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Abstract: *Transition from one economic equilibrium to another as a consequence of shocks is often associated with sunk adjustment costs. Firm specific sunk market entry investments (or sunk market exit costs) in case of a reaction to price shocks are an example. These adjustment costs lead to a dynamic supply pattern similar to hysteresis. In analogy to "hysteresis losses" in ferromagnetism, we explicitly model dynamic adjustment losses in the course of market entry and exit cycles. We start from the micro level of a single firm and use explicit aggregation tools from hysteresis theory in mathematics and physics to calculate dynamic losses. We show that strong market fluctuations generate disproportionately large hysteresis losses for producers. This could give a reason for the implementation of stabilizing measures and policies to prevent strong (price) variations or, alternatively, to reduce the sunk entry and exit costs. However, the explicit inclusion of uncertainty (associated with an option value of waiting) is shown to reduce economic hysteresis losses.*

JEL-* Classifications*: *B59, C61, D21, D69*

Keywords: sunk-cost hysteresis, adjustment costs, dynamic losses, path-dependence, persistence, option value of waiting

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1 Introduction and a brief literature overview

In many cases firms must incur sunk costs to enter new markets (e.g., for gathering information on the new market and for market research, setting up distribution and service networks, for advertising or establishing a brand name, or for hiring new workers to start the production, etc.). Since these entry investments are firm specific, the firms cannot recoup these costs if they exit. Analogously, a market exit results in exit costs, e.g. for severance payments for fired employees if production is stopped. These sunk entry and exit costs result in a path-dependent behavior of firms, which is called “hysteresis” (see Baldwin, 1989, Baldwin, Krugman, 1989, Dixit, 1989).[1]

Directly after a firm has entered a market, firm-specific entry investments in fact have to be treated as sunk costs, but this investment is not really lost – as long as the firm continues to be active on the market. However, in case of a later market exit, the sunk entry investment actually has to be written off; and sunk exit costs must be paid. In a complete market entry and exit cycle, both sunk entry and exit costs finally have to be written off. As a consequence, during the complete entry and exit cycle of a firm, a dynamic loss is generated consisting of the sum of sunk entry and exit costs that were paid. This is analogous to a phenomenon in physics called the “*hysteresis loss*”, where heat is produced by magnetization-demagnetization cycles. In contrast to the standard microeconomic market model, where welfare effects (producer and consumer surpluses, deadweight losses, etc.) are analyzed for static market equilibrium situations, our paper deals with theoretical issues of *dynamic losses* directly caused by *variations in the economic environment* during the adjustment process towards equilibria, or by fluctuations around and switches between different equilibria. We model this along the lines of the hysteresis loss in magnetization; and we will show that – as in the case of magnetism – these losses caused by a “loading-unloading” (i.e., market entry-exit) cycle are proportional to the area inside the so-called “hysteresis loop” (see Mayergoyz, 2003, pp. 50).

In our paper we analyze the dynamic hysteresis losses in a systematic way – starting from the microeconomic level of a single firm and explicitly modelling the aggregation to hysteresis

[1] Hysteresis originally stems from physics (ferromagnetism, plasticity, etc.) and also occurs in several phenomena in chemistry, biology, engineering (see Visintin, 2006, p. 3), and in economics, especially in international trade and unemployment (see Göcke, 2001). Amable et al. (1992), Cross (1993), Göcke (2002) and Cross et al. (2009, 2010) provide an overview of hysteresis in economics. The term hysteresis is derived from a Greek verb „hysteros“ meaning “lagging behind” or “that which comes later” (Cross, 1993, p. 53) and describes an effect that persists after the cause that brought it about has been removed.

effects on entire markets, using explicit tools from mathematical/physical hysteresis theory – which is novel in economics.

To shift the focus from welfare effects in an equilibrium state (as it is done, e.g., in the case of deadweight losses) to welfare effects of fluctuations around equilibria, given that adjustment costs are relevant, is a promising topic. We found out that large economic fluctuations are generating disproportionately high dynamic adjustment costs, due to (1) the cubic effect of price changes on the size of the dynamic losses, and since (2) “wait-and-see” filters away reactions on small stochastic price changes. Especially the strong economic fluctuations of the last years (in exchange rates, share and real estate prices, commodity and oil prices, etc.) should have led to a dissipation effect for many sunk investments (that had to be written off in the end), which are likely to show similarities to the hysteresis loss described in our paper. From an economic policy point of view, this could give an additional reason for the implementation of stabilizing measures and policies, first and foremost, to prevent strong variations on markets. Examples of such stabilizing (dynamic loss-avoiding) policies could be (stable) fixed exchange rates, financial market regulations, or even two-price buffer stock schemes on commodity markets. However, an alternative policy to reduce the hysteresis losses would be to preserve flexible markets, i.e., to reduce the sunk costs that act like barriers for market entry and exit.

The relevance of our analysis can be illustrated for agricultural and commodity markets, as an example: agricultural markets exhibit a relatively high volatility due to their strong links with natural shocks, associated with high costs for the economy. A number of studies regarding the development of commodity prices and their volatility state that the price volatility in the last decade was higher than in the nineties (e.g., see Huchet-Bourdon, 2011, von Braun and Tadesse, 2012, FAO, 2011, OECD-FAO, 2014, World Bank, 2015). Existing literature regarding the welfare impacts of commodity price volatility mostly deals with static welfare losses in terms of consumer income changes and concentrates on the demand side (see e.g., Bellemare et al., 2013). In contrast, we analyze dynamic losses of producers (farmers) caused by sunk adjustment costs. The fact that the (food) price volatility in the past was, and tends to stay high in the future underlines the relevance of the sunk adjustment costs in form of investments or disinvestments that producers have to face after every price shock. The markets with high sunk costs are those with the greatest barriers to entry and exit, while once the sunk costs are incurred, they cannot be recovered. Together with the presence of uncertainty the existence of sunk costs significantly changes the “normal” economic behavior of producers, resulting in hysteresis. Since hysteresis effects are an empirically proven phenomenon in economics (see e.g., Belke and Göcke, 2001, Kannebley, 2007, Mota and Vasconcelos, 2012, Mota et al., 2012, Piscitelli et al., 1999, 2000, Piscitelli and Hallett, 2002, Prince and Kannebley, 2013, Belke et al. 2013, 2014), the consequences in terms of economic hysteresis losses resulting from fluctuations are a relevant question.

This paper is organized as follows: In subsection 2.1, sunk cost hysteresis is introduced and illustrated by a discrete one-period model on the micro level of a single firm. In subsection 2.2, based on the firm-level model the hysteresis loss resulting from price variations is considered. In subsection 2.3, the micro model is extended to a multi-period optimization, leading to a modification of the entry and exit trigger prices, and thus, to a modification in the calculation of the hysteresis loss. In subsection 2.4, based on the micro model, the adequate aggregation procedure to a hysteresis loop on the macro-/market level is briefly described. In the course of aggregation the path-dependent characteristics and the resulting hysteresis loop change their pattern. Section 2.5 deals with the graphical illustration and calculation of the hysteresis losses on the aggregate level. Based on the aggregate loop, the hysteresis losses of an entire market resulting from variations in the price level are illustrated. Subsections 3.1 to 3.4 introduce uncertainty (and option value effects) on the micro and the aggregate macro levels, calculate the hysteresis losses corrected for uncertainty effects and illustrate these losses graphically. The findings in the subsections 2.5 and 3.4 are completely novel in economics. Section 4 concludes.

2 Hysteresis losses under certainty

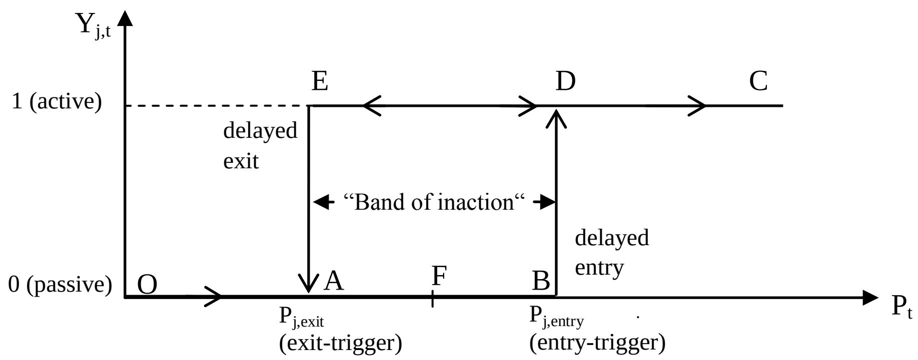
2.1. Sunk costs and hysteresis on a firm level

2.1.1. Hysteresis band and the non-ideal-relay

First, the simplest form of hysteresis called “non-ideal-relay” (Krasnosel’skii and Pokrovskii, 1989, p. 263) is considered. In international trade this hysteresis phenomenon occurs due to sunk entry and exit costs (see Baldwin 1989, 1990), which induce a “band of inaction” related to changes in the economic environment. A firm observes the development of a forcing variable (e.g., the price level, or in the special case of international trade the exchange rate) and does not change its economic behavior – i.e., its state of market (in)activity – until the price (or exchange rate) changes significantly and passes certain trigger values specific to each (heterogeneous) firm. In other words, a firm delays its entry and exit due to sunk entry and exit costs. Moreover, once the economic behavior of firm j has changed (due to large *past* price changes), it will not completely go back to the initial state, even if the forcing variable (price) returns to its initial level (see Göcke, 2002, p. 168). This kind of after effect is called the remanence property (see Göcke, 2002, p. 171). It plays an important role in our subsequent analysis of the dynamic losses associated with sunk adjustment costs.

Fig. 1 illustrates the decision process of a one-product firm j which can be described as a “non-ideal relay” (see Belke, Göcke, 1999, p. 266). The ordinate in Fig. 1 captures the state of activity of firm j or its supply in current period t. The abscissa reflects the net price received by firms on the market for one unit of the sold good. Depending on the size of sunk costs, the threshold value for an exit is on the left-hand side and the threshold value for an entry is on the right-hand side of the variable/unit costs (see point F in Fig. 1). There are also two potential equilibria between the exit/entry-thresholds. The currently valid equilibrium can only be determined if the state of activity of a previous period is known. Between both triggers there is a “band of inaction”, since only a move outside this band, passing one of the triggers, will result in a switch in the state of activity. Thus, if the market price varies within the hysteresis band, firm j remains in its state, which can either be active or passive.

Fig. 1: Non-ideal-relay with one-period optimization under certainty



Thus, the economic behavior of a price taker in period *t* depends not only on the price development, but also on the previous state of its activity. There are two states of activity a firm may have had in the previous period (*t-1*), namely passive or active. A passive firm does not produce any goods, so its production level is zero ($y_{j,t-1}=0$). An active firm produces one unit of goods ($y_{j,t-1}=1$). If a firm was passive, it can choose to stay passive or to enter the market in a current period and become active. The firm stays passive as the market price varies in the interval between zero and the entry-trigger. Thus, the supply curve of a previously passive firm j corresponds to the line OAB in Fig. 1. However, if the price rises further so that the entry-trigger value is passed - the firm enters the market (paying sunk entry costs) and is active. After the entry, the supply of firm j corresponds to the line EDC (see in Fig. 1) and stays on the same level as long as the price varies between the exit trigger and infinity. In this case, the firm does not change its behavior and continues producing. However, as later on the price falls below the exit trigger, the firm shuts down (pays exit costs) becoming passive again. The cycle ABDEA represents a complete hysteresis loop which can also be described as a switch between different (path-dependent) supply curves of the firm. In the next section this adjustment pattern is modeled explicitly for a simple case of a one-period optimization of a single firm.

2.1.2. Sunk costs in a discrete one-period model

A one-period model, with the firm's time horizon reduced to the current period, is the simplest microeconomic perspective on the hysteresis phenomenon. It is used in order to formalize and illustrate a firm's economic behavior taking the changes of a forcing variable into account. The forcing variable in this model is the price. The drivers of food price changes are different supply and demand side shocks, speculation in commodity prices, exchange rate changes or even tariff and non-tariff barriers. A firm is able to produce and sell just one product (i.e., it is a single-product firm) and the sales volume always equals one unit (single-unit). A real firm producing several units can be seen as fictitiously disaggregated into single units. Each of them is characterised by a non-ideal relay, if sunk costs are relevant to changing the activity state of the respective unit. Entering and exiting the market is associated with the *sunk entry* (k_j) and *exit* (l_j) *costs*. This induces a difference between the entry and exit threshold prices (P_{entry} and P_{exit}, respectively). Depending on both, the previous and the current state of activity, the following unit cost function of firm j is assumed for the current period t (Göcke, 2002, p. 170):

$$K_{j,t} = \begin{cases} c_j, & if\ \ y_{j,t} = y_{j,t-1} \quad = 1 \\ c_j + k_j, & if\ \ y_{j,t} = 1, y_{j,t-1} = 0 \\ l_j, & if\ \ y_{j,t} = 0, y_{j,t-1} = 1 \\ 0, & if\ \ y_{j,t} = y_{j,t-1} \quad = 0, \quad with\ \ c_j, k_j, l_j > 0 \end{cases} \tag{1}$$

c_j represents the unit variable costs of firm j, $y_{j,t}$ is the production and sales volume of firm j within the current period and $y_{j,t-1}$ is a measure for the sales in the previous time period. In case of activity of firm j the output is normalized to 1. k_j quantifies the sunk entry costs which must be paid for an increase in sales by this additional unit of production. l_j denotes sunk exit costs that must be paid if sales are reduced by this unit. Consequently, the following output function of firm j can be written depending on the previous level of production and the current price level (p_t):

$$y_{j,t} = \begin{cases} 1, & if \ \ y_{j,t-1} = 1,\ p_t \geq c_j - l_j \\ 1, & if \ \ y_{j,t-1} = 0,\ p_t \geq c_j + k_j \\ 0, & if \ \ y_{j,t-1} = 1,\ p_t \leq c_j - l_j \\ 0, & if \ \ y_{j,t-1} = 0,\ p_t \leq c_j + k_j \end{cases} \tag{2}$$

If firm j was not selling in the previous period and stays inactive in the current period, there are no costs (fourth line in eq. (1)). As eq. (2) states, in this case the firm j is not selling as long as the price doesn't exceed the threshold for an entry (P_{entry}), which is the sum of the variable and the sunk entry costs (fourth line in eq. (2)). If however, firm j encounters a price change stimulating an entry into the market in period t (second line in eq. (2)), it has to pay sunk entry costs and the variable costs (as described by the second line of eq. (1)). After the entry, it will not exit immediately if the price falls below the entry trigger value. Firm j keeps selling unless the price falls below its exit threshold value (P_{exit}), which is the difference between variable and sunk exit costs. Firm j pays variable costs c_j if it remains active in period t. If the price level decreases below the exit threshold, the third line of eq. (2) becomes valid, and firm j leaves the market, paying only the sunk exit costs (third line in eq. (1)).

Thus, firm j has two threshold prices. Market entry results for $p > P_{j,entry} = c_j + k_j$, and the firm exits the market, if $p < P_{j,exit} = c_j - l_j$. Every heterogeneous firm with its specific cost structure can be characterized by a combination of its individual thresholds (prices) inducing market entry and/or exit. The distance between the entry and exit threshold values equals the sum of the sunk entry and exit costs ($k_j + l_j$). This zone is called the "band of inaction" and is the wider, the higher the sunk costs are.

The non-ideal-relay model illustrated in Fig. 1 allows for an analysis of only one element of an aggregate economic system, which consists of a multitude of heterogeneous units sold on the entire aggregate market. Below we will present an adequate aggregation procedure in order to derive path-dependent behavior (and adjustment/hysteresis losses) of an entire market.

2.2. Hysteresis loss in a discrete one-period model

The non-ideal relay model (see Fig. 1) is now applied in order to determine the loss caused by price dynamics that changes the activity state of a firm. This loss arises due to the sunk costs a firm has to bear if it changes its production volume. The entry investment (from a previous period) cannot be used further in case of an exit and has to be written off. Additionally, sunk exit costs have to be paid. Thus, the hysteresis loss induced by a complete entry-exit cycle becomes effective with the market exit. Therefore, starting from a low price level, we need both to complete a full loading-unloading cycle: a significant price increase inducing the firm's entry into the market and a subsequent drastic price fall passing the exit threshold; i.e., a closed hysteresis loop, such as ABDEA in Fig. 1, is required for a hysteresis loss to occur.

The assumption of a single-product and single-unit firm still holds. Therefore, after every change of activity of firm j an output change of one unit is observed ($|\Delta y_j| = 1$). The area enclosed by the hysteresis loop ABDEA can be quantified as follows:

$$\left[c_j + k_j - \left(c_j - l_j\right)\right] \cdot |\Delta y_j| = k_j + l_j \tag{3}$$

According to eq. (3) the area within the hysteresis loop in the non-ideal-relay model equals the sum of the sunk entry and exit investments. Thus, the geometric area enclosed by the hysteresis loop, i.e., the area between the triggers in the non-ideal relay, represents the dynamic loss of a complete entry-exit cycle in a one-period model under certainty.

2.3. Hysteresis loss in the discrete multi-period model

Following Belke and Göcke (1999), the threshold values that stimulate expansion or reduction in production are now calculated based on a multi-period optimization. The non-ideal-relay model presented in the previous sections is used again. The present value ($V_{j,t}$) from the activity of a firm j in the current period and expected for the infinite future is now a sum of two components – the profit in period t ($R_{j,t}$) and the present value of annuity due to future revenues, whereby the latter is discounted by the factor $\delta = \frac{1}{1+i}$ with an interest rate i (Belke, Göcke, 1999, pp. 265):

$$V_{j,t} = R_{j,t} + \frac{1}{1+i} \cdot V_{j,t+1} \tag{4}$$

The operating profit in the current period ($R_{j,t}$) can be calculated as the difference between income and operating costs (see eq. (5)). The latter depends on the sales volume of firm j, which can either be higher (lower) than in previous period $(t-1)$ and include the sunk entry (exit) costs in addition to the unit variable costs, or it can be unchanged compared to the past production and contain no sunk adjustment costs. Given that firm j can either produce one unit or be inactive, the following current profit results in:

$$R_{j,t} = \begin{cases} p_t - c_j - k_j, \; if \; y_{j,t} = 1 > \; y_{j,t-1} = 0 \\ p_t - c_j, \; if \; y_{j,t} \quad = \; y_{j,t-1} = 1 \\ -l_j, \; if \; y_{j,t} = 0 < \; y_{j,t-1} = 1 \end{cases} \tag{5}$$

We assume that firm j expects the same price as in the current period for the entire infinite future, so the time indices for price and operating costs can be omitted. Thus, the present value of future profits can be calculated as follows:

$$\delta \cdot V_{j,t+1} = \begin{cases} \frac{\delta \cdot (p-c_j)}{1-\delta}, \; if \; y_{j,t-1} < y_{j,t} = \; y_{j,t+1}, \; with \; \delta = \frac{1}{1+i} \\ 0, \; if \; y_{j,t} \quad < y_{j,t-1} \end{cases} \tag{6}$$

Eq. (6) captures two scenarios – [1] entry in *t*, followed by a positive (expected) present value of future profits, which is the discounted value of operating profit as an annuity, or [2] exit in t followed by zero future profits. Therefore, in case of an entry in t the benefit of firm j is the sum of the operating profit and the present value of future profits in $t+1$ (see first line in eq. (7)). The operating profit of a firm entering the market is defined in the first line of eq. (5). If a previously active firm j stays active in t and in $t+1$, it does not have to pay sunk adjustment costs in any period and the annual profit of firm j is defined in the second line of eq. (7). If firm j exits the market in t, it has neither an operating nor any future profits. However, it experiences sunk exit costs (see the last line in eq. (7)).

$$V_{j,t} = R_{j,t} + \delta V_{j,t+1} = \begin{cases} p - c_j - k_j + \frac{\delta(p-c_j)}{1-\delta} = \frac{p-c_j-(1-\delta)k_j}{1-\delta}, \; if \; y_{j,t-1} < y_{j,t} = \; y_{j,t+1} \\ p - c_j + \frac{\delta(p-c_j)}{1-\delta} = \frac{p-c_j}{1-\delta}, \; if \; y_{j,t-1} = y_{j,t} = \; y_{j,t+1} \\ -l_j, \; if \; y_{j,t-1} > \; y_{j,t} \end{cases} \tag{7}$$

Firm j enters the market or expands in production at time t (first case of eq. (7)) if the present value from activity ($V_{j,t}$) by entry in *t* is positive:

$$\frac{p-c_j-(1-\delta)k_j}{1-\delta} > 0 \quad \Leftrightarrow \quad P_{entry} = c_j + (1-\delta)k_j \tag{8}$$

Thus, on the one hand, for a previously inactive firm a price above P_{entry} (see eq. (8)) stimulates expansion in production at time t. On the other hand, firm j leaves the market if its loss from continuing production exceeds the loss from shutting down. Therefore, the exit threshold is calculated by:

$$\frac{p-c_j}{1-\delta} < -l_j \quad \Leftrightarrow \quad P_{exit} = c_j - (1-\delta)\cdot l_j \tag{9}$$

If the price falls below P_{exit} (see eq. (9)), firm j reduces its production.

If we look back at Fig. 1 and substitute the trigger values from the one-period model by the trigger values defined in eqs. (8) and (9), a modified hysteresis band results. In the one-period model it was shown that the hysteresis loss equals the geometric area inside the non-ideal relay loop between the triggers. This area can be computed as follows:

$$\big(1-\delta\big)\big(k_j+l_j\big)\cdot\big|\Delta y_j\big| = \big(1-\delta\big)\big(k_j+l_j\big) \tag{10}$$

This expression can be interpreted as the *interest costs* on the sunk entry investment and exit disinvestment. Compared to the sum of sunk costs $(k_j + l_j)$ as the actual dynamic loss if an entire entry and exit cycle is run through, the area inside the non-ideal relay loop now is not equivalent but only *proportional* to the hysteresis loss: the geometric area inside the loop has to be divided by $(1-\delta)\approx i$, or multiplied by $1/(1-\delta)\approx 1/i$. The multiplier typically has a value higher than *1*. Thus, due to multi-period optimization the hysteresis loss typically is larger than the area inside the hysteresis loop.

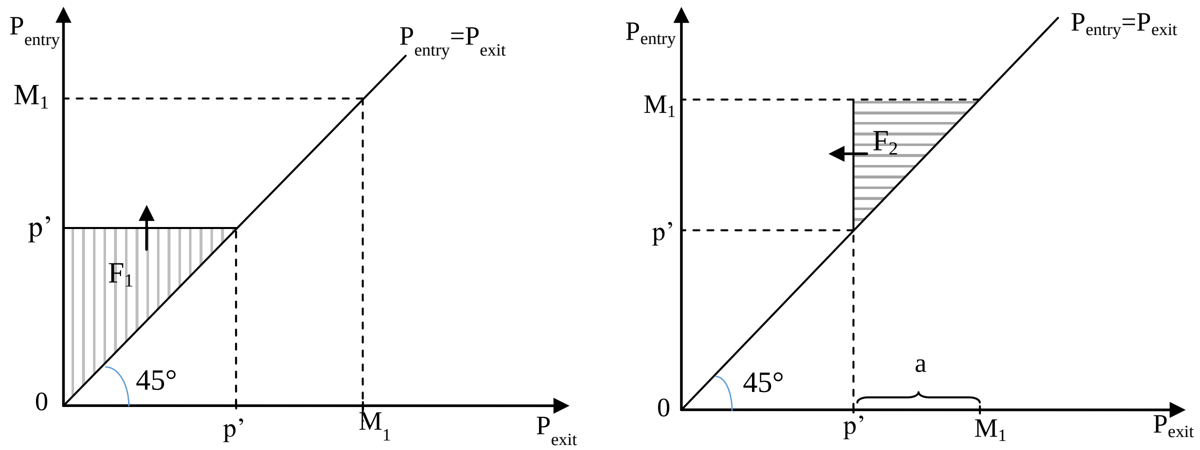
2.4. Aggregated hysteresis loop for heterogeneous firms

As stated above, every firm *j* has a specific cost structure which implicates heterogeneity in entry and exit thresholds (this means every firm has an individual non-ideal-relay operator $\hat{\gamma}_{j,P_{entry}P_{exit}}$). In the mathematical Preisach-Mayergoyz-Krasnosel'skii aggregation procedure (see Preisach, 1935, Cross, 1993, pp. 85, Mayergoyz, 2003, pp. 1, Mayergoyz, 2006, pp. 293) the non-ideal-relay is the elementary hysteresis operator (Mayergoyz, 1986, p.604). It illustrates a micro element of an aggregate macro system. Based on the Preisach-Mayergoyz-Krasnosel'skii procedure we aggregate the supply of heterogeneous firms or sum up firms entering and/or exiting the market following a certain price change.

Fig. 2 contains two entry-exit-trigger diagrams, each with the exit threshold price variable on the abscissa and the entry price trigger on the ordinate. Each heterogeneous unit/firm is represented by an individual $(P_{entry}; P_{exit})$-point in such a diagram. The 45°-line represents firms without any sunk adjustment costs, for which the entry trigger equals the exit threshold value ("non-hysteretic" firms, with $P_{entry} = P_{exit}$). The area above the 45°-line captures all the firms with sunk adjustment costs („hysteretic" firms) since for these firms the entry trigger price is higher than the exit trigger. The area below the 45°-line represents impossible combinations of the entry and exit threshold values, since $P_{exit} > P_{entry}$. The behavior of every hysteretic firm is assumed to be a non-ideal relay as illustrated in Fig. 1. Given that the output of every active firm is one, the aggregate output volume corresponds to the number of active firms on the market. Real firms, producing more than one unit, can be considered as artificially disaggregated into single unit firms, each represented by a point in the diagram. We assume that all the firms have the same time horizon and cannot re-enter or re-exit the market. As a simplification, a continuous uniform distribution[2] of firms in the upper triangle area of the (P_{entry}/P_{exit})-diagram is assumed, so that each geometrical area in the diagram is proportional to the number of represented firms.

Fig. 2: Cumulated output changes induced by a positive and a negative price shocks

a) a positive price shock ($0 \rightarrow M_1$) *b) a subsequent negative shock ($M_1 \rightarrow p'$)*



The left-hand diagram of Fig. 2 illustrates the cumulated output change induced by a positive price shock and the right-hand graph shows the effect of a subsequent negative shock. In case of a positive price change, starting at *0* and going up to a maximum $M_1(0 \rightarrow p' \rightarrow M_1)$, the triangle area F_1 is generated, representing the cumulated output increase or the number of

[2] The distribution of firms in entry-exit diagram determines the curvature of the aggregated hysteresis loop.

firms that have entered the market due to the favorable change of the forcing variable. The numeric value of the triangle area F_1 is:

$$F_1 = \rho \cdot \frac{1}{2} \cdot (p' - 0)^2 = \rho \cdot \frac{1}{2} \cdot p'^2 \qquad (11)$$

with ρ as a density parameter of the firm-distribution inside the area above the ($P_{entry} = P_{exit}$)-line and p as a price change if the price increase starts at *0*. M_1 is the maximum price where the initial price increase has turned around. The right-hand diagram of Fig. 2 captures the effect of a later price reduction which follows the price increase to M_1. Thus, the triangle area F_2 represents the cumulated output reduction or the multitude of firms that have exited the market after the subsequent price fall from M_1 to p' (with an absolute price change equal to a):

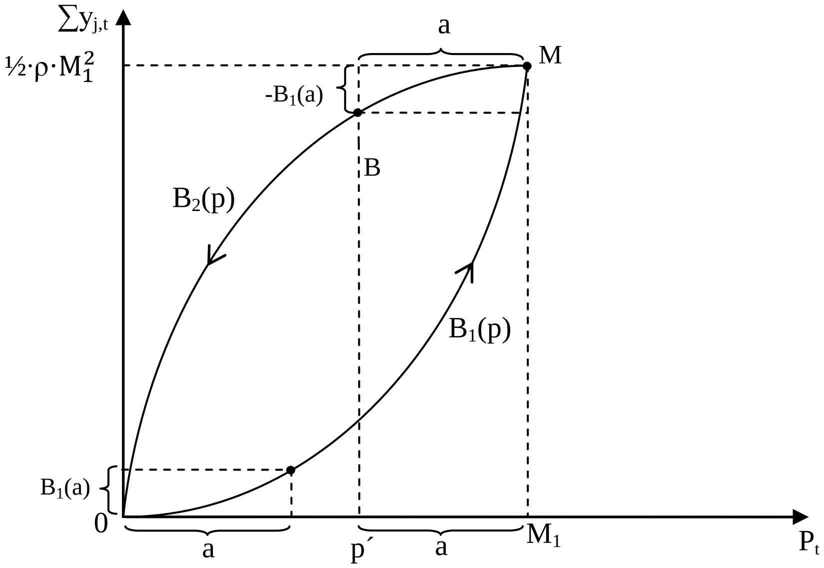
$$F_2 = \rho \cdot \frac{1}{2} \cdot (M_1 - p')^2 = \rho \cdot \frac{1}{2} \cdot a^2 \qquad (12)$$

Fig. 3 illustrates the aggregate output or the sum of active firms depending on price changes ($0 \rightarrow p \rightarrow M_1$ and $M_1 \rightarrow p \rightarrow 0$).[3] This constitutes a complete aggregate/macro hysteresis loop. The aggregate hysteresis loop has the form of a lens and consists of an upward ($B_1(p)$) and a downward leading ($B_2(p)$) branch. Each part of the macro hysteresis loop in Fig. 3 corresponds to an area in the (P_{entry}/P_{exit})-diagram in Fig. 2. The upward branch captures the quantitative effects of a positive price change, so every point on this line can be calculated by means of eq. (11). The downward branch captures the impact of the subsequent negative price change. Each point on this curve can be quantified via eq. (12). Given the uniform distribution of firms, this effect is again equivalent to a triangle area (as in the case of the initial increase), and the following relation results:

$$F_2 = B_2(M_1) - B_2(p') = B_1(a) \qquad (13)$$

Fig. 3: Macro hysteresis loop resulting from price change $M_1 \rightarrow 0 \rightarrow M_1$ or $0 \rightarrow M_1 \rightarrow 0$

[3] For a detailed aggregation procedure see Amable et al. (1991), Göcke (2002).



The upward branch captures the aggregate output change by increasing prices and equals the area F_1 from Fig. 2. Thus, the maximum of this upward branch (point *M*), resulting from the price increase from *0* to M_1, equals:

$$B_1(M_1) = \rho \cdot \frac{1}{2} \cdot (M_1 - 0)^2 = \rho \cdot \frac{1}{2} \cdot M_1^2 \quad (14)$$

If the price level later falls from M_1 to *p´*, the number of exit-firms or aggregate supply reduction equals the area F_2 in Fig. 2. The aggregate output changes as a result of a negative price shock are captured by the part of the downward branch, going down from point *M* to *B* (in Fig. 3). In point *B*, the following level of production results:

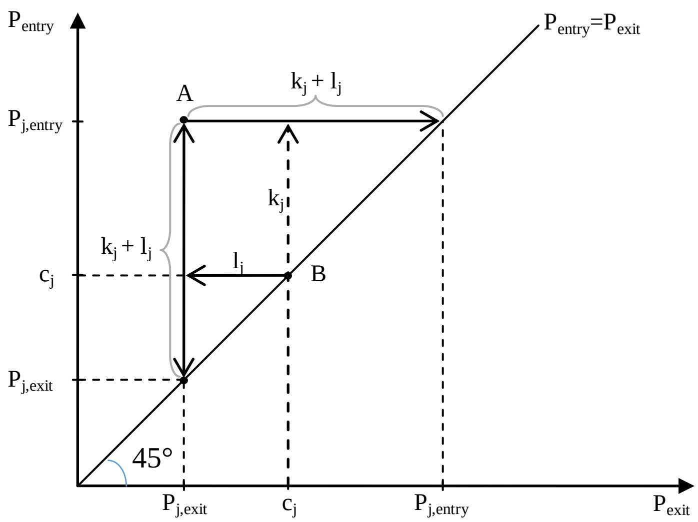
$$B_2(p') = B_1(M_1) - B_1(M_1 - p') = \rho \cdot \frac{1}{2} \cdot M_1^2 - \rho \cdot \frac{1}{2} \cdot a^2 \quad (15)$$

In sum, if the price rises from *0* to M_1, the aggregated output change is illustrated by the upward branch going from *0* to $B_1(M_1)$. The aggregate output reduction due to the subsequent price fall from M_1 to *p´* is captured by the part of the downward-leading branch falling from $B_1(M_1)$ to $B_2(p')$.

2.5. Aggregated hysteresis loss of heterogeneous firms in entry-exit cycles

The aim of this section, which is one of the most important contributions of this paper, is to cumulate the hysteresis losses (l_j+k_j) of all exiting firms $j=1,2,\ldots,N$ during a price cycle $0 \rightarrow (p > P_{entry}) \rightarrow (p < P_{exit})$. For reasons of simplicity the interpretation of the aggregation procedure is based on the one-period model introduced in section 2.1. The assumptions made in section 2 still hold.[4] As stated above, the sunk entry and exit costs are presumed to be written off after the market exit has come about. Thus, only the effect of a subsequent negative shock following a previous positive shock is relevant for calculating hysteresis losses based on the cumulating procedure. The dynamic loss in economics can be modeled in a similar way as the hysteresis loss in magnetization (Mayergoyz, 2003, pp. 50). The basis for this is the feasibility of a geometric interpretation of the sunk entry and exit costs as depicted in Fig. 4. Fig. 4 has a structure similar to Fig. 2: The ordinate captures the entry trigger and the abscissa the exit trigger. The 45°-line represents the “non-hysteretic“ firms while all „hysteretic“ firms are located in the triangle above the 45°-line. Point *A* illustrates the non-ideal relay of a particular hysteretic firm *j* with the sunk entry and exit costs (k_j and l_j respectively), the variable costs c_j and the resulting entry and exit price triggers $P_{j,entry}$ and $P_{j,exit}$. A (hypothetical) non-hysteretic firm with entry/exit trigger c_j is represented by point *B*, if it has the same variable costs as the hysteretic firm *j* but no sunk costs. The existence of the sunk entry costs (k_j) leads to the shift of a firm’s vertical position upwards by an extent of k_j and results in a higher entry trigger value ($P_{j,entry} > c_j$). This is illustrated by a dashed upward arrow starting at point *B*. Due to the sunk exit costs (l_j) the horizontal position of a sunk exit cost firm is shifted to the left by an extent of l_j. This is illustrated by the solid arrow starting at point *B* and pointing to the left. This shifts the hysteretic firm’s point A to a lower exit price trigger ($P_{j,exit} < c_j$).

Fig. 4: Sunk costs of firm j in the P_{entry}/P_{exit}-diagram



[4] We assume one-product and one-unit firms with the same time horizon and no possibility to re-enter or re-exit the market.

Thus, the inclusion of sunk (dis)investments allocates firm j from point B to A. The higher the sum of the sunk costs, the further from the 45°-line is a firm located and the larger is difference in its economic behavior in comparison to the “non-hysteretic” firms located on the 45°-line. Consequently, both the vertical and the horizontal distance between point *A* and the 45°-line equals the sum of sunk entry and exit investments. This distance can also be calculated as the difference between the entry and exit trigger of the firm. It is the hysteresis loss of this firm, if a price cycle leads to an entry and later on to an exit of this firm:

$$P_{j,entry} - P_{j,exit} = c_j + k_j - (c_j - l_j) = k_j + l_j \tag{16}$$

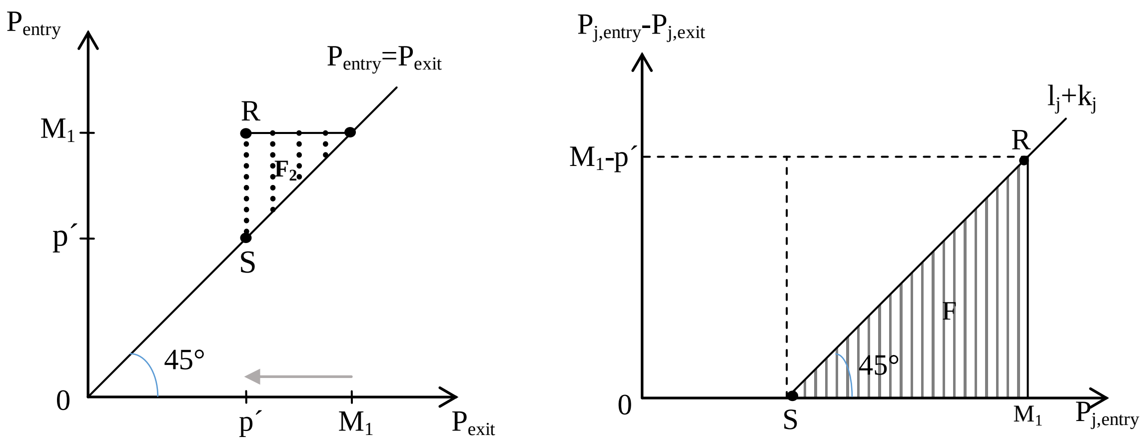
If, however, the multi-period model forms the basis of the aggregation, a proportional correction factor based on the interest cost rate $[1/(1-\delta)]$ has to be applied in order to determine the hysteresis losses (see eq. (10) in section 2.3) of exit-firms. Since we want to keep the aggregation process as simple as possible, the one-period micro model is used in the following to interpret the aggregation procedure results.

Fig. 5 consists of two graphs, where the right-hand diagram is used to interpret a part of the left-hand diagram. The (P_{entry}/P_{exit})-diagram illustrates firms exiting the market in consequence of the price reduction from M_1 to p' as a scatter plot. Every vertical line in the left graph in Fig. 5 (e.g., line RS) represents a continuum of firms with a different extent of hysteresis loss $(P_{entry} - P_{exit})$ and an exit trigger that is larger than p'. The right-hand graph in Fig. 5 captures the relation between the hysteresis loss and the entry trigger. Point *S* in this graph corresponds to point *S* in the left-hand diagram. The dashed area in the right-hand graph represents the cumulated hysteresis loss of all firms located on the line RS in the left-hand (P_{entry}/P_{exit})-diagram. The quantitative expression for the dashed area is:

$$F = \rho \cdot \int_{p'}^{M_1} p \, dp = \rho \cdot \frac{1}{2}(M_1 - p')^2 \tag{17}$$

Thus, in order to sum up the hysteresis loss of all heterogeneous firms on a certain vertical line in the (P_{entry}/P_{exit})-diagram, a “vertical integration” over P_{entry} must be executed.

Fig. 5: Cumulation of hysteresis losses in P_{entry}/P_{exit}-diagram after a price reduction $M_1 \rightarrow p$`



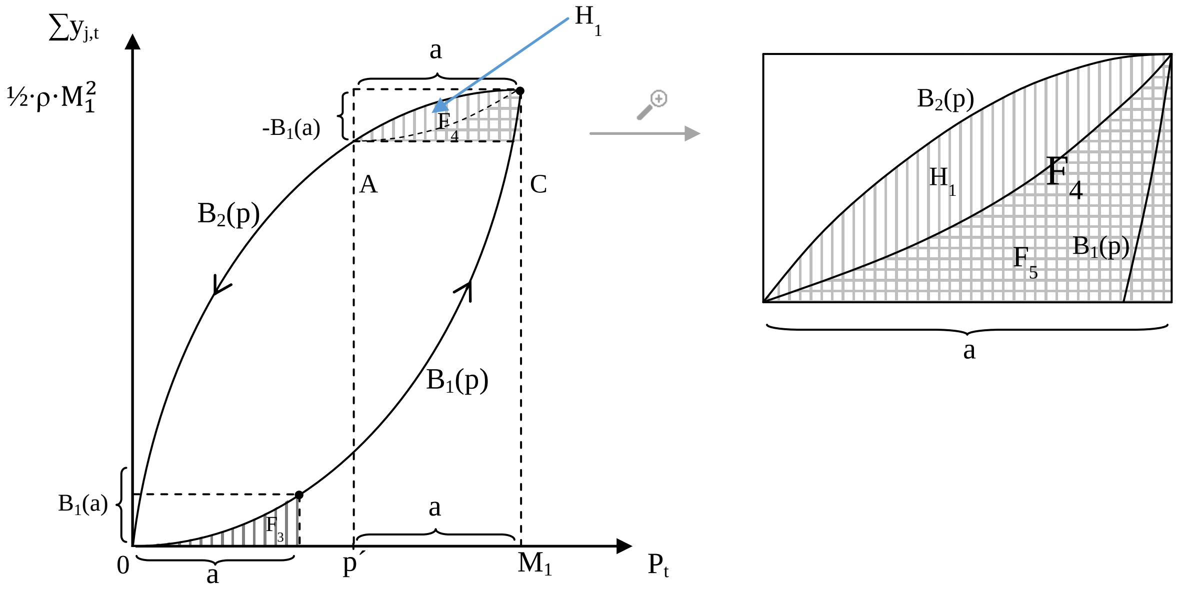
The cumulated hysteresis loss H of all firms located on any vertical line in the left-hand graph in Fig. 5 can be calculated using the same expression as in eq. (17) for different levels of p. In our example all the relevant p values are in the interval [p`; M_1] (see Fig. 5). Therefore, in order to calculate the hysteresis loss of all firms in the area F_2 (see the left-hand graph in Fig. 5) a subsequent "horizontal integration" (over P_{exit}) of all vertical lines in (P_{entry}/P_{exit})-diagram over different prices (p) in the above mentioned interval has to be executed:

$$H = \int_{p'}^{M_1} \rho \cdot \frac{1}{2}(M_1 - p)^2 \, dp = \frac{\rho}{6}(M_1 - p')^3 \qquad (18)$$

Now we come back to the hysteresis loop derived in section 2.4 and illustrated by Fig. 3 in order to graphically interpret the hysteresis loss calculated in eq. (18). Fig. 6 depicts the aggregated output depending on price variations. The illustrated loop is the closed loop generated by the price changes $0 \rightarrow M_1 \rightarrow p' \rightarrow 0$. The right-hand graph is simply an optically enlarged upper part of the hysteresis loop on the left-hand side. If the system passes through the complete hysteresis loop, the hysteresis loss H is graphically represented by the geometrical area enclosed by the loop (see Mayergoyz, 2003, p. 50) as illustrated in Fig. 6a.

Fig. 6: Hysteresis loss as an area enclosed by the maximum (extreme) loop

a) complete loop *b) enlarged upper part of the graph*



If, however, the loop is not closed due to the fact that the forcing variable does not completely change back to the initial level *0* but only decreases to the level *p´* as illustrated in Fig. 5, the hysteresis loss can be graphically interpreted by fictitiously assuming an artificially closed "inside" loop. This can be done if a subsequent fictitious prices increase from *p´* back to the maximum M_1 is assumed. As a result of this fictitious price cycle $M_1 \to p' \to M_1$ a small (inside) loop is generated enclosing the area H_1, which represents the hysteresis loss induced by the price change $0 \to M_1 \to p'$. According to eq. (13), the parity of the areas F_3 and F_5 can be claimed, where F_3 and F_5 are the squared areas and F_4 is the whole dashed area in Fig. 6. Consequently, the lens-shaped hysteresis loss (H_1) can be computed as the difference between F_4 and F_3. The area F_4 is an integral of the downward leading branch $B_2(p)$ in the interval $[p';M_1]$ minus the area of the rectangle $Ap'M_1C$. The downward branch $B_2(p)$ corresponds to the area F_2 specified in eq. (12).

$$F_4 = \left[\int_{p'}^{M_1} B_2(p)dp\right] - B_2(p') \cdot (M_1 - p') = \frac{\rho}{3} \cdot (M_1 - p')^3 \qquad (19)$$

The area F_3 can be quantified as an integral of an upward branch $B_1(p)$ in the interval [*0; a*]. The upward branch $B_1(p)$ corresponds to the area F_1 specified in eq. (11).

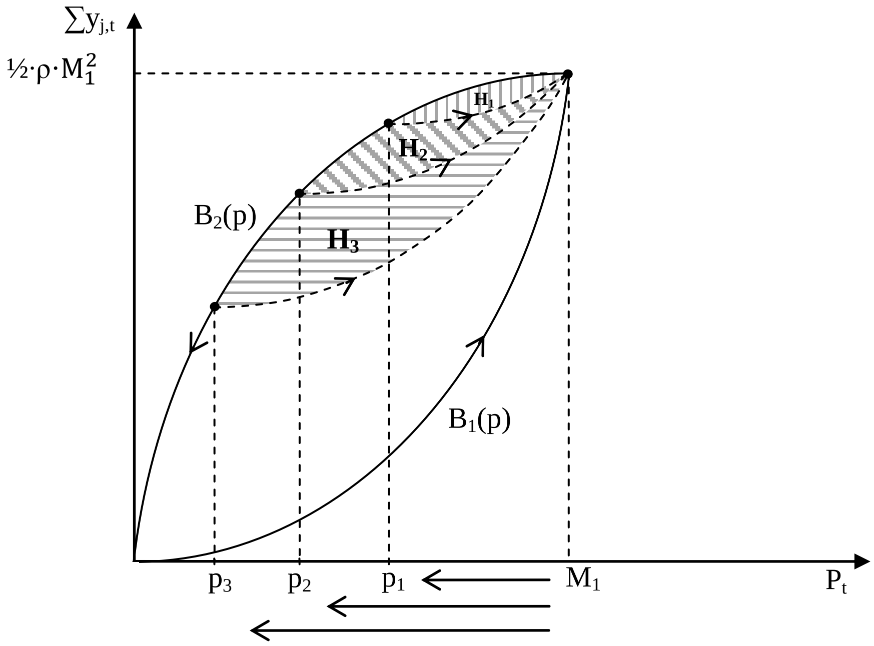
$$F_3 = \int_0^a B_1(p)dp = \int_0^{M_1-p'} \left(\rho \cdot \frac{1}{2} \cdot p^2\right) dp = \frac{\rho}{6} \cdot (M_1 - p')^3 \qquad (20)$$

Therefore, the area H_1 in Fig. 6 is quantified as follows:

$$H_1 = F_4 - F_3 = \frac{\rho}{3} \cdot (M_1 - p')^3 - \frac{\rho}{6} \cdot (M_1 - p')^3 = \frac{\rho}{6} \cdot (M_1 - p')^3 \tag{21}$$

The equality between eq. (18) and eq. (21) confirms the correctness of the hysteresis loss interpretation as an area inside the hysteresis loop. Fig. 7 provides an example with three price changes of different extents: $M_1 \rightarrow p_1$, $M_1 \rightarrow p_2$ and $M_1 \rightarrow p_3$.

Fig. 7: Hysteresis losses generated by different price changes under certainty



As stated above, in order to determine the hysteresis loss graphically we have to imagine an artificially closed inner loop. This can be done by adding the fictitious loop which leads back to the initial maximum M_1. By doing so, fictitious inner loops (lenses) are generated for every potential price reduction. Thus, H_1 is the lens capturing the hysteresis loss in case of a price reduction to p_1, H_2 represents the lens with a hysteresis loss after a price change to p_2, and H_3 is the hysteresis loss resulting from a price reduction to p_3. If the price would fall completely back to *0*, the whole area inside the outer maximum loop would describe the hysteresis loss of this complete ($0 \rightarrow M_1 \rightarrow 0$) cycle of the price level.

As obvious from eq. (21) and from the illustration of the hysteresis loss lenses in Fig. 7, the size of this loss, as a cubic function of the price variation ($M_1 – p'$), is increasing by degree 3, if we assume a uniform distribution of firms in the (P_{entry}/P_{exit})-diagram. I.e., doubling [or tripling] the size of a price cycle ($p' \rightarrow M_1 \rightarrow p'$) results in an increase of the generated hysteresis loss by a factor 8 [or 27]. Thus, large economic fluctuations are generating disproportionately high dynamic adjustment costs.

3 Hysteresis losses under uncertainty

3.1. Effects of uncertainty on the width of the microeconomic "band of inaction"

In the previous sections it was assumed that the firms ignore the uncertain stochastic nature of the future price level when they decide about market entry or exit. However, if the price is stochastic, a real option approach applies. E.g., an inactive firm deciding on a present entry has to consider the alternative option of a later entry. A current price level which is covering costs may decrease in the future, and by staying passive the firm can avoid future losses for this potential future situation. Moreover, a current entry kills the option to enter later and to "wait-and-see" whether the future price will turn out to be favorable or not. As a result, additional to the sunk costs an option value of waiting has to be covered in order to trigger an entry. Thus, uncertainty implies an upward shift of the entry trigger price.

The opportunity of a market entry is analogous to an *American call option* which can be defined as a *right to buy* a stock at a present strike price (see Dixit, 1992). In our model, the sunk entry investments represent the "strike price" of the market entry. Intrinsically the option has a value of waiting, called "holding premium" (Dixit, 1992, p. 116). Entering the market kills that option and causes the loss of the "holding premium" representing the opportunity costs of the entry decision. This offers an incentive to enter the market only if this loss is covered by future profits. Analogously, a disinvestment (market exit) is comparable with an *American put option* (Belke and Göcke 1999, p. 263). Therefore, the existence of uncertainty requires a correction in modelling the hysteresis losses which takes these option effects into account.

Dixit (1989) models entry and exit decisions in a stochastic situation assuming a Brownian motion of a price level, which is a standard assumption in the option pricing theory. We assume that in the next period a single change in the price level will happen, which can either be positive ($+\varepsilon$) or negative ($-\varepsilon$) with the same probability of *0.5 (*see Belke and Göcke, 2005). The firm is only anticipating the effects of stochastic variations on the next entry (or exit) decision. A later re-exit (or re-entry) due to ongoing stochastic fluctuations is not considered for reasons of simplicity.

The option value for an individual firm j depends on its previous state of activity. Thus, we have to analyze the behavior of both, a previously active and passive firm. In the first case, a previously active firm j has to decide between immediate exit in t and staying active in t with an option to exit in the future (*t+1*) if the price change is unfavorable ([-ε] - realization). Using the same notation as in previous sections the following expected present value of a "wait-and-see" (value of a put option) strategy results:

$$E_t\left(V_{j,t}^{wait}|y_{j,t-1}=1\right)=\left(p-c_j\right)-\left(\frac{1}{2}\cdot\delta\cdot l_j\right)+\left(\frac{1}{2}\cdot\delta\cdot\frac{p+\varepsilon-c_j}{1-\delta}\right) \tag{22}$$

The first expression in parentheses is the current profit from staying active, the second captures the probability-weighted and discounted sunk exit costs in case of a [-ε] – realization (leading to an exit in *t+1*) and the last one represents a probability-weighted present value of the annuity resulting from a future continuation of activity in case of a [+ε] – realization. A firm is indifferent between immediate exit and waiting if the expected present value equals the payment resulting from the exit costs ($-l_j$). Solving this equation results in the following exit trigger:

$$p_{j,exit}^{u}=c_j-(1-\delta)\cdot l_j-\frac{\delta\cdot\varepsilon}{2-\delta} \tag{23}$$

By combining eqs. (9) and (23), the relationship between the exit trigger values under certainty and under uncertainty become obvious[5]:

$$p_{j,exit}^{u}=p_{j,exit}^{c}-\frac{\varepsilon}{1+2i} \tag{24}$$

If we now consider a previously passive firm j that has to decide between immediate entry in *t* and staying passive in *t* with an option to enter in the future if the price change is favorable ([+ε] - realization), the following expected present value of a "wait-and-see" strategy results:

$$E_t\left(V_{j,t}^{wait}|y_{j,t-1}=0\right)=-\left(\frac{1}{2}\cdot\delta\cdot k_j\right)+\left(\frac{1}{2}\cdot\delta\cdot\frac{p+\varepsilon-c_j}{1-\delta}\right) \tag{26}$$

The first expression in parentheses captures the probability-weighted and discounted sunk entry costs in case of the [+ε] – realization (leading to an entry in *t+1*), the second one represents the probability weighted and discounted annuity value resulting from an entry in *t+1*.

[5] The discount factor δ equals $\frac{1}{1+i}$.

The expected value of an immediate entry in *t* is:

$$E_t\left(V_{j,t}^{entry}\right) = -k_j + \frac{p-c_j}{1-\delta} \tag{26}$$

The firm is indifferent between staying passive and entering the market if both expected present values are equal ($E_t\left(V_{j,t}^{wait}\right) = E_t\left(V_{j,t}^{entry}\right)$), resulting in the following entry trigger value:

$$p_{j,entry}^{u} = c_j + (1-\delta)\cdot k_j + \frac{\delta\cdot\varepsilon}{2-\delta} \tag{27}$$

Combining eqs. (8) and (27) the following relationship between the entry trigger values under certainty and under uncertainty results:

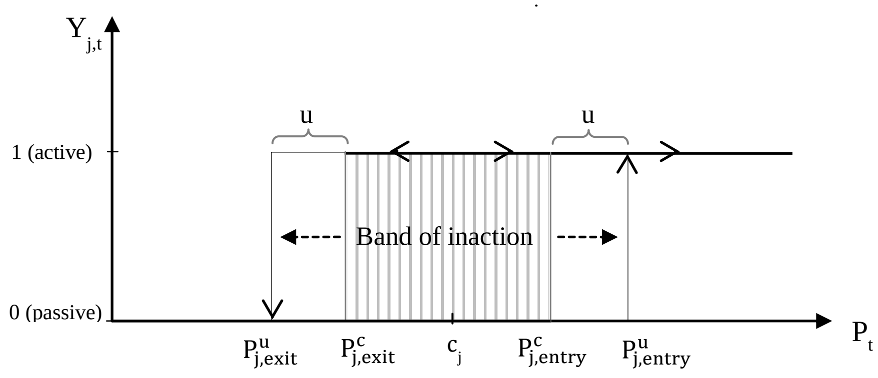
$$p_{j,entry}^{u} = p_{j,entry}^{c} + \frac{\varepsilon}{1+2i} \tag{28}$$

3.2. Hysteresis loss in a discrete multi-period model

As the simple model has shown (see eqs. (24) and (28)), the presence of uncertainty and the associated option value effects shift the entry trigger upwards/to the right and the exit trigger downwards/to the left (see Fig. 8). This widens the "band of inaction".

Fig. 8 (which is an extension of Fig. 1) illustrates two complete hysteresis loops – one under certainty (with threshold values $P_{j,entry}^{c}$ and $P_{j,exit}^{c}$) and one under uncertainty (with $P_{j,entry}^{u}$ and $P_{j,exit}^{u}$).

Fig. 8: Non-ideal-relay and hysteresis loss under (un)certainty



As we know, the area enclosed by a complete micro hysteresis loop under certainty (the hatched area in Fig. 8) equals the interest costs of the sum of sunk entry and exit investments.

In other words, the area inside the loop is proportional to the hysteresis loss. In case of uncertainty we get a different result. If we define the stochastic part of each trigger value as $u \equiv \left(\frac{\varepsilon}{1+2i}\right)$, (see eqs. (24) and (28)), the area enclosed by the uncertainty loop F^U (which is the whole white area between P^u_{exit} and P^u_{entry} in Fig. 8) equals:

$$F^U = (1-\delta)\cdot\left(k_j + l_j\right) + 2\cdot u = (1-\delta)\cdot H + 2u \tag{29}$$

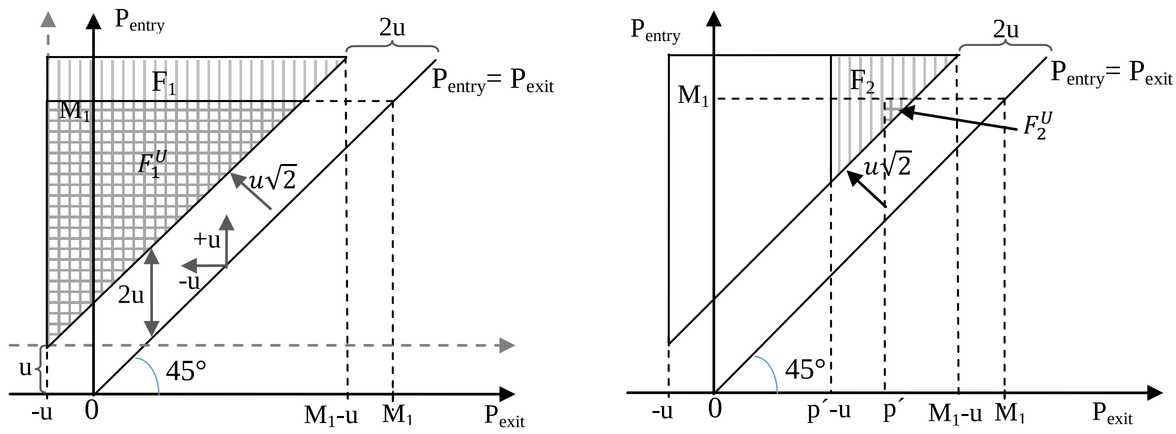
Thus, the inclusion of stochastic effects increases the area inside the non-ideal relay loop relative to the sum of the sunk costs H. Since a "wait-and-see" strategy in a stochastic environment sometimes prevents sunk entry and exit costs to be actually written off, option value effects reduce dynamic hysteresis losses H in relation to the area inside the hysteresis loop. The area inside the loop is no longer proportional to the hysteresis loss. In that regard our model differs from the original (non-stochastic) case of hysteresis losses in ferromagnetism.

3.3. Aggregated hysteresis loop of heterogeneous firms in entry-exit cycles

In this section the effects of uncertainty on the aggregation of heterogeneous firms (see Fig. 8) are shown (see Belke and Göcke, 2005). For the illustration of this problem we will again use entry-exit-trigger diagrams capturing all hysteretic firms in an area above the 45°-line (see Fig. 9). For reasons of simplicity, in order to illustrate the principal effects of uncertainty on the aggregation procedure, we explicitly analyze only a simplified situation: we assume that all firms are affected by sunk costs, so that there are no firms on the 45°-line. Moreover, all firms are assumed to be affected by uncertainty in the same way, resulting in the same widening effect on the band of inaction for all firms. Fig. 9 is an extension of Fig. 2 and illustrates the aggregate effects of an inclusion of the option value effects (as calculated by the simple stochastic model above). Widening the "band of inaction" of each firm by $2u$ (see Fig. 8) due to uncertainty means that the coordinate system of the entry-exit diagram with all hysteretic firms has to be shifted to the left by u and upwards by u in order to account for the "outward" shifts in both triggers. This results in a horizontal shift of the 45°-line by $2u$ or an orthogonal shift by $u\sqrt{2}$. The resulting area between the 45°-line and the shifted triangle area is now "depopulated" (i.e., there are no hysteretic firms that would enter or exit the market if the market price varies within this area).

Fig. 9: Cumulated output changes induced by price shocks under uncertainty

a) a positive price shock ($0 \rightarrow M_1$) *b) a subsequent negative shock ($M_1 \rightarrow p'$)*



If we compare Fig. 2 and Fig. 9, it becomes obvious that under uncertainty both positive and negative output changes of the same extent as under certainty require a larger price change, or, vice versa, for a given price change the output reaction is weaker. E.g., under certainty the maximal output gain F_1 (see Fig. 2a)) results if the price rises from 0 to M_1. If, however, the stochastic nature of prices is taken into account, a price increase from $p=0$ to $p=M_1$ results in an output change under uncertainty F_1^U which is comparable to the reaction under certainty for a price increase only up to $p=(M_1-u)$ (see squared triangle in Fig. 9a). The reaction of a subsequent negative price shock under uncertainty is also weakened by option effects. E.g., for a subsequent price decrease from M_1 to p' the output reduction is described by the small triangle area F_2^U instead of $F_{2,}$ as in the case of certainty (see Fig. 2b)). For a small price decrease (smaller than the option value effect u), there would even be no negative reaction of the output. The aggregate reaction shows similarities to *'play'* (or 'backlash') phenomena (as known from mechanics or engineering).

The output gain under uncertainty after a price increase from 0 to M_1 (see Fig. 9a)), described by triangle F_1^U, and adjusted by the density parameter ρ is:

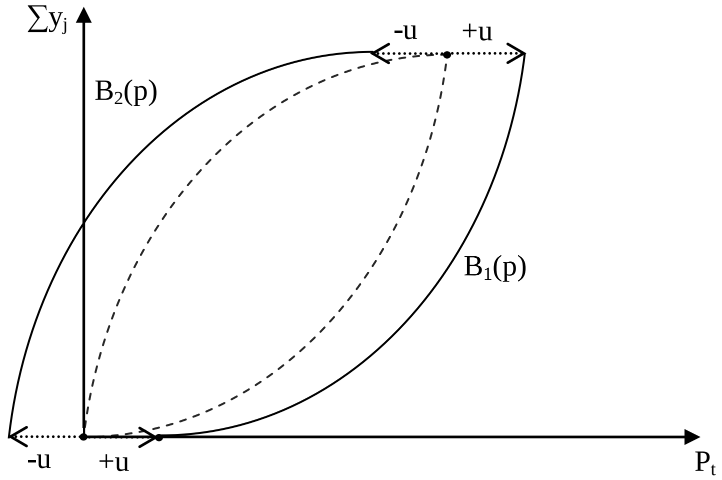
$$F_1^U = \rho \cdot \frac{1}{2} \cdot (M_1 - u)^2 \tag{30}$$

If the market price subsequently falls from the local maximum to p´, the output loss under uncertainty equals the density-adjusted triangle area F_2^U (see Fig. 9b)):

$$F_2^U = \rho \cdot \frac{1}{2} \cdot (M_1 - 2u - p')^2 \tag{31}$$

Fig. 10 schematically illustrates the effects of uncertainty on the aggregated behavior of firms (hysteresis loop) for all prices in the interval [*0; M_1*]. The aggregation procedure is analogous to the one described in section 2.4. As a result of the inclusion of uncertainty the downward leading branch (*$B_2(p)$*) shifts to the left and the upward leading branch (*$B_1(p)$*) shifts to the right; each by the absolute extent of *u*. The dotted lines illustrate the hysteresis loop in the case of certainty and the solid lines represent the hysteresis loop under uncertainty, if option effects are considered.

Fig. 10: Macro hysteresis loop including option value effects



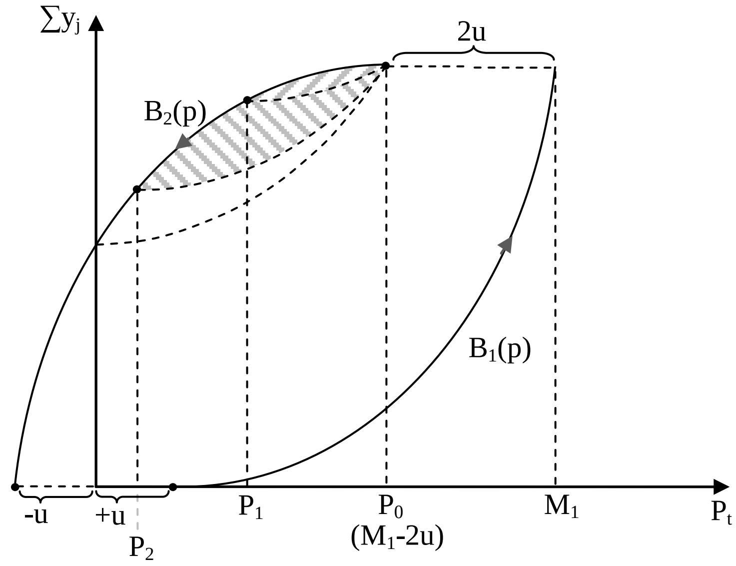
3.4. Aggregated hysteresis loss of heterogeneous firms

As Fig. 11 in combination with Fig. 9 in combination illustrate, a price increase from *0* to *M_1* now changes the aggregate output by $Y = \rho \cdot \frac{1}{2} \cdot (\mathrm{M}_1 - \mathrm{u})^2$, which is remarkably smaller than in the case of no option value effects. Fig. 11 provides some examples of hysteresis losses associated with a subsequent price decrease of different extents: $M_1 \rightarrow p_0$, $M_1 \rightarrow p_1$ and $M_1 \rightarrow p_2$. If the price falls from maximum M_1 only a bit (by less than the option value effect of uncertainty *u*) to p_0, there are no hysteretic firms in this “play area” who exit the market. Only if the market price falls below p_0 (i.e., by more than u) firms start to leave the market. The resulting hysteresis losses under uncertainty can be calculated analogous to hysteresis losses under certainty (see section 2.5). Assuming a price fall from M_1 to the level *p´* (corresponding to *p´* in Fig. 3, 5 and 6) the following function capturing the hysteresis losses results:

$$H = \begin{cases} \frac{d}{6} \cdot (M_1 - 2u - p')^3, \; if \; (M_1 - 2u - p') > 0 \\ 0, \; if \; else. \end{cases} \tag{32}$$

As the second case of eq. (32) captures and Fig. 11 illustrates, (analogous to "backlash" in mechanics) "play" areas for price changes of an extent of $2u$ arise due to option value effects. In such areas no hysteresis losses occur, since as a consequence of a wait-and-see-strategy, no firm will actually leave the market. If, however, a positive price shock is followed by a price reduction larger than $2u$ (in Fig. 11 for a market price lower than p_0) hysteresis losses are generated and can be graphically illustrated and interpreted analogous to Fig. 7. However, in the case of substantial uncertainty effects the resulting hysteresis loss-areas are considerably reduced – due to the trimming by the 'play' area (by $2u$).

Fig. 11: Hysteresis losses generated by different price changes under uncertainty



Even, if the market price for their products is negative (which is theoretically conceivable), some firms will stay active in the market because the firms are considering an expected potential positive price change – hoping not to experience hysteresis losses.

In sum, an inclusion of uncertainty effects results in a "play area" in which no hysteresis losses (and actually no quantitative output reactions) are generated due to waiting. The size of this inaction area is positively correlated with the degree of uncertainty (in our extremely simple model the size of play $2u$ is even in a linear way related to the size ε of the stochastic shock). In other words, when the stochastic nature of prices is taken into account, firms become more cautious and delay their entry and exit decision. On the one hand, falling prices generate smaller hysteresis losses since the exit triggers are lower under uncertainty, but on the other hand, a certain output gain requires a higher price increase, which makes economic reactions under uncertainty more "sticky" on the micro as well as on the aggregate level.

4 Conclusions

This paper deals with economic hysteresis on the supply side caused by sunk adjustment (e.g., entry and exit) costs. Our aim was to model and to calculate the dynamic losses in the entire market in case of price fluctuations. As a first step, a hysteretic dynamics of firm-level reactions based on one- and multi-period optimization was presented. Here the hysteretic behavior of one firm was explained according to the existing literature regarding hysteresis in economics (see Krasnosel'skii, Pokrovskii, 1989, Baldwin 1989, 1990, Göcke, 2002). Since the unit/marginal costs as well as sunk entry and exit (dis)investments are firm specific, for heterogeneous firms individual entry and exit price trigger values result, leading to a "non-ideal-relay" reaction pattern to price changes. The distance between these triggers/thresholds constitutes a so called "band of inaction" (see Fig. 1) and is proportional to the sum of sunk entry and exit costs. Thus, the area inside the non-ideal-relay triggers represents the firm's dynamic loss during a complete entry-exit cycle. Considering that firms are heterogeneous, we applied an adequate aggregation procedure to describe the aggregate supply hysteresis loop of all heterogeneous firms related to price changes (see Amable et al., 1991, Göcke, 2002). As an innovation in economics, we showed how the hysteresis loss of the entire market is calculated based on the aggregated hysteresis loop. If the system passes through the complete hysteresis loop, the dynamic loss is graphically represented by the geometrical area enclosed by the loop (see Fig. 6a). Since this enclosed area is a cubic function of the price variation, hysteresis losses are over proportionally increasing in the size of price fluctuations. However, the size of this area depends on the curvature of the hysteresis loop – the wider the loop is the larger are the hysteresis losses. The curvature of the aggregated hysteresis loop is determined by the distribution of firms in the entry/exit-trigger diagram (see section 2.4). For simplicity reasons we assume a uniform distribution of firms in the upper-left area in this diagram. If, however, most of the hysteretic firms had low sunk entry and exit cost in their cost structure, these firms would be more concentrated close to the 45°-line (which represents non-hysteretic firms). As a consequence, in this case of a more flexible market, the aggregated behavior of hysteretic firms would be represented by a more "narrow" shape of the aggregated loop and, thus, to lower hysteresis losses during a cycle. In an opposite scenario of a very inflexible market, where most of the firms experience very high sunk costs for entry and exit, this would result in a distribution of firms with a higher density in the north-east part of the diagram ("far apart" from the 45° line). Consequently, the result would be a wider (more "inflated") shape of the aggregate hysteresis loop, c.p. resulting in relatively large hysteresis losses resulting from an entry-exit cycle.

In order to allow for the uncertain stochastic nature of the future price level, uncertainty was explicitly included into the model. Related to the standard theory of hysteresis (in mathematics or physics), the inclusion of economic option effects on the size of hysteresis losses is a second novelty of our paper. Including stochastic effects results in "wait-and-see" strategies based on option values (see Pindyck, 1988, 1991, Dixit, 1989, 1990, 1992, 1995, Krugman, 1989, Dixit and Pindyck, 1994, Belke and Göcke, 1998, 1999, Sarkar, 2000, Wong, 2007). Since a wait-and-see strategy in a stochastic environment may prevent sunk entry and exit costs to be actually written off, option value effects reduce dynamic hysteresis losses in relation to the area enclosed by the hysteresis loop. This dynamic loss-reducing effect was demonstrated for the micro firm level as well as on the macro aggregate level. Especially for small fluctuations in the price level, option value effects result in a kind of "play" (or "backlash") type of a sticky reaction of the market to price changes, and thus prevent the actual generation of hysteresis losses. As a consequence, in a situation with uncertainty only large price fluctuations will generate severe hysteresis losses.

Taken as a whole, large economic fluctuations are generating disproportionately high dynamic adjustment costs, due to (1) the cubic effect of price changes on the size of the dynamic losses, and since (2) "wait-and-see" filters away reactions on small stochastic price changes. This may be part of the problems related to large economic fluctuations in markets where substantial sunk investments are relevant – the "crises" of the last decade, with financial market instability and recent oil price fluctuations (where many fracking oil producers have to write off their investments due to low oil prices) as examples. From an *economic policy* point of view, there are two ways to reduce hysteresis losses. The first one is to reduce/prevent variations on markets implementing the stabilizing measures and policies. Examples of such stabilizing (dynamic loss-avoiding) policies could be (stable) fixed exchange rates, financial market regulations, or even two-price buffer stock schemes on commodity markets (though with a wide ceiling and floor difference). The second alternative is to preserve flexible markets, i.e., to reduce the sunk costs as barriers for market entry and exit, leading to a "narrow" aggregated hysteresis loop and smaller hysteresis losses. Aray (2015) argues that the reduction of "institutional uncertainty" through information policies on the part of the government (e.g., promoting the exchange of information among firms) would reduce sunk entry costs.

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