6 is also concerned with the third question and shows that changes in the parameters for the persistence of demand ρ and the hardness of capacity constraints η do not affect the economic interpretation of our results.

Our analysis starts with calculating the competitive and collusive equilibria for all combinations of parameters with $r \in \{0.01, 0.02, ..., 0.1\}$, $\delta \in \{0.01, 0.02, ..., 0.1\}$, $\rho \in \{0.5, 0.7, 0.9\}$, and $\eta \in \{2.5, 10, 40\}$. For every combination of these parameters our calculation proceeds in the following steps:

1. Given a set of values for r, δ, ρ, and η together with the profit functions described in subsection 3.1, we use a variant of the Pakes and McGuire (1994) algorithm to compute the optimal investment $x_j(\boldsymbol{s},d)$ and the resulting firm value $V_j(\boldsymbol{s},d)$ of some firm j for each of the $M \cdot M \cdot D$=324 combinations of demand d and capacity $\boldsymbol{s}$. These policy and value functions are calculated both in competition and collusion. Additional information on our implementation of the algorithm including a pseudocode is provided in Appendix A.

2. Given the policy function of optimal investments, we simulate the probabilities $prob(\boldsymbol{s},d)$ to observe a particular combination of capacities $\boldsymbol{s}=(s_1,s_2)$ in demand state d. These probabilities form a limiting distribution of capacities (see Figure 4 on page 24 as an example).

   On the one hand, we determine limiting distributions that are unconditional on the recent history of an industry. To simulate such a limiting distribution we choose a random starting state $(\boldsymbol{s},d)$ and let the industry evolve over 10^7 periods with firms investing in capacities according to their policy functions. Recording the frequency with which each state was visited provides the unconditional limiting distribution $prob(\boldsymbol{s},d)$.

   On the other hand, this paper is concerned with the question whether a perceptible, negative demand shock is particularly likely to facilitate cartel formation. Therefore, we calculate the conditional probabilities $prob(\boldsymbol{s},d_t|d_{t-\tau}=d_t+\tau)$ of observing capacity combination $\boldsymbol{s}$ in demand state d_t under the condition that demand has previously made $\tau \in [1,4]$ consecutive downward movements. In other words, we assume that demand τ periods earlier was τ states higher ($d_{t-\tau}=d_t+\tau$) and we calculate the conditional limiting distributions $prob(\boldsymbol{s},d_t|d_{t-\tau}=d_t+\tau)$.

3. In the third step, we calculate the $M \cdot M \cdot D$=324 values of the incremental value to collude Ω, i.e. for all combinations of capacity $\boldsymbol{s}$ and demand d, by subtracting the competitive firm value $V_{c,j}$ from the collusive firm value $V_{k,j}$.

Depending on the values of the depreciation probability δ and the discount rate r, we find three types of competitive equilibria (labeled P, A, and S; also see Besanko et al. 2010.1 and 2010.2) whose properties are explored in greater detail in subsections 3.3 to 3.5. Table 3 shows the distribution of competitive equilibria across the parameter space for δ (in rows) and r (in columns) given ρ=0.5 and η=10.[11] It appears useful to think about the equilibria as being observed most frequently in one of four regions for δ and r:

δ=low, r=low:	Multiple equilibria of types P, A, and S
δ=high, r=low:	Type P equilibria (preemption races; the competing firms invest into fairly asymmetric capacities)
δ=high, r=high:	Type A equilibria (asymmetric competition; the competing firms invest into moderately asymmetric capacities)
δ=low, r=high:	Type S equilibria (symmetric competition; the competing firms invest into fairly symmetric capacities)

The criterion for distinguishing different competitive equilibrium types is the most frequently observed, i.e. modal, capacity state $\boldsymbol{s}_c$ in demand state d=9. In d=9 capacities can be symmetric (type S), asymmetric by no more than one unit of capacity (type A), or asymmetric by more than one unit of capacity (type P).[12] In contrast to competition, the type of the collusive equilibria does not change fundamentally with δ and r. Therefore, differences in the incremental value to collude across these regions are typically caused by the different types of competitive equilibria rather than differences in the collusive equilibria.

Table 3 shows that we sometimes find multiple equilibria for a certain set of parameters. These can be of different types (P, A, or S) or of the same type. For example, the entry A S for δ=0.03 and r=0.07 means that we find competitive equilibria of types A and S. Value and policy functions that are obtained in different restarts of the algorithm are treated as multiple equilibria of the same type if they

11 Table 3 does not report results for r=0.01 where the firms would engage in type P preemption race equilibria. With r=0.01 every firm puts a high weight on the option of earning high profits by becoming dominant in the future. Therefore, as long as the firms are fairly symmetric they invest into capacities way beyond market demand until an asymmetry emerges that is sufficiently strong to establish one firm as dominant and the other as dominated. This strategy is curtailed by the assumption of an upper bound for capacity, i.e. assuming a maximum number of M=6. Our evaluations indicate that choosing a fairly high maximum number for capacity states M can solve this issue but – because of the curse of dimensionality – results in drastically increased computation times. Therefore, we continue with the assumption of M=6 and refrain from reporting results for r=0.01.

12 In Table 3, the entry P/A for, e.g., δ=0.08 and r=0.07 means that we find an equilibrium that has characteristics of type P and type A equilibria and does not allow for a clear distinction.

satisfy the above criteria of this type but a distance measure for the value functions and policy functions is higher than a predefined threshold as is explained in Appendix A.[13] Competitive equilibria of the same type are typically very similar so that the economic interpretation of their properties is the same. Nonetheless, in sections 3.3 to 3.4 we illustrate the properties of our model for combinations of δ and r where we find a unique competitive equilibrium. Our sample combinations of δ and r are indicated in Table 3 by bold frames. In the case of collusion we also find multiple equilibria for some parameter combinations that, however, are all of the same type. Section 3.5 establishes that a cartel with Nash bargaining prices, grim trigger punishments and price wars, cannot be stabilized for some combinations of (asymmetric) capacities and demand when the competitive equilibrium is of type P. In the introduction, this is termed the *symmetry effect*. The respective combinations of δ and r are shown by the shaded areas in Table 3.

δ | r	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0.01	A S	A S	A S	A S	A S	A S	S	S	S
0.02	P A S	A	A S	A S	A S	S	A S	S	S
0.03	P	P A	A	A	A	A S	A S	S	S
0.04	P S	P A	P A	A	A	A	A S	S	S
0.05	P	P	P	A	A	A	A	A S	S
0.06	P	P	P	P A	A	A	A	A	A
0.07	P	P	P	P	A	A	A	A	A
0.08	P	P	P	P	P	P/A	A	A	A
0.09	P	P	P	P	P	P/A	A	A	A
0.10	P	P	P	P	P	P/A	A	A	A

Table 3: Distribution of competitive equilibrium types with ρ=0.5, η=10

3.3 Type A: Asymmetric Competition

Subsections 3.3 to 3.5 are concerned with the first two questions formulated above, i.e. what competitive and collusive capacity distributions are chosen by the firms for specific parameters of δ and r and how do these capacity distributions affect the incremental value to collude? We start with competitive type A equilibria which are typically found when capacity depreciates quickly (high values

13 In order to detect multiple equilibria we run the algorithm multiple times with different starting values and random dampening factors for each of the combinations of parameters. In case of the three combinations of parameter values reported in sections 3.3 to 3.4 we use 100 restarts of the algorithm to establish that a unique competitive equilibrium exists for the reported parameter combinations (see Appendix A).

of δ) and when the firms strongly discount future profits (high values of r). The results presented in this section are obtained by setting δ=0.08 and r=0.08 with ρ=0.5 and η=10.

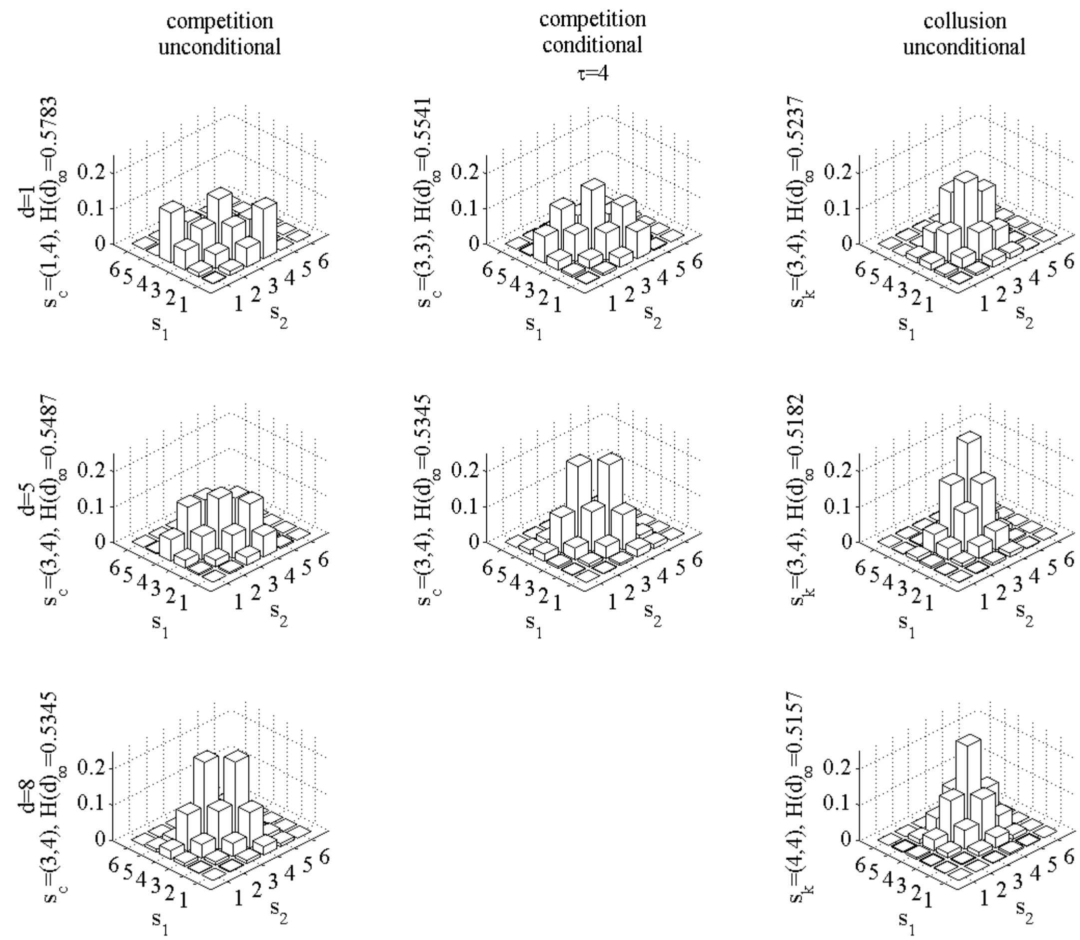

Figure 4: Limiting distributions with δ=0.08, r=0.08, ρ=0.5, η=10

Figure 4 characterizes the nature of the equilibria by showing the limiting distribution of capacities $prob(\boldsymbol{s},d)$ for $d \in \{1,5,8\}$ (1st to 3rd row). Column 1 shows the unconditional competitive limiting distribution. Column 2 shows the competitive limiting distribution conditional on demand having previously made τ=4 consecutive downward movements. With ρ=0.5 such an event occurs with a probability of about 4%. Column 3 shows the unconditional collusive limiting distribution. Figure 4 also provides the modal capacity state $\boldsymbol{s}_c$ (respectively $\boldsymbol{s}_k$ in the case of collusion) and the Herfindahl-index $H(d)_\infty$. The Herfindahl-index as a measure of asymmetry is defined in (20).

$$H(d)_\infty = \sum_{\boldsymbol{s}} \left[\left(\frac{\bar{q}_1}{\bar{q}_1 + \bar{q}_2} \right)^2 + \left(\frac{\bar{q}_2}{\bar{q}_1 + \bar{q}_2} \right)^2 \right] \cdot prob(\boldsymbol{s}|d) \tag{20}$$

Given the parametrization of our model, $H(d)_\infty$ is defined in the interval [0.5, 0.755] with higher values implying a more asymmetric industry structure.[14] The modal capacity states $\boldsymbol{s}_c$ and $\boldsymbol{s}_k$ are defined as the combinations of capacities that are most frequently observed in the competitive or the collusive limiting distribution.

We start by characterizing the properties of the limiting distribution in the unconditional competitive equilibrium (see the 1st column in Figure 4). One sees that every competing firm j aims at the capacity where its competitive profit peaks (see Figure 2 in subsection 3.1). This is the case for a capacity in the vicinity of s_j=4. When demand falls the firms let their capacities deplete which results in lower and – as depreciation follows a stochastic process – more asymmetrically distributed capacities. The 2nd column in Figure 4 shows the conditional limiting distribution of capacities after demand has dropped by τ=4 states in a row. In this case, capacities cannot follow demand quickly and remain at elevated levels for some time.

The collusive limiting distribution of capacities (see the 3rd column in Figure 4) reveals that the colluding firms invest into higher and more symmetric capacities in collusion than in competition. A similar observation has been made by Fershtman and Gandal (1994) in a model where firms invest into cost-reducing research and development. Capacity rises because in collusion the increase in capacity does not result in a price decrease (see Figure 2) while at the same time production costs are lowered (see conditions (5) and (6)). Hence, the increase in capacity raises the collusive profit $\pi_{k,j}$ (see Figure 2). The capacity expansion also causes a more symmetric distribution of capacities. The result of higher and more symmetric collusive capacities is driven by the assumption of semi-collusion, i.e. the firms collude in prices but do neither coordinate their investment policies nor their market shares (Fershtman and Pakes 2000).

Figure 5 presents the value function of firm 1 in competition ($V_{c,1}$, 1st column) and collusion ($V_{k,1}$, 2nd column) in demand states $d \in \{1,5,8\}$ (1st to 3rd row). Both value functions look pretty much like the competitive and collusive profit functions that are displayed in Figure 2 and that are interpreted economically in section 3.1. In particular, one finds the *demand effect* to apply: The competitive firm value $V_{c,1}$ and the collusive firm value $V_{k,1}$ fall when demand declines. In the following, we describe the properties of the incremental value to collude Ω_1=$V_{k,1}$-$V_{c,1}$ (see the 3rd column in Figure 5) and demonstrate the influences of the *competition effect* and the *symmetry effect* that are introduced in section 1.

14 The lower bound of 0.5 is found when the firms set symmetric capacities q_1=q_2 in all cases. The upper bound assumes the maximum asymmetry q_1=5 $\wedge$ q_2=30 respectively q_1=30 $\wedge$ q_2=5 to occur with a 50% probability each.

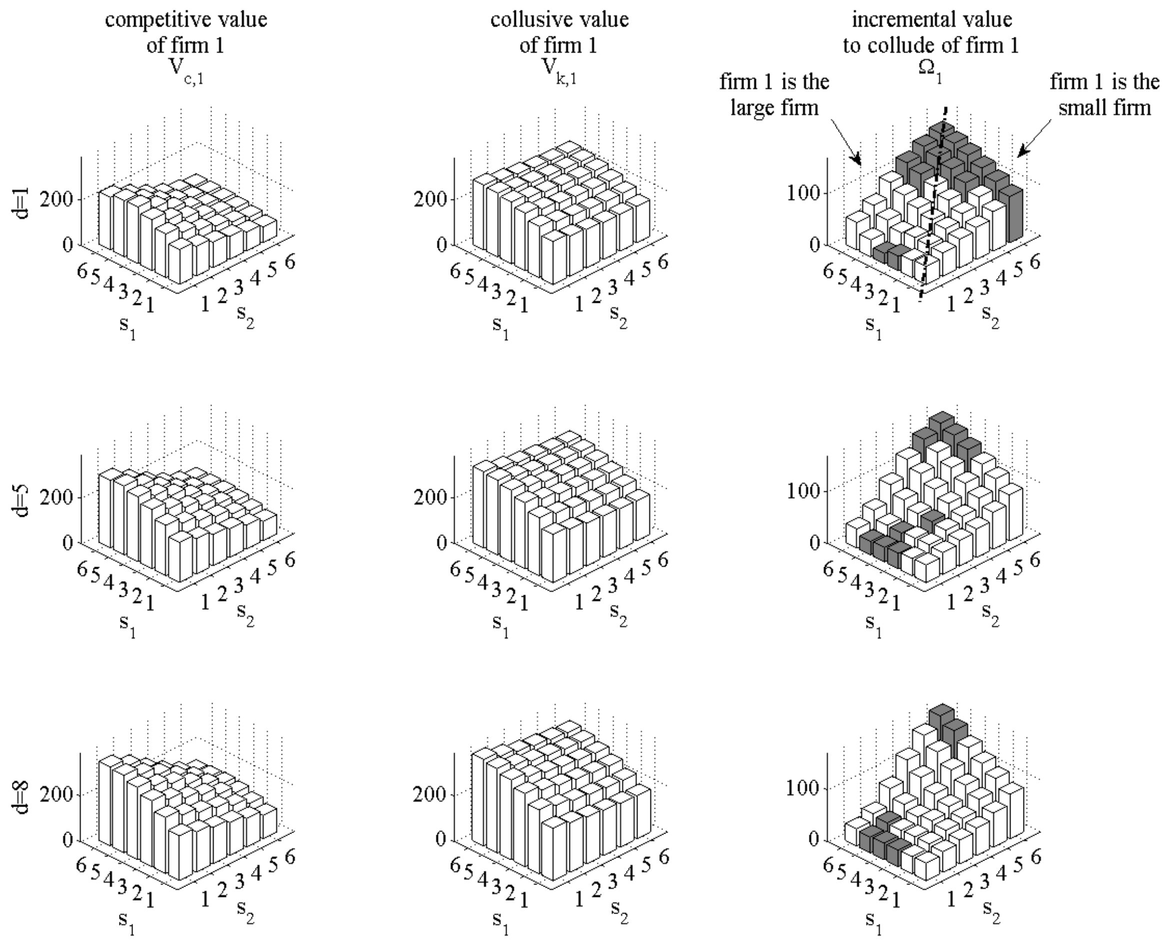

Figure 5: Firm values and Ω with δ=0.08, r=0.08, ρ=0.5, η=10

The *competition effect* postulates that the incremental value to collude is particularly high in states with intense competition. (i)[15] Such a situation can be brought about by a drop in demand when prior to the shock the firms had produced at binding capacity constraints, i.e. with low or intermediate capacities. Ω_1 rises when due to the negative demand shock the capacity constraints become less binding and, thus, competition will be more intense. This can be seen by the white bars in the 3rd column of Figure 5 which indicate an increase in Ω_1 when demand falls by one state (d=9→8, d=6→5, d=2→1). This is in line with our discussion of the short-run gain from collusion $\pi_{k,1}$-$\pi_{c,1}$ in subsection 3.1.

(ii) The competitive limiting distribution (1st column in Figure 4) shows that due to the high depreciation probability (δ=0.08) capacities are reduced rather quickly in competition when the industry is hit by a negative demand shock. As the incremental value to collude typically falls when

15 The enumeration (i), (ii), and (iii) refers to the numbering used for the *competition effect* in the introduction.

capacity is reduced[16] the quick depreciation counteracts the intensification of competition and, thus, works against cartel formation. This is different when the demand shock is pronounced and occurs quickly. When demand drops by τ=4 states in a row capacity cannot follow suit quickly and remains at elevated levels (2nd column in Figure 4) where competition is intense and the incremental value to collude is high. This finding provides evidence for cartels that were formed in response to a sudden shock in demand. An example might be the French beef cartel where demand had fallen substantially in 2000 due to the discovery of the mad cow disease while production capacity and livestock remained the same. This led to a sudden and substantial decrease in prices.

(iii) The collusion-enhancing effect of falling demand vanishes when the firms had produced at fairly high, non-binding capacities even before the shock. In this case, competition is intense before and after the shock and the *demand effect* dominates the *competition effect*. Our sample-industry provides evidence of these patterns, i.e. the gray bars in Figure 5 show the capacity combinations where a drop in demand (d=9→8, d=6→5, d=2→1) lowers Ω_1.[16] However, in these states the absolute value of the incremental value to collude is still quite high which makes collusion desirable for the firms.

The *symmetry effect* postulates that a symmetric capacity distribution facilitates cartel formation. Therefore, we analyze the effect of capacities on the incremental value to collude. One finds that Ω_1 rises in the capacity s_2 of firm 1's rival. This pattern has already been described in subsection 3.1 for the profit difference $\pi_{k,1}$-$\pi_{c,1}$ and is a result of the more intense competition and marginalization of firm 1 in competition when firm 2 becomes more dominant. However, Ω_1 can be u-shaped in the own capacity s_1 of firm 1. Ω_1 is particularly low when firm 1 is dominant producing in the range of capacity state s_1=4 while facing a smaller rival (s_1>s_2). In states with asymmetric capacities the incremental value to collude is mainly shaped by the competitive value function $V_{c,1}$ which is inversely u-shaped in firm 1's own capacity s_1 (also see the discussion of $\pi_{c,1}$ in section 3.1). Moreover, the incremental value to collude of a dominant firm often falls along with demand as can be seen by the gray bars in Figure 5.

When the firms are asymmetric with capacities $s_j \in \{s,s'\}$ and $s<s'$ the incremental value to collude of the large firm is lower than that of the small firm, i.e. $\Omega_j(s)>\Omega_j(s')$. This finding is supported by all numeric evaluations of our model. This is because of the shift in the collusive limiting distribution towards higher and more symmetric capacities as compared to competition. With more symmetrically distributed collusive capacities, expected future profits are distributed more

16 Exceptions are explained below in this section in the context of the *symmetry effect*.

symmetrically in collusion than in competition. The previously dominant firm would loose its supremacy in the cartel. Therefore, collusion creates a higher benefit for a small firm than for a large firm. Since a cartel is only formed when both firms find it profitable to collude it is the incentive of the high-capacity firm that is decisive for cartel formation. This finding is consistent with the observation that cartels are most often observed among firms with similar market shares.

In the competitive limiting distribution asymmetric capacity states with a large and a small firm are observed rather frequently. This implies two findings for our type A sample industry. First, asymmetry makes cartel formation in such an industry fairly hard. Second, as the incremental value to collude of a dominant firm often falls along with demand the risk of cartel formation is typically lowered when demand falls in this industry with non-durable capacities (δ=0.08) and impatient firms (r=0.08).

3.4 Type S: Symmetric Competition

In this subsection, we analyze firms' incremental value to collude for parameter values that yield a type S competitive equilibrium. Type S equilibria are typically found when capacity depreciates slowly (low values of δ) and the firms strongly discount future profits (high values of r). In particular, we use the parameter values δ=0.02 and r=0.09 while setting ρ=0.5 and η=10.

Figure 6 shows the competitive limiting distributions both unconditional on the recent history of the industry (1st column) and after τ=4 negative demand shocks in a row (2nd column). For the above parameter values we find two collusive equilibria (labeled *type kA* and *type kB*) whose limiting distributions are shown in the 3rd and 4th column. We do not find evidence of further competitive or collusive equilibria. Figure 6 also provides the modal capacity state and the Herfindahl-index for each of these limiting distributions. Figure 7 shows the value of firm 1 in competition (1st column), in the collusive equilibria of type kA (2nd column) and of type kB (4th column). It also provides the incremental value to collude for all capacity states ***s*** in demand states d=1, 5, and 8 for the collusive equilibria of types kA (3rd column) and kB (5th column).

The competitive type S equilibrium is shaped by the low depreciation probability δ. The long useful life of production facilities prevents capacities from being adjusted quickly to situations of lowered demand. Therefore, the capacities are chosen to cater average demand and respond fairly little to demand shocks. They are kept fairly constant as long as demand is in states $d\geq5$ and decrease in response to lower demand mainly in states d<5. The firms invest into quite symmetric capacities. This is because a preemption race for a dominant position (see subsection 3.5) would bear the risk that – as a result of the infrequent depreciation events – the firms involuntarily end up in an unfavorable situation with very high and symmetric capacities which are associated with intense competition and low profits.

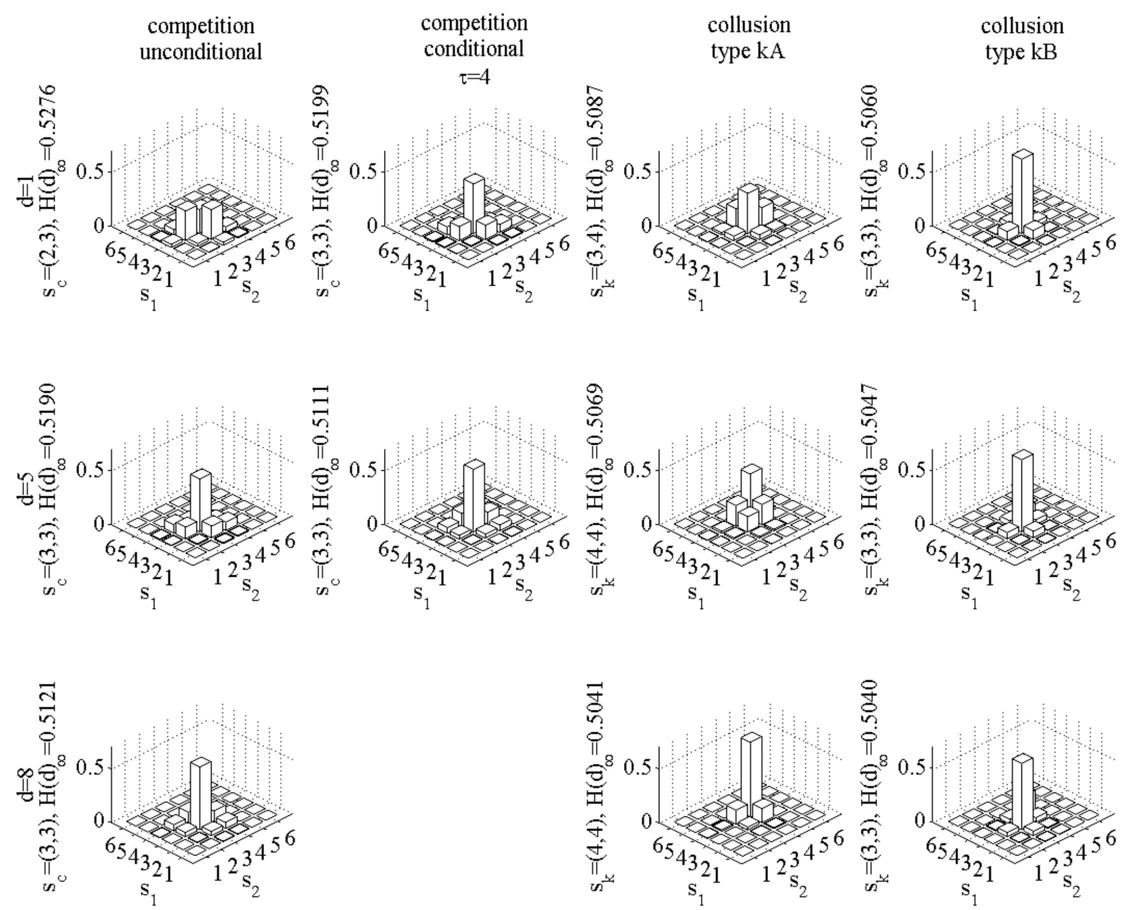

Figure 6: Limiting distribution for region 4 (δ=0.02, r=0.09, ρ=0.5, η=10)

The collusive limiting distributions reveal the same effects that are described in subsection 3.3. The colluding firms invest into higher and more symmetric capacities than in competition. In the collusive type kA equilibrium the firms most frequently invest into capacity states (3,4) and (4,4), i.e. both firms expand their capacity. The modal capacity state in the collusive type kB equilibrium is (3,3) for all demand states, i.e. the small firm expands its capacity while the large firm maintains about its competitive capacity. The effects of fairly durable capacities that are described above for the competitive limiting distribution also apply to the collusive limiting distribution: As compared to the collusive equilibrium with δ=0.08 (see subsection 3.3) the assumption of durable capacity (δ=0.02) results in more symmetric collusive limiting distributions where capacity varies fairly little when demand changes.

Figure 7 shows that the shape of the competitive and the collusive value functions resembles the shape of the underlying profit functions (see Figure 2) as was also described for the value functions of type A equilibria (see subsection 3.3). The *demand effect* applies, i.e. lower demand lowers both the competitive and the collusive firm values. With regard to the *symmetry effect* one finds that the incremental value to collude Ω_1 of firm 1 rises in the capacity s_2 of its rival and may be u-shaped in its own capacity s_1. Moreover, the incremental value to collude of a large firm is lower than that of a small firm.

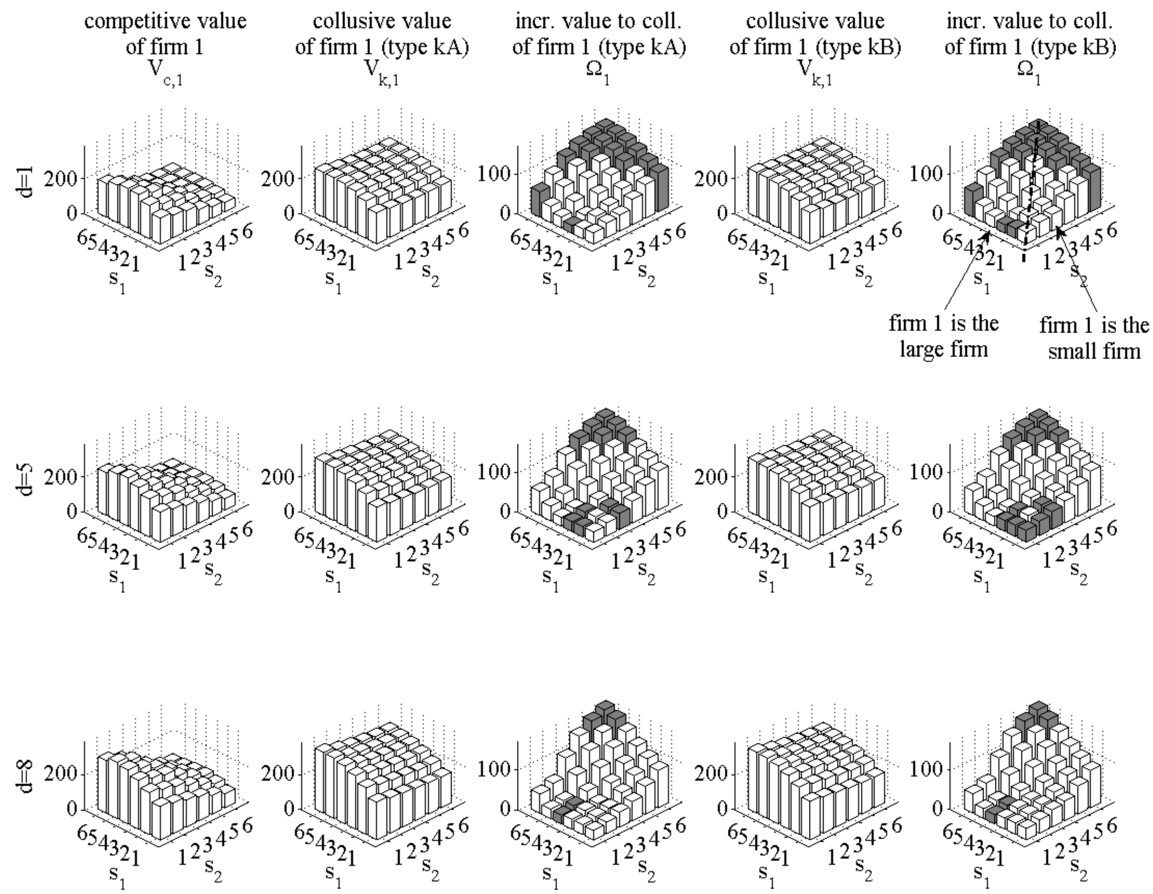

Figure 7: Firm values and Ω in region 4 (δ=0.02, r=0.09, ρ=0.5, η=10)

We turn to the *competition effect*. (i), (iii)[17] One sees that a negative demand shock may raise the incremental value to collude (white bars in the 3rd and 5th column of Figure 7) when capacities are not too high and when firm 1 did not have a dominant position prior to the drop in demand. This is the case when firm 1 produces at about capacity s_1=3 or s_2=4 while facing a smaller competitor 2. (ii) The conditional limiting distribution (2nd column in Figure 6) shows that durable capacities cannot follow

17 The enumeration (i), (ii), and (iii) refers to the numbering used for the *competition effect* in the introduction.

suit quickly when demand falls by τ=4 steps in a row. Hence, the firms are likely to be stuck with suboptimally high capacities where a drop in demand results in more intense competition and a high incremental value to collude.

The competitive limiting distribution in Figure 6 shows that the firms most frequently invest into capacity states with symmetric capacities of a medium size where a decrease in demand raises the intensity of competition and, thus, the incremental value to collude. As a consequence, with durable capacity (δ=0.02) a drop in demand is more likely to result in a rising incremental value to collude than in the case of shorter-lived capacities (δ=0.08, subsection 3.3). This is especially the case for demand states $d \geq 5$ where capacities respond fairly little to changes in demand. When demand falls further ($d<5$) the competitive limiting distribution in Figure 6 shows that the firms let their capacities deplete and become more unevenly sized. The emergence of dominant firms and the lower capacities mitigate the competition effect and those capacity states are observed more frequently where the incremental value to collude Ω_1 falls along with demand.

These effects are similar for the collusive type kA and type kB equilibria. The main difference is the choice of higher capacities in the type kA equilibrium. This results in both a lower collusive firm value $V_{k,1}$ and a lower incremental value to collude Ω_1 in the collusive type kA equilibrium than in the type kB equilibrium. To summarize, a negative demand shock facilitates cartel formation especially in industries with durable capacities.

3.5 Type P: Preemption Races

Besides competitive equilibria with moderately asymmetric (type A, see subsection 3.3) and fairly symmetric capacity distributions (type S, see subsection 3.4) we find a third competitive equilibrium (type P) where firms engage in preemption races that result in fairly asymmetric capacity distributions. This can be seen in Figure 8 which shows the competitive limiting distribution of capacities and the value $V_{c,1}$ of firm 1 for all capacity states $\boldsymbol{s}$ in demand states d=1, 5, and 8. We find competitive equilibria of type P when capacities can quickly be adjusted downwards (the depreciation probability δ is high) and the firms are sufficiently patient (the discount rate r is low). Figure 8 is derived by setting δ=0.08, r=0.02, ρ=0.5, and η=10.

Competitive preemption race strategies that result in fairly asymmetric capacities have been described by Besanko and Doraszelski (2004). During a preemption race, firms continue to invest as long as their capacities are similar. This competition in investments is intense because the low discount rate r induces the firms to put a large weight on future profits – when a firm expects to have attained a dominant position – as opposed to current investment expenditures. The race comes to an end once there is a sufficient difference in firms' capacities that can occur more quickly for higher values of the depreciation probability δ, which generates depreciation events more frequently. The depreciation events contribute to the creation of asymmetric capacities. Once sufficiently asymmetric capacities have emerged, the investment race stops and the small firm reduces some of the excess capacity that has been built during the race (Besanko et al. 2010.2: 1179). In the resulting product market equilibrium the high-capacity firm generates profits by selling a high quantity at a low price while the low-capacity firm benefits from selling a low quantity at a higher price (see Figure 2).

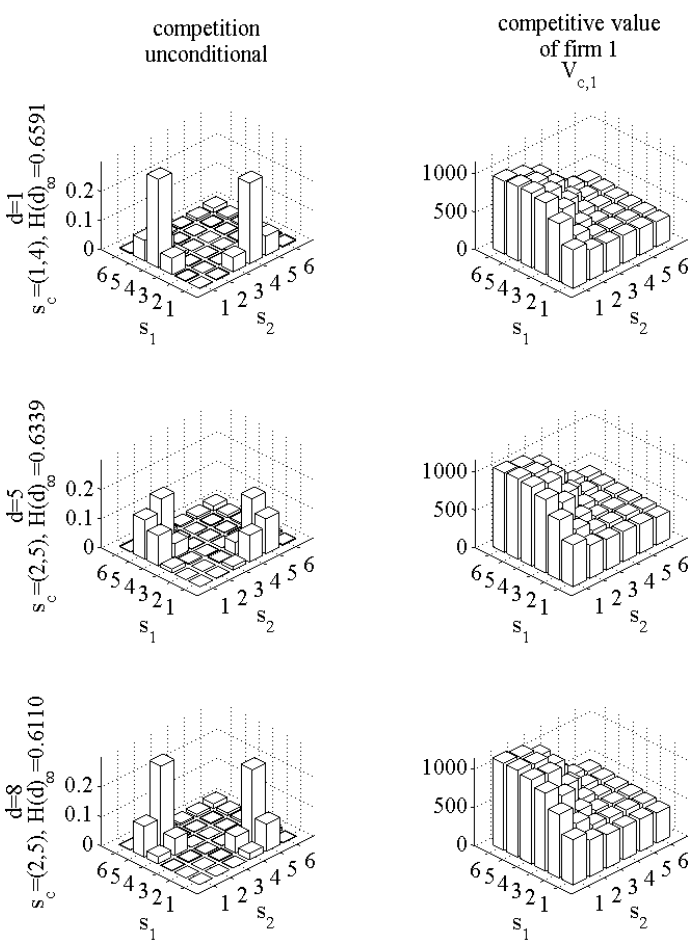

Figure 8: Limiting distribution and competitive firm value for region 2 (δ=0.08, r=0.02, ρ=0.5, η=10)

No graphs or other results are provided for collusion. This is because with this parametrization and the resulting asymmetric capacities collusion cannot be stabilized in all capacity states ***s***. In particular, a large firm that faces a very small competitor prefers being a dominant competitor, i.e. it already earns the maximum competitive profits shown in Figure 2, over sharing collusive profits with another firm. In this context, consider the above result that in our model colluding firms invest into above-competitive and fairly symmetric capacities. This may make collusion unattractive for a previously dominant firm. Collusion that cannot be stabilized by a grim trigger strategy with price wars goes beyond the assumptions of our collusive model (see subsection 2.4). This is a variant of the *symmetry effect*.

Noticeably, this instability of collusion occurs when the discount rate is low, which is typically considered a stabilizing factor for cartels. This finding illustrates the importance of analyzing firms' decision to collude in the context of firms' entire strategy set that also includes investment strategies. Analyzing the interactions between different strategies yields new insights such as the above one: A low discount rate r does not always contribute to stabilizing collusion as it may also cause firms to invest into asymmetric capacities which are associated with collusion being unstable.[18]

3.6 Robustness Checks

In the following, we address the third question raised in subsection 3.2: How do changes in the parameters δ, r, η, and ρ affect the results presented above? When evaluating the properties of the type A, S, and P equilibria for combinations of r and δ other than the ones used in subsections 3.3 to 3.5 one finds that the equilibria obtained for these adjacent parameter combinations differ from the presented ones only in their numerical values. They do not reveal any new properties, effects or insights.

In the following, we show how different assumptions on the hardness of capacity constraints affect our results. The above results assume a marginal cost function with η=10 that rises steeply when the output $\tilde{q}_j$ of firm j approaches or exceeds the capacity $\bar{q}_j$. i.e. the capacity constraints are fairly hard (see Figure 1 in subsection 2.2). Now, we assume $\eta \in \{2.5, 10, 40\}$ and find that varying η does not generate new equilibrium types other than those of types A, S, and P. To illustrate the effect of different

18 Table 3 shows that this effect does not apply to all parameter combinations that yield a type P equilibrium. Stable cartels are found for values of $r \geq 0.04$ because the limiting distributions are typically more symmetric when firms are less patient, i.e. they share more characteristics with type A equilibria where collusion is stable (see subsection 3.3). This result is economically meaningful. Stable cartels in competitive type P equilibria can also be found for some values of δ when $r<0.04$. This should be considered an artifact of the choice of the state space especially the upper bound for capacity, i.e. assuming a maximum number of M=6. Our evaluations indicate that choosing a fairly high maximum number for capacity states can solve this issue but – because of the curse of dimensionality – results in drastically increased computation times. Therefore, we continue with the assumption of M=6 and do not interpret the respective results economically (also see Footnote 11 on this issue).

assumptions on η, Figure 9 presents the incremental value to collude of firm 1 in our type A sample industry with δ=0.08, r=0.08, ρ=0.5 (see subsection 3.3) for $\eta \in \{2.5, 10, 40\}$. Assuming even harder capacity constraints (η=40) does not change our results much as can be seen in Figure 9. The incremental value to collude is fairly similar for η=10 and η=40. The main difference is that with harder capacity constraints being dominant is of greater value for the firm. This can be seen in the more pronounced u-form of Ω_1 for η=40 and the more frequent capacity states where a drop in demand lowers the incremental value to collude (gray bars).

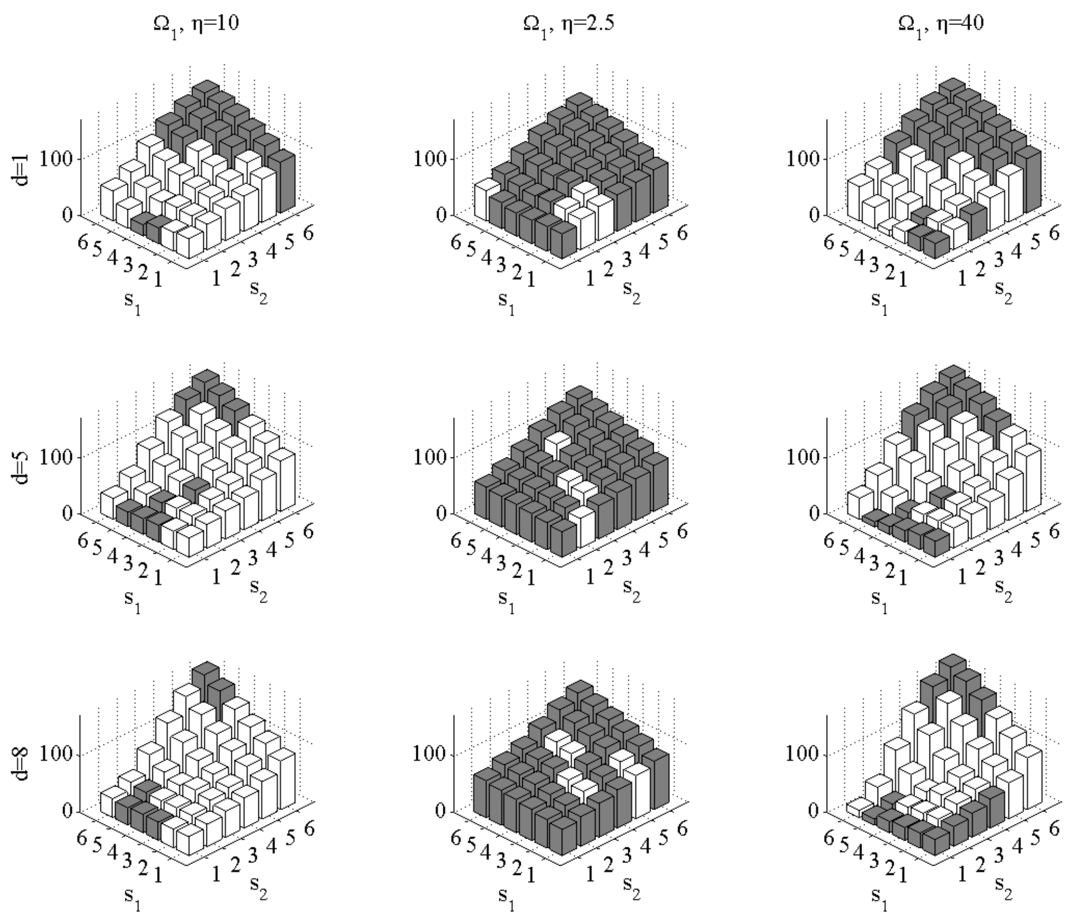

Figure 9: Incremental value to collude of firm 1 (Ω_1) with δ=0.08, r=0.08, ρ=0.5

More differences can be seen when assuming softer capacity constraints (η=2.5). In this case, changes of demand and firms' capacities have a smaller effect on marginal costs, price, and profit of some firm j. One also finds that with soft capacity constraints the firms do not vary capacity as much across demand states as in the case of harder capacity constraints. Therefore, the *competition effect*, i.e. the intensification of competition in response to a decline in demand, is relatively moderate while the *demand effect*, i.e. the decline in profits and firm values due to a drop of demand, is comparatively strong. Hence, a negative demand shock lowers the additional profits from collusion ($\pi_{k,j}$-$\pi_{c,j}$) even for medium-sized capacities. Similarly, the incremental value of collusion in Figure 9 falls along with demand for almost all capacity states. This makes cartel formation in response to a decline in demand less likely and underlines the importance of capacity constraints for the patterns of cartel formation.

Moreover, when capacity constraints are soft (η=2.5) we also find a greater multiplicity of equilibria within the types A, S, and P. This is because with smaller diseconomies of scale the following trade-off becomes more relevant for the firms: Either they invest into higher production capacities and, thus, lower production costs. Or they save on investments but produce at higher marginal costs. This greater substitutability of investment costs and production costs gives rise to a greater number of equilibria without, however, affecting our qualitative conclusions.

Appendix B shows that our results are robust to changes in the persistence of demand ρ. More persistent demand gives the firms more time to adjust to new demand conditions such that a greater number of equilibria (of the types A, S, and P) may emerge for every combination of parameters. However, these equilibria are very similar. None of our qualitative conclusions is affected by changes in ρ.

4 CONCLUSION

To conclude this paper, we would like to respond to some frequently asked questions:

1. *What can we learn from the above model about cartel formation?*

   The above model identifies two markers for cartel formation. First, symmetry among firms is a risk factor for cartel formation (*symmetry effect*). Second, firms' gain from collusion is the higher the more intense competition would be absent the conspiracy (*competition effect*).

2. *Isn't the answer to question 1 trivial?*

   No. It is not so much the finding that symmetry and intense competition facilitate cartel formation which makes this article interesting. Its contribution is to identify factors that make competition intense and/or contribute to a symmetric capacity distribution. Moreover, it shows how changes in these factors affect the intensity of competition and symmetry.

3. *What makes competition intense and how can we identify such circumstances?*

   In the above model with near-homogeneous products competition is intense and, thus, the incremental value of collusion is high when the production capacity of the firms exceeds the level that is needed to satisfy demand at a price of zero. Evidence of such a situation is provided by low values of the price-cost margin or a low return on sales. A situation where demand is small relative to production capacities can be brought about by an exogenous shock that lowers demand. It can also be caused endogenously by firms' decision to invest into higher capacities.

4. *Can negative demand shocks be used as a screening device to identify industries that are prone to cartel formation?*

   No, not in isolation. Changes in demand can have ambiguous effects. The pure *demand effect* suggests that a decrease in market size makes collusion less profitable in absolute terms. However, the firms' incremental value to collude may rise when the drop in demand contributes to making competition more intense. This is the case when previously binding capacity constraints become less binding in the smaller market (*competition effect*). Given the interaction of demand and capacity, demand alone is not a good screening device. This can also be seen by the case evidence provided in the introduction. It shows that many kinds of demand shocks have already been observed prior to cartel formation.

5. *Can excess capacities be used as a screening device to identify industries that are prone to cartel formation?*

   No, not in isolation. The existence of high capacities contributes to intense competition which is a facilitating factor for cartel formation. However, when capacities had been high even before a negative demand shock the intensity of competition will not be raised much more due to such a shock. The incremental value to collude will even fall because of the lower market size. Moreover, it depends on the distribution of capacities across firms whether the risk of cartel formation is high or low in an industry. Suppose an industry where one firm is dominant with a high capacity while facing a small rival. In this situation, the large firm has a small incentive to form a cartel despite its high production capacity.

6. *Are there predictors for the distribution of capacities?*

   Yes. The model shows that the firms choose fairly different investment strategies depending on their discount rate and the useful life of capacities. (a) When capacity is quite durable the firms invest into capacity that is appropriate to cater average demand. Therefore, the firms are fairly symmetric and capacities are not adjusted much in response to demand conditions. Under such circumstances changes in demand have a strong impact on competition such that cartels may easily be triggered by negative demand shocks. (b) When capacity is less durable, asymmetric capacity distributions arise more often which the firms try to exploit in order to attain a dominant position. Asymmetry makes cartel formation less likely. Moreover, striving for dominance leads to high capacities and intense competition. Therefore, the demand effect is often stronger than the competition effect and a decrease in demand tends to lower the incremental value to collude. (c) When the firms have a low discount rate and produce with short-lived capacities their desire for dominance is even stronger which results in fairly asymmetric capacity distributions where collusion cannot necessarily be stabilized (at least under the assumptions of our model). This is interesting because a low discount rate is typically considered a facilitating factor for collusion as it helps to stabilize cartels.

7. *What are the main takeaways from this model?*

The model shows clearly that product market strategies like collusion and competition must not be analyzed in isolation. At the same time, firms pursue other strategies like investments into production capacity which are affected by the same factors like the strategies in the product market. Here, it is the discount rate which affects both types of strategies. To date, a low discount rate was quite undisputedly considered a facilitating factor for collusion. After analyzing the interaction between investment strategies and product market strategies one must conclude that a low discount rate can even obstruct collusive strategies by making firms go for a dominant position in the market.

8. *What might be done next?*

The above model can be extended in a variety of ways. For example, it may be worthwhile to relax the assumption of semi-collusion and analyze firms' strategies when they also coordinate their investment strategies. Instead of price-fixing the firms might also be assumed to allocate customers or markets, or to allocate market shares. Additionally, one may assume a competition authority with the ability to detect, prosecute and fine cartels. This will also complement the model by introducing costs of collusion. Moreover, the type of investment strategy may be broadened by allowing for product innovations instead of or in addition to capacity investments. The firms might also be allowed to choose among a greater set of strategies in the product market besides competing or colluding. For example, they might also be allowed to engage in exclusionary strategies with the goal of driving competitors out of the market instead of conspiring with them. Moreover, in addition to building capacity the firms might also be allowed to actively reduce capacity.

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APPENDICES

Appendix A The Algorithms and Multiplicity of Equilibria

Pseudocode 1: Competitive optimization

1: Initialization
2: Calculate competitive equilibrium $\boldsymbol{p}_c(\boldsymbol{s},d)$, $\boldsymbol{q}_c(\boldsymbol{s},d)$, $\boldsymbol{\pi}_c(\boldsymbol{s},d)$ $\forall$ $\boldsymbol{s}$, d
3: Policy and value function
4: Policy function: $x0_{c,j}(\boldsymbol{s},d) = \zeta(\boldsymbol{s},d,j)\cdot \pi_{c,j}(\boldsymbol{s},d)$ $\forall$ $\boldsymbol{s}$, d, j
5: $x1_{c,j}(\boldsymbol{s},d) = x0_{c,j}(\boldsymbol{s},d)$
6: Value function: $V0_{c,j}(\boldsymbol{s},d) = \zeta(\boldsymbol{s},d,j)\cdot \pi_{c,j}(\boldsymbol{s},d)/r$ $\forall$ $\boldsymbol{s}$, d, j
7: $V1_{c,j}(\boldsymbol{s},d) = V0_{c,j}(\boldsymbol{s},d)$
8: With values of $\zeta(\boldsymbol{s},d,j)$ being drawn pseudo-randomly from a uniform distribution $\in [0,1]$
9: Program controls
10: $tolV = tolV\text{-}1 = tolx = tolx\text{-}1 = 20$
11: $\lambda = 1$
12: Optimization
13: while $(tolV > tol) \wedge (tolx > tol)$
14: Use $\boldsymbol{x0}_c$ and $\boldsymbol{V0}_c$ (Gauss-Jacobi scheme) to calculate optimal policy function $\boldsymbol{x1}_c$ according to equations (12) and (13)
15: Update value function $\boldsymbol{V1}_c$ with values of $\boldsymbol{x1}_c$ and perform a policy iteration on $\boldsymbol{V1}_c$
16: Update distance measures
17: $tolV\text{-}1 = tolV$
18: $tolx\text{-}1 = tolx$
19: $tolV = \max |\, (\boldsymbol{V1}_c\text{-}\boldsymbol{V0}_c)/(1+|\boldsymbol{V0}_c|)\, |$
20: $tolx = \max |\, (\boldsymbol{x1}_c\text{-}\boldsymbol{x0}_c)/(1+|\boldsymbol{x0}_c|)\, |$
21: Determine dampening factor λ
22: if $(tolV > tolV\text{-}1) \vee (tolx > tolx\text{-}1)$, draw λ pseudo-randomly from a uniform distribution $\in [0,1]$
23: else, set $\lambda = 1$
24: Update value and policy function with dampening
25: $\boldsymbol{V0}_c = \lambda\cdot \boldsymbol{V1}_c + (1\text{-}\lambda)\cdot \boldsymbol{V0}_c$
26: $\boldsymbol{x0}_c = \lambda\cdot \boldsymbol{x1}_c + (1\text{-}\lambda)\cdot \boldsymbol{x0}_c$
27: end
28: Return $\boldsymbol{V1}_c$ and $\boldsymbol{x1}_c$

Pseudocode 1 presents our implementation of the Pakes-McGuire (1994) algorithm used for the optimization in the competitive model. The main characteristics of the algorithm are the use of a Gauss-Jacobi scheme (14:), i.e. the policy function in iteration l is calculated by use of information from the value and policy function obtained in iteration l-1 only. Moreover, we use a policy iteration scheme for updating the value function (15:), i.e. we iterate on the value function for (typically) 3 steps while using the same candidate policy function. The optimization is stopped when both sup norm distance measures $tolV$ and $tolx$ fall below a tolerance of tol=5e-8 (12:), i.e. when the modification in the entries of the value function and the policy function between any two iterations is very small.

The policy function $\boldsymbol{x0}_c$ is initialized (4:) by multiplying the profits $\pi_{c,j}(\boldsymbol{s},d)$ of firm j that correspond to the entries in the policy function by a pseudo-random number $\zeta(\boldsymbol{s},d,j)$ drawn from a uniform distribution in the interval [0,1]. The value function $\boldsymbol{V0}_c$ is initialized (7:) in a similar fashion by multiplying a naïve estimate of firm j's value, i.e. $\pi_{c,j}(\boldsymbol{s},d)/r$, with a pseudo-random number.

We use a dampening scheme to prevent the algorithm from visiting a sequence of policy and value functions all over again. Hence, the value and policy function used in iteration l+1 are generated as weighted averages of the functions from iterations l and l-1 (25: and 26:). When using a fixed dampening factor λ the circling behavior of the algorithm is not always avoided completely. Therefore, we draw a new value of $\lambda \in [0,1]$ from a uniform distribution in every iteration. This dampening scheme is only applied in iterations where one of the distance measures *tolV* or *tolx* exceeds its respective value from the previous iteration (22:). Otherwise, we use a value of λ=1 (23:). This helps the algorithm to stay on a convergence path while avoiding jumps away from the previous candidate solutions. Such jumps often indicate that the algorithm oscillates between different types of candidate solutions rather than converging to an equilibrium of the game. The occasional use of undampened updating (λ=1) results in a faster convergence than in the case of continuously employed dampening.

The randomness in the initialization of the value function $\boldsymbol{V0}_c$ and the policy function $\boldsymbol{x0}_c$ as well as the randomness in the choice of the dampening factor λ ensures that the algorithm generates a unique sequence of candidate solutions in every restart with otherwise identical industry parameter values. When a game has multiple equilibria our algorithm possesses the ability to converge towards them. We explore this multiplicity by running the algorithm several times on the same set of parameter values.

Pseudocode 2 presents our implementation of the Pakes and McGuire algorithm (1994) used for the collusive model. Its main structure mimics that for the optimization of the competitive model. However, it is in some instances more complex. We need to calculate (or load) the product market equilibria in competition, collusion and in deviation periods (3: to 6:). The collusive policy and value functions $\boldsymbol{x0}_k$ and $\boldsymbol{V0}_k$ are initialized (8: to 14:) by multiplying each entry of their competitive counterparts by a number $\zeta(s,d,j)$ drawn pseudo-randomly from a uniform distribution in the interval [0.5,1.5]. We assess in every iteration whether some firm would want to deviate from a collusive equilibrium at Nash bargaining prices and, if yes, set prices, quantities and profits at competitive, i.e. price war levels, and at Nash bargaining levels otherwise (21: to 22:). The stopping criterion of the algorithm (19:) is determined by the convergence of the collusive policy function and value function ($\boldsymbol{x0}_k$ and $\boldsymbol{V0}_k$) only. This is because the deviation policy function and value function ($\boldsymbol{x0}_d$ and $\boldsymbol{V0}_d$) closely resemble their competitive versions so that convergence of $\boldsymbol{x0}_d$ and $\boldsymbol{V0}_d$ is achieved more quickly than convergence of $\boldsymbol{x0}_k$ and $\boldsymbol{V0}_k$.

Pseudocode 2: Collusive optimization

Initialization
  Product market equilibria
    Load competitive equilibrium $\boldsymbol{p}_c(\boldsymbol{s},d)$, $\boldsymbol{q}_c(\boldsymbol{s},d)$, $\boldsymbol{\pi}_c(\boldsymbol{s},d)\ \forall\ \boldsymbol{s}, d$, $\boldsymbol{V1}_c$ and $\boldsymbol{x1}_c$
    Calculate Nash bargaining equilibrium $\boldsymbol{p}_{NBS}(\boldsymbol{s},d)$, $\boldsymbol{q}_{NBS}(\boldsymbol{s},d)$, $\boldsymbol{\pi}_{NBS}(\boldsymbol{s},d)\ \forall\ \boldsymbol{s}, d$
    Calculate deviation equilibrium $\boldsymbol{p}_{dev}(\boldsymbol{s},d)$, $\boldsymbol{q}_{dev}(\boldsymbol{s},d)$, $\boldsymbol{\pi}_{dev}(\boldsymbol{s},d)\ \forall\ \boldsymbol{s}, d$
    Initialize collusive equilibrium $\boldsymbol{p}_k(\boldsymbol{s},d)=\boldsymbol{p}_{NBS}(\boldsymbol{s},d)$, $\boldsymbol{q}_k(\boldsymbol{s},d)=\boldsymbol{q}_{NBS}(\boldsymbol{s},d)$, $\boldsymbol{\pi}_k(\boldsymbol{s},d)=\boldsymbol{\pi}_{NBS}(\boldsymbol{s},d)\ \forall\ \boldsymbol{s}, d$
  Policy and value function
    Policy function: $x0_{k,j}(\boldsymbol{s},d) = \zeta(\boldsymbol{s},d,j)\cdot x1_{c,j}(\boldsymbol{s},d)\ \forall\ \boldsymbol{s}, d, j$
    $x1_{k,j}(\boldsymbol{s},d) = x0_{k,j}(\boldsymbol{s},d)$
    $x1_{d,j}(\boldsymbol{s},d) = x0_{d,j}(\boldsymbol{s},d) = x0_{c,j}(\boldsymbol{s},d)$
    Value function: $V0_{k,j}(\boldsymbol{s},d) = \zeta(\boldsymbol{s},d,j)\cdot V1_{c,j}(\boldsymbol{s},d)\ \forall\ \boldsymbol{s}, d, j$
    $V1_{k,j}(\boldsymbol{s},d) = V0_{k,j}(\boldsymbol{s},d)$
    $V1_{d,j}(\boldsymbol{s},d) = V0_{d,j}(\boldsymbol{s},d) = V0_{c,j}(\boldsymbol{s},d)$
    With values of $\zeta(\boldsymbol{s},d,j)$ being drawn pseudo-randomly from a uniform distribution $\in [0.5,1.5]$
  Program controls
    $tolV = tolV\text{-}1 = tolx = tolx\text{-}1 = 20$
    $\lambda = 1$
Optimization
  while $(tolV > tol) \wedge (tolx > tol)$
    Price war assessment
      If for some combination of $\boldsymbol{s}$ and d deviation is profitable for at least one firm, set $\boldsymbol{p}_k(\boldsymbol{s},d)=\boldsymbol{p}_c(\boldsymbol{s},d)$, $\boldsymbol{q}_k(\boldsymbol{s},d)=\boldsymbol{q}_c(\boldsymbol{s},d)$, $\boldsymbol{\pi}_k(\boldsymbol{s},d)=\boldsymbol{\pi}_c(\boldsymbol{s},d)$
      Otherwise set $\boldsymbol{p}_k(\boldsymbol{s},d)=\boldsymbol{p}_{NBS}(\boldsymbol{s},d)$, $\boldsymbol{q}_k(\boldsymbol{s},d)=\boldsymbol{q}_{NBS}(\boldsymbol{s},d)$, $\boldsymbol{\pi}_k(\boldsymbol{s},d)=\boldsymbol{\pi}_{NBS}(\boldsymbol{s},d)$
    Use $\boldsymbol{x0}_k$ and $\boldsymbol{V0}_k$ (Gauss-Jacobi scheme) to calculate optimal collusive policy function $\boldsymbol{x1}_k$
    Use $\boldsymbol{x0}_d$ and $\boldsymbol{V0}_d$ (Gauss-Jacobi scheme) to calculate optimal deviant policy function $\boldsymbol{x1}_d$
    Update value function $\boldsymbol{V1}_k$ with values of $\boldsymbol{x1}_k$ and perform a policy iteration on $\boldsymbol{V1}_k$
    Update value function $\boldsymbol{V1}_d$ with values of $\boldsymbol{x1}_d$ and perform a policy iteration on $\boldsymbol{V1}_d$
    Update distance measures
      $tolV\text{-}1 = tolV$
      $tolx\text{-}1 = tolx$
      $tolV = \max \left| (\boldsymbol{V1}_k - \boldsymbol{V0}_k)/(1+|\boldsymbol{V0}_k|) \right|$
      $tolx = \max \left| (\boldsymbol{x1}_k - \boldsymbol{x0}_k)/(1+|\boldsymbol{x0}_k|) \right|$
    Determine dampening factor λ
      if $(tolV > tolV\text{-}1) \vee (tolx > tolx\text{-}1)$, draw λ pseudo-randomly from a uniform distribution $\in [0,1]$
      else, set $\lambda = 1$
    Update value and policy function with dampening
      $\boldsymbol{V0}_k = \lambda\cdot \boldsymbol{V1}_k + (1-\lambda)\cdot \boldsymbol{V0}_k \wedge \boldsymbol{x0}_k = \lambda\cdot \boldsymbol{x1}_k + (1-\lambda)\cdot \boldsymbol{x0}_k$
      $\boldsymbol{V0}_d = \lambda\cdot \boldsymbol{V1}_d + (1-\lambda)\cdot \boldsymbol{V0}_d \wedge \boldsymbol{x0}_d = \lambda\cdot \boldsymbol{x1}_d + (1-\lambda)\cdot \boldsymbol{x0}_d$
  end
  Return $\boldsymbol{V1}_k$, $\boldsymbol{x1}_k$, $\boldsymbol{V1}_d$ and $\boldsymbol{x1}_d$

The numeric and to some extent stochastic nature of our algorithm causes some slight variation in the policy and value functions that are obtained in different runs of the algorithm. As a consequence, one requires a method for distinguishing this *normal* variation attributable to the numeric nature of the algorithm from the variation caused by the existence of multiple equilibria. To make this distinction we compute the sup norm distance measure

$$tolx_{u,y} = \max_{s,d} \left| \frac{x_u(\boldsymbol{s},d) - x_y(\boldsymbol{s},d)}{1 + |x_u(\boldsymbol{s},d)|} \right| \tag{21}$$

for every pair of policy functions (indexed by u and y) that are obtained in all runs of the algorithm for the same set of parameter values. An analogous measure $tolV_{u,y}$ is calculated for the value function.

(a) We conclude that two solutions are (imperfect) representations of the same equilibrium if the values of the distance measures $tolx_{u,y}$ and $tolV_{u,y}$ are sufficiently small, i.e. differences in the values of the policy and value functions are attributed to the numeric nature of the search. We use the threshold values $tolx_{u,y}$≤5e-5 and $tolV_{u,y}$≤5e-6 with our stopping criterion in the optimization being tol=5e-8.

(b) When the distance measures are above the thresholds $tolx_{u,y}$≤5e-5 and $tolV_{u,y}$≤5e-6 we conclude that the respective equilibria are distinct.

For each of the parameter combinations evaluated in sections 3.3 to 3.5 there exists only one type of competitive equilibrium. This is checked by restarting the algorithm 100 times for every combination of parameter values. The maximum tolerances between these solutions obtained in different restarts for the same set of parameters are as follows.

	max($tolx_{u,y}$)	max($tolV_{u,y}$)
type A (section 3.3, δ=0.08, r=0.08, ρ=0.5)	6e-7	1e-7
type S (section 3.4, δ=0.02, r=0.09, ρ=0.5)	6e-6	1e-6
type P (section 3.5, δ=0.08, r=0.02, ρ=0.5)	7e-7	9e-7

We do not find evidence of a further collusive equilibrium than the one presented in subsection 3.3 for δ=0.08, r=0.08, ρ=0.5. This is also true for ρ=0.7 and ρ=0.9. Moreover, we do not find evidence of further collusive equilibria than the ones of type kA and kB presented in subsection 3.4 for δ=0.02, r=0.09, ρ=0.5. This is because the maximum distances of policy or value functions of the same type are very small. The two equilibria are distinct from each other because distances between equilibria of different types are no smaller than $tolV_{u,y}$=0.027 and $tolx_{u,y}$=0.24. We find the following distances between the collusive value or policy functions obtained in the 100 different restarts of the algorithm.

	max($tolx_{u,y}$)	max($tolV_{u,y}$)	
section 3.3, δ=0.08, r=0.08, ρ=0.5	3e-7	1e-6	
section 3.4, δ=0.02, r=0.09, ρ=0.5	2e-6	5e-7	eq. of same type A or B

Appendix B The Effects of Demand Persistence on Competitive and Collusive Equilibria

We have calculated the competitive and collusive policy and value functions of the firms for the parameter combinations $r \in \{0.01, 0.02, ..., 0.1\}$, $\delta \in \{0.01, 0.02, ..., 0.1\}$, $\rho \in \{0.5, 0.7, 0.9\}$, and $\eta \in \{2.5, 10, 40\}$ but only present results for ρ=0.5 in section 3. Here, we show that the presented results are robust to changes in ρ. Tables 4 and 5 correspond to Table 3 in the main text and show the distribution of competitive equilibrium types across the parameter space spanned by δ and r when assuming ρ=0.7 or ρ=0.9 together with η=10. The existence of multiple equilibria typically deteriorates the convergence properties of the algorithm and may result in lengthy computation times. Therefore, for ρ=0.7 and ρ=0.9 we do not analyze parameter combinations with $\delta<0.05 \wedge r<0.06$, where multiple equilibria are most likely. In order to detect multiple equilibria, we run the algorithm 10 times (20 times in case of ρ=0.5) on every combination of parameters. Analyzing Tables 4 and 5 yields the following findings:

1. Changes in the persistence of demand only have a small impact on the distribution of equilibrium types across the parameter space spanned by δ and r. One merely observes a faint effect that type A equilibria somewhat spread out into the regions of type P and type S equilibria.

2. A higher persistence of demand ρ is found to result in a greater number of equilibria (typically of the same type) for some combinations of parameter values. For example, for δ=0.08, r=0.08 we find one competitive equilibrium in the case of ρ=0.5 and for ρ=0.7 and two competitive equilibria for ρ=0.9 that are denoted A.1 and A.2. The longer time spent in every demand state allows the firms to choose from a greater variety of similar policies. The competitive limiting distributions of these equilibria are shown in Figure 10.

3. Figure 10 shows that all four competitive equilibria are very similar. Higher persistence of demand only somewhat alters the characteristics of the equilibrium. The longer time, which a firm expects to stay in any demand state, makes the firms adjust their capacities more closely especially to states of low demand. However, these differences are minor. The collusive equilibria share an even greater degree of similarity. (No graph is provided for this case to keep the presentation concise.) Therefore, the function of the incremental value to collude is very similar for the evaluated values of ρ.

We conclude that the level of demand persistence has a surprisingly small effect on the strategic patterns in our model. The most visible effect of more persistent demand is the emergence of a greater number of equilibria.

$\delta - r$	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.01						A S	A S	A S	S	S
0.02						A S	A S	A S	S	S
0.03						A	A S	A S	A S	S
0.04						A	A	A	A S	S
0.05	P	P	P	P.1 P.2	P/A	A	A	A	A	A S
0.06	P	P	P	P	P.1 P.2	A	A	A	A	A
0.07	P	P	P	P.1 P.2	P.1 P.2 A	A	A	A	A	A
0.08	P	P	P	P	P	P/A, P/S	A	A	A	A
0.09	P	P	P	P	P	P	P/A	A	A	A
0.1	P	P	P	P	P	P	P/A	A	A	A

Table 4: Distribution of competitive equilibrium types (ρ=0.7)

$\delta - r$	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.01						A S	A S	A S	A S	S.1 S.2
0.02						A S	A S	A S	A S	S.1 S.2
0.03						A	A	A S	A S	A S
0.04						A.1 A.2	A	A	A S	A S
0.05	P.1 P.2	P	P	P	P/A.1 P/A.2 A	A	A	A	A	A
0.06	P	P	P	P	P	A	A.1 A.2	A	A	A
0.07	P	P	P	P	P	A	A.1 A.2	A	A	A
0.08	P	P	P	P	P	P A	A	A.1 A.2	A	A
0.09	P.1 P.2	P	P	P	P	P A	P/A	A.1 A.2	A	A
0.1	P.1 P.2	P	P	P	P	P	P/A	A	A	A

Table 5: Distribution of competitive equilibrium types (ρ=0.9)

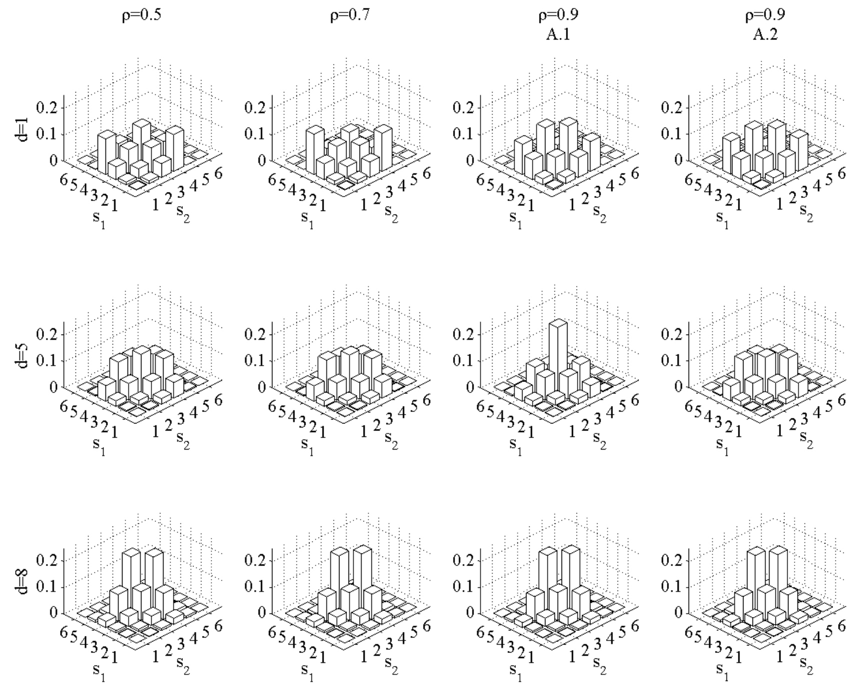

Figure 10: Competitive limiting distributions with δ=0.08, r=0.08, η=10

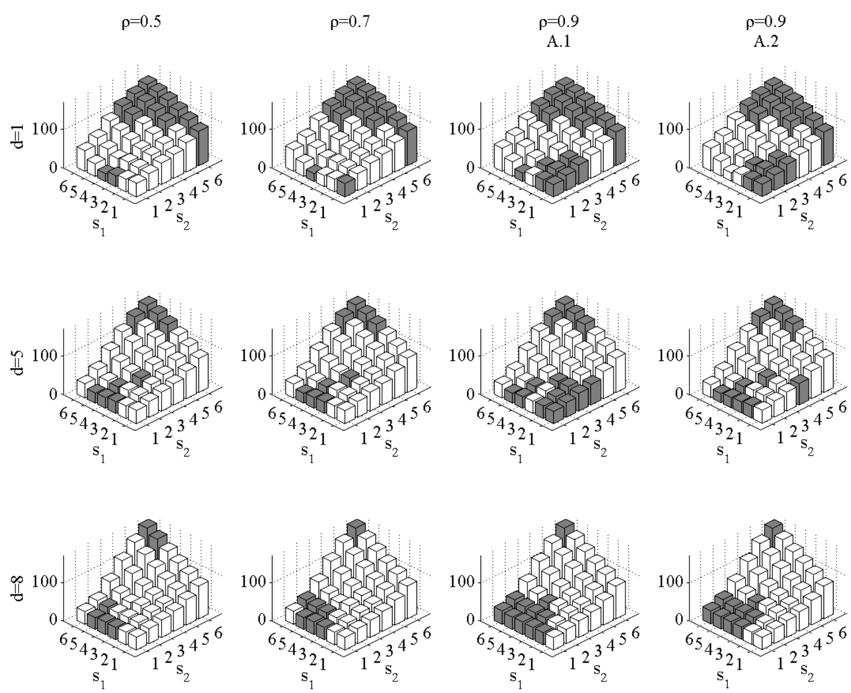

Figure 11: Incremental value to collude with δ=0.08, r=0.08, η=10