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Abstract

This paper proposes somatic capital as a hitherto neglected variable in the discussion of factors impacting the timing of the Neolithic transition. It develops an evolutionary growth theory that builds on the trade–off between the quantity and the quality of offspring. The theory suggests that harsh climatic conditions during the ice age raised skill intensity of the environment and altered the evolutionary optimal allocation of resources from offspring quantity to offspring quality. Higher somatic investment in offspring increased the innovation capability of individuals and ultimately accelerated the rate of technological progress. Thus, the adaptive response triggered within human populations living in cold and harsh climate for thousands of years had a significant impact on the timing of the Neolithic transition. The theory further suggests that differential somatic investment can be identified as deep–rooted determinant of comparative economic development.

Keywords: Economic Growth, Human Evolution, Ice Age Climate, Neolithic Revolution, Out–of–Africa Expansion, Somatic Capital, Skill Intensity, Technological Progress

JEL Classification Numbers: J10, O10, O30

1 Introduction

The Neolithic transition to agriculture was a remarkable episode in human history. The shift from hunting and gathering to plant cultivation and animal husbandry significantly changed the long–run economic outcome of mankind. For the first time human societies were capable of supporting a non–food producing class of specialized craftsmen and knowledge creators. Agriculture is therefore a necessary condition for modern civilization itself. The Neolithic transition independently occurred in at least five regions of the world. Agriculture began about 11,500 years ago (11.5 kya) in the fertile crescent in the Middle East from where it spread to Europe, followed by about 9.5 kya in North China, 8 kya in South China, 5.5 kya in Central Mexico, and 4.5 kya in Sub–Saharan Africa. Apparently, there is a rough ecogeographic patterning in the timing of the transition: societies from over 30°N latitude experienced on average an earlier transition than societies from under 30°N latitude.

Current research on long–run growth attempts to identify deep–rooted determinants of comparative economic development. The aims are to explain the transition from a period of Malthusian stagnation to one of sustained economic growth, and to identify factors that generated the differential pattern of development across the world (Galor and Weil, 2000; Galor and Moav, 2002; Galor, 2005, 2011). The Neolithic transition is important in this respect because per–capita incomes, life expectancies, and population densities seem to be correlated with the region–specific timing of the transition. Diamond (1997) argues that favorable biogeographic conditions expedited the shift from foraging to farming in well endowed areas. Olsson and Hibbs (2005) provide empirical support for the Diamond hypothesis, and they show that current variations in economic prosperity still reflect the effects of the timing of the transition. Putterman (2008) and Putterman and Weil (2010) find that the number of years since the transition positively effects incomes today, where the latter are more precise, using *ancestry–adjusted* Neolithic transition timing to reflect post–1500 worldwide migration flows.[1] Galor and Moav (2007) demonstrate that a significant portion of contemporary variation in life expectancy across countries is a result of the differential timing pattern of the Neolithic transition. Finally, Ashraf and Galor (2011) show that population densities between 1 CE and 1500 CE are positively effected by the time span since the transition.

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[1]See Appendix C for details on the calculation of *ancestry–adjusted* years since the Neolithic transition.

Apparently, a significant part of comparative economic development can be traced back to the differential timing of the Neolithic transition. Therefore, it is important to identify factors that influenced the timing. Here we want to add a novel twist to the discussion. According to the present paper, it is not only biogeographic factors (Diamond, 1997; Hibbs and Olsson, 2004) or climatic fluctuations (Ashraf and Michalopoulos, 2015; Dow, Reed and Olewiler, 2009) that had an impact on the timing, but also climate induced variation in human somatic capital. Somatic capital is–in a physical sense–embodied energy. In a functional sense, somatic capital includes body size, physical stature, and strength, but also factors like immune function, coordination, and skill, all of which affect the profitability of human activities like resource acquisition (Kaplan et al., 1995).

The theory builds on two key assumptions. First, we assume that individuals are naturally selected to make an evolutionary optimal decision with respect to the quantity and the quality of children.[2] That is, individuals choose somatic investment in offspring such as to maximize reproductive fitness. In fact, the human system of fertility regulation seems to be well organized to respond adaptively to variable environments (Kaplan, 1996: 92). This suggests that natural selection has provided humans with some kind of built–in feedback mechanism that allows fitness maximizing behavior in varying environments.[3] Second, we assume that somatic investment increases the innovation capability of individuals and ultimately leads to a higher rate of productivity growth. This assumption appears plausible, since there is a correlation between body size and brain size on the one hand (Koh et al., 2005) and brain size and intelligence on the other hand (McDaniel, 2005). Hence, it is reasonable to think of somatic capital not only as body size, but also as brain size and intelligence.

In this paper we argue that the migration of anatomically modern humans out

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[2]See Robson (2010) for a similar approach. Fitness–maximizing trade–offs have long been studied within the context of life history theory in biology (Lessels, 1997; Stearns, 1992; Roff, 1992, 2002; Charnov, 1993) and anthropology (Hawkes and Paine, 2006; Hill, 1993; Lummaa, 2007). In economics, Becker was the first to introduce both a qualitative and a quantitative dimension to the demand for children (Becker, 1960; Becker and Lewis, 1973; Willis, 1973).

[3]It is possible to explicitly model the process of natural selection on a genetically heterogeneous population with respect to somatic investment. Galor and Moav (2005) use this approach to explain the rise in life expectancy since the Neolithic transition. Similar to Robson (2010), this paper relocates the process of natural selection into the individual which is assumed to act evolutionary optimal. This keeps the model simple and generates identical long–run results.

of Africa into Western Asia and Eastern Europe between 40 and 45 kya and then into Europe between 40 and 36 kya triggered a behavioral adaptation to the harsh climatic conditions in Eurasia during the ice age. Lower hunter–gatherer productivity in areas above 30°N latitude was compensated by increased somatic investment in offspring, reallocating parental investment from child quantity to child quality. Higher somatic investment in offspring increased the innovation capability of individuals and accelerated the rate of technological progress, at least as long as the rise in environmental harshness was no too large. Eventually, the Neolithic transition to agriculture was significantly accelerated.

The point of this paper is twofold. First, it adds somatic capital as a hitherto neglected variable to the discussion of factors impacting the timing of the Neolithic transition. This variable is interesting because it potentially fits as an omitted variable causing both an early switch to agriculture and rapid subsequent growth. Interestingly, the link between body mass and comparative development has been explored by Dalgaard and Strulik (2018). Second, differential somatic investment and its link to the innovation capability of individuals is suggested as a deep–rooted determinant of comparative economic development.

It is a key feature of this model that the optimization problem of an adult individual is formulated using reproductive fitness as objective function. That way, the model is deeply rooted in the evolutionary biology of our species. The resulting optimal behavior regarding quantity and quality of offspring, however, would be identical if the optimization problem were formulated using Unified Growth Preferences (Galor and Weil, 2000; Galor, 2011). This is an important result. Abstracting from adult consumption, humans seem to be naturally selected to maximize the aggregate income of their children. Hence, the theory provides a deep-rooted foundation for the utility function commonly used in Unified Growth Theory.

This paper is related to several strands of literature. On the one hand, it is associated with the theoretical literature on economic growth in the very long run, specifically Galor and Weil (2000), Galor and Moav (2002), Lucas (2002), Cervellati and Sunde (2005), Cervellati and Sunde (2015); see Galor (2005, 2011) for a summary. On the other hand, the paper is related to a growing strand of empirical literature that aims to understand the deep–rooted determinants of comparative economic development as recently surveyed by Spolaore and Wacziarg (2013) and by Nunn (2014). Especially relevant are papers that share the physiological focus,

i.e., Dalgaard and Strulik (2015; 2016; 2017;2018).

This paper is also related to publications that intend to explain the advent of Neolithic agriculture, especially economic publications (cf. for surveys Pryor, 1983; Weisdorf, 2005). Early work stresses excessive hunting (Smith, 1975; Locay, 1989) or population pressure coupled with the institutional shift from common to exclusive communal property rights (North and Thomas, 1977). More recently, Marceau and Myers (2006) argue that the breakdown of cooperative structures at a certain level of technology leads to food–crises and ultimately to agricultural food production. Weisdorf (2003) suggests that the emergence of non–food specialists is critical for the transition to agriculture. With respect to the timing of the transition, Diamond (1997) argues that the biogeographic endowment of a society with domesticable plants and animals expedited the shift from foraging to farming, a view that is theoretically supported by Olsson (2001) and empirically documented by Olsson and Hibbs (2005). Baker (2008) finds that societies located farther from the core centers of agricultural food production experienced a later transition. A similar pattern is empirically established in the work of Ashraf and Michalopoulos (2015), who suggest that a foraging society's history of climatic shocks shaped the timing of its adoption of farming. Complementary, Dow, Reed and Olewiler (2009) hold the view that a single abrupt climatic reversal forced a large regional population through a small geographic bottleneck, leading to a spike in labor supply at the best food production sites, thereby initiating the Neolithic transition. Hence, this paper is the first to explore the role of geographically induced differences in somatic investment for the timing of the transition.

The main argument of this paper is based on anthropological, archeological, and climatological evidence. It is helpful to summarize the respective findings before developing the model. Thus, the paper is organized as follows. Section 2 sketches the Out–of–Africa expansion and the climate of the past, gathers evidence for differential somatic investment, and demonstrates its influence on the timing of the Neolithic transition. Sections 3 and 4 formalize the key assumptions and incorporate them into a simple model. Section 5 characterizes the dynamical system and analyzes the economy with respect to the timing of the Neolithic transition. Section 6 concludes.

2 Motivating Evidence

2.1 The Out–of–Africa Hypothesis and the Climate of the Past

The Out–of–Africa hypothesis for modern human origins is widely accepted today: living humans exhibit remarkably little genetic diversity and the implication is that they share a recent common African ancestor (Klein, 2009: 615). However, mounting evidence points to a lag between the emergence of modern human anatomy and the emergence of modern human behavior. Recent studies suggest that modern anatomy evolved at least 160–195 kya (White et al., 2003). By contrast, the total package of modern human behaviors that indicates a capacity for abstract thought–language, art, personal ornamentation, specialized tools, complex social organization, extensive trade networks–, was not in place until about 50–45 kya (Nowell, 2010). At this time, human populations in Africa gained a significant fitness advantage that led to the Out–of–Africa expansion. The spread of African populations to western Asia and eastern Europe is currently dated to between 45 and 40 kya, whereas western Europe was reached between 40 and 36 kya (Klein, 2009: 743). However, the timing of the human dispersal is still a fundamental issue and earlier routes of expansion are actively debated (e.g., Armitage et al., 2011).

When modern humans migrated into Eurasia, they had to cope with challenging climatic conditions. The average temperature in the northern hemisphere at the height of the last ice age was around 12 to 14° lower than today. This is not true for the tropics, where average temperatures seem to have been much closer to current figures (Burroughs, 2005: 41). The climate record for the northern hemisphere in Figure 1 draws a detailed picture of the climatic challenges faced by African migrants in Eurasia after 45 kya. Ice age conditions reached a peak about 24 kya (Burroughs, 2005: 40), where the growth of ice sheets had reached their maximum positions. Northern Europe, the central East European Plain, and much of Siberia seem to have been abandoned at this time. Average temperatures rose to modern values only around 10 kya. This date constitutes the beginning of the Holocene and finally marks the break from ice age conditions that characterized Eurasia before (Burroughs, 2005: 47).
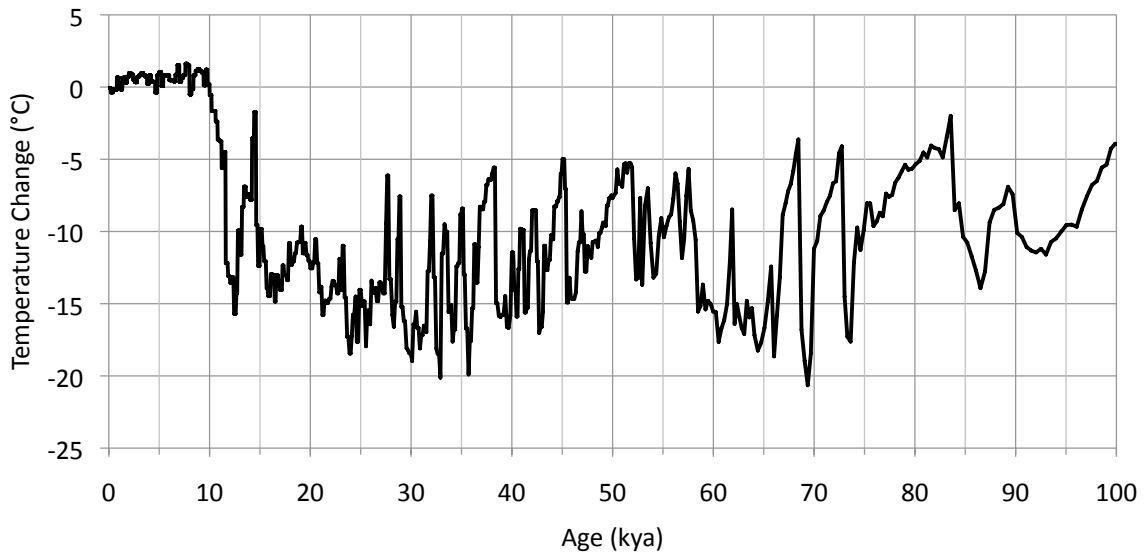
Figure 1: Temperature Change in the Northern Hemisphere of the Past 100,000 Years. The Time Series Uses GISP2 Ice Core Data with Average Values for Every 200 years (World Data Center for Paleoclimatology, Boulder, Colorado, USA).

2.2 Evidence for Differential Somatic Investment

As modern humans spread out across the globe, their physical size changed in response to climatic challenges. Ruff (1994) demonstrates an ecogeographic patterning in body mass and body height in contemporary humans, with populations from higher latitudes being larger on average than those from lower latitudes. The relationship between absolute latitude and body mass is depicted in Figure 2 for pre–1500 societies.[4] Ruff, Trinkaus and Holliday (1997) show that the ecogeographic patterning can be confirmed for Early Upper Paleolithic societies as well: specimens from over 30°N latitude were significantly larger than those from under 30°N latitude. The correlation between absolute latitude and body size is typically viewed as the result of Bergmann's and Allen's eco–geographic rules: mammals living in cold climates tend to have a larger body size (Bergmann, 1847) and shorter extremities (Allen, 1877) than their warm climate conspecifics in response to thermoregulatory effects. However, as discussed by Formicola and Giannecchini (1999: 326), shorter

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[4]The data on body mass is from the Goldman data set (Auerbach and Ruff, 2004), which compromises morphological observations from Holocene skeletons from 1500 CE or earlier. See Appendix C for details.
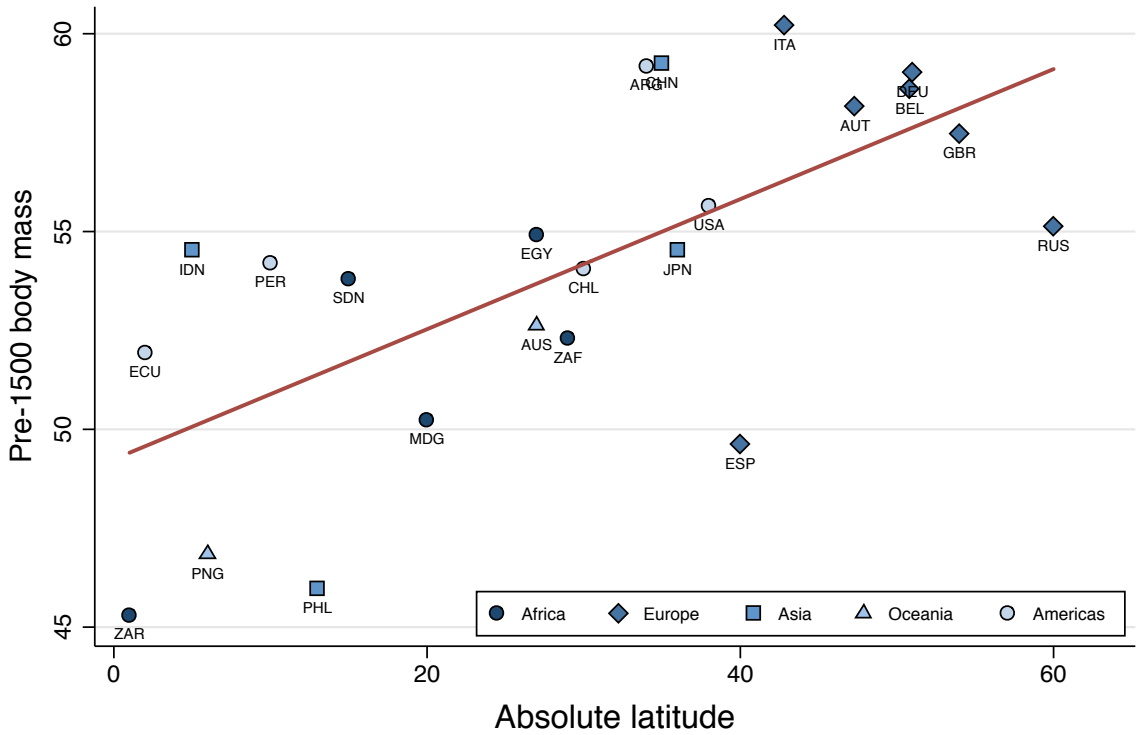
Figure 2: Body Mass and Absolute Latitude Across Pre–1500 Societies.

extremities imply shorter stature. Hence, this rule cannot explain the large stature that was common during the Early Upper Paleolithic. Moreover, contrary to Allen's rule, early modern Europeans possessed body proportions that are typical for tropical climates (Holliday, 1997).

In contemporary humans, differences in body proportions correspond to differences in brain size. Koh et al. (2005) calculate a significant correlation of 0.67 between brain volume and body height in healthy 20–year old Koreans.[5] Witelson, Beresh and Kigar (2006) show body height to account for 1 to 4 % of the variation in cerebral volume within each sex. There is also a significant relationship between brain volume and intelligence. In a meta analysis of 37 samples across 1530 people, McDaniel (2005) calculates a correlation of brain volume and intelligence of 0.33. Recent genome wide association studies confirm this link further. Stein et al. (2012)

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[5]The effect differed with respect to the sex. The whole brain volume was significantly correlated with body height in male ($r = 0.37$, $p < 0.05$), but not in female participants ($r = 0.24$, $p > 0.05$) (Koh et al., 2005: 2030).

identify a locus within the HMGA2 gene that is associated with larger intracranial volume across lifespan. Specifically, the C allele of the locus increases the intracranial volume on average by 9 cm^3 per allele and is weakly associated with increased general intelligence by approximately 1.29 IQ points. Moreover, the HMGA2 gene is also a strong biological candidate for body height. Weedon et al. (2007) calculate that each copy of the C allele is associated with an average increase of 0.4 cm. Interestingly, Stein et al. (2012) find evidence that the genetic correlation between body height and intracranial volume is more general in nature.

Apparently, people from more northerly latitudes are, on average, not only larger, but have also bigger brains than people from lower latitudes. Ruff, Trinkaus and Holliday (1997) show that the link between body size and cranial capacity holds for early modern humans as well. Moreover, the frequency of certain brain regulating genes in living populations might unexpectedly support the hypothesis. Microcephalin, a gene regulating brain size, is currently discussed in this respect (Evans et al., 2005; Mekel–Bobrov et al., 2007).

A key element of the proposed theory is a relationship between geographic latitude and somatic investment on the one hand and somatic investment and technological progress on the other hand. Interestingly, the archeological record supports this link, as a significant correlation has been found between technological complexity and geographic latitude (Hoffecker, 2005: 191). Although it seems logical to assume that declining temperatures directly acted as a stimulus to technological change, Hoffecker (2005: 190) notes that, first, many early inventions probably were not critical to survival in these latitudes and, second, a series of important innovations took place after climates in northern Eurasia began to warm. Thus, archeological findings support an indirect link between environmental harshness and technological progress, as proposed in this paper.

2.3 Somatic Investment and the Timing of the Neolithic Transition

In this paper, we make use of two simple concepts. First, we propose the term *skill intensity of the environment* to suggest that exogenous variation in geographical conditions can be a source of variation in the amount of somatic capital of an adult hunter–gatherer. Second, we use the term *somatic investment* to emphasize that the amount of somatic capital of an adult hunter–gatherer can be mainly understood as

the result of parental investment during childhood. In this subsection, we want to operationalize both concepts in a reasonable way. In the first case, we use absolute latitude as a measure for the skill intensity of the environment because it captures the ecogeographic patterning in worldwide climatic conditions during the last glacial period close to and far from the equator. In the second case, we use pre–1500 *body mass* as an approximate variable.[6] Across taxa, body mass is known to be the most appropriate measure of an animal's overall size (Darveau et al., 2002). Furthermore, it shows much variation in human evolution (Holliday, 2015). Pre–1500 body mass is not ideal, as the theory is based on variation in the body size of late Pleistocene *Homo sapiens*. However, the respective sample size is–to our knowledge–not large enough to compile reasonable cross–country data. Therefore, pre–1500 body mass is as close as possible to the relevant time span.

According to the theory, absolute latitude should have a significant positive effect on pre–1500 body mass. This is exactly what Table 1 reports in a 23 country sample (column 1). Impressively, the result is robust to continental fixed effects in spite of the small sample size (column 2). The remainder of Table 1 establishes– again in line with the theory–a significant positive effect of body mass on the timing of the Neolithic transition. Following Putterman and Weil (2010), we use *ancestry– adjusted* years since the Neolithic transition as response variable that corrects for the worldwide migration flows after 1500 CE.[7] Column 3 shows that body mass accounts for a significant part of the variation in the timing of the transition. The result is again robust to continental fixed effects (column 4). Once we control for absolute latitude (columns 5 and 6) the strength of the association between body mass and the outcome variable slightly weakens. This is to be expected given the strong latitude gradient in body mass. Interestingly, however, absolute latitude remains insignificant, suggesting that body mass is the relevant channel for the transmission of absolute latitude on the timing of the transition. The association between pre–1500 body mass and ancestry–adjusted years since the Neolithic transition is depicted in Figure 3.

Part of the strong correlation between pre–1500 body mass and the timing of the

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[6]The data on body mass is from the Goldman data set (Auerbach and Ruff, 2004), which compromises morphological observations from Holocene skeletons from 1500 CE or earlier. See Appendix C for details.

[7]See Appendix C for details on the calculation of *ancestry adjusted* years since the Neolithic transition.

Table 1: Latitude, Pre-1500 Body Mass, and the Timing of the Neolithic Transition.

	Pre-1500 body mass		Neolithic transition timing (ancestry adjusted)					
	OLS (1)	OLS (2)	OLS (3)	OLS (4)	OLS (5)	OLS (6)	2SLS (7)	2SLS (8)
Pre-1500 body mass			263.779*** (0.001)	212.827** (0.025)	227.728* (0.055)	204.178* (0.071)	306.505*** (0.000)	229.861** (0.037)
Absolute latitude	0.164*** (0.001)	0.194*** (0.009)			12.948 (0.586)	4.972 (0.865)		
Cont. FE	No	Yes	No	Yes	No	Yes	No	Yes
Obs.	23	23	23	23	23	23	23	23
R^2	0.458	0.494	0.425	0.506	0.434	0.507	0.413	0.505
Anderson-Rubin (*p-value*)							0.003	0.102
Kleibergen-Paap (*F-statistic*)							15.864	8.629

Notes: This table establishes a significant positive effect of absolute latitude on pre-1500 body mass in a 23-country sample (columns 1 and 2). Furthermore, it documents a significant positive effect of pre-1500 body mass on the ancestry adjusted timing of the Neolithic transition (columns 3 and 4). The association between body mass and the transition timing weakens while controlling for absolute latitude but remains significant (columns 5 and 6). Columns 7 and 8 deal with reverse causality and provide IV estimates. The instrument is absolute latitude and it is clearly relevant. All regressions with continental fixed effects do not employ the Oceania dummy due to limited observations for this continent. Heteroskedasticity-robust standard errors are reported in parentheses.

 *** Significant at the 1 percent level.

  ** Significant at the 5 percent level.

   * Significant at the 10 percent level.

Neolithic transition could be reverse causality: an early switch to agricultural food production might be associated with a larger body size due to better nutrition.[8] Therefore, column 7 deals with reverse causality and provides IV estimates. The instrument is absolute latitude. The instrument is clearly relevant. It stays relevant if continental fixed effects are introduced in column 8, which again is surprising given the small sample size. The Kleibergen–Paap F–statistic strongly rejects weak identification. The slightly higher p–value of the Anderson–Rubin test for the significance of endogenous regressors in column 8, however, is probably due to non–significant continental dummies. Table 2 in Appendix B provides additional robustness checks with respect to the diamond hypothesis (Diamond, 1997), and with respect to the

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[8]Note that Robson (2010) argues that the advent of agricultural should be associated with a *decline* in body mass from a bioeconomic point of view, in contrast to the reverse causality argument.
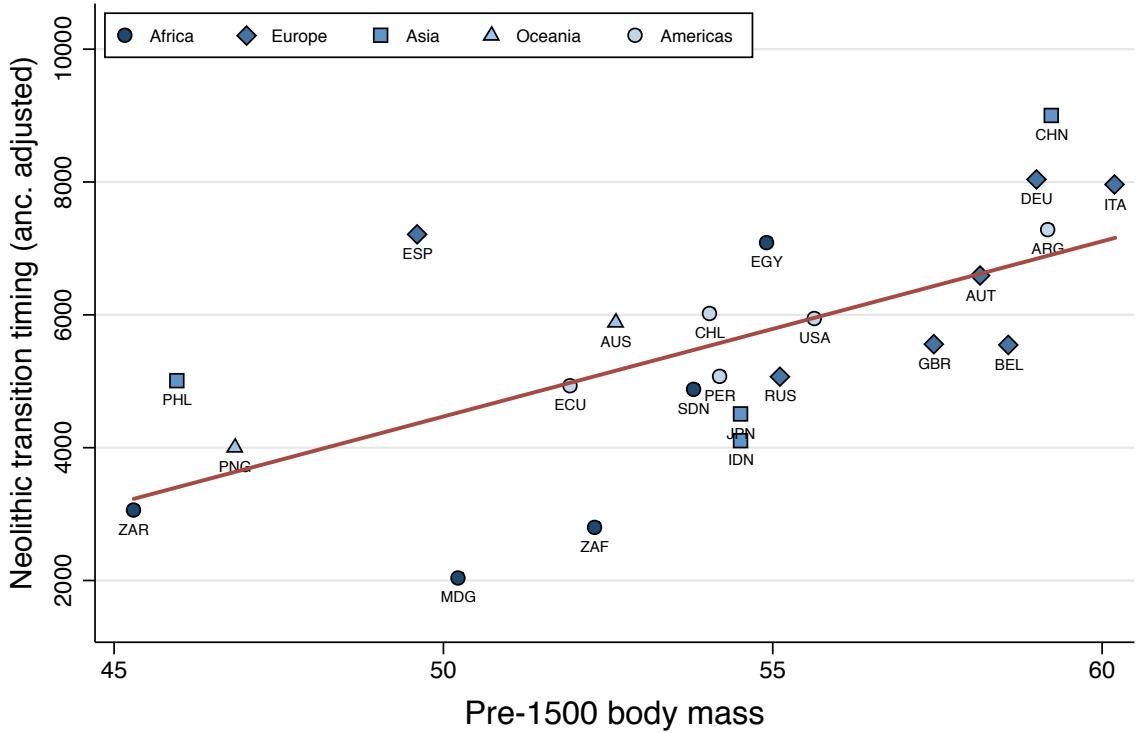
Figure 3: The Timing of the Neolithic Transition and Pre–1500 Body Mass.

ancestry adjustment of the response variable.

Overall, the results suggest that body mass has a strong causal influence on the timing of the Neolithic transition. Using absolute latitude as an instrument for body mass reveals that much of the effect of ancestral geographical conditions–as proxied by absolute latitude–operates through somatic capital as an embodied, indirect transmission channel. In the remainder of the paper, we develop a theoretical model that is consistent with these results. The model suggests that differential somatic investment is a key transmission channel of ice age conditions on historical comparative development in terms of the timing of the Neolithic transition. Interestingly, Dalgaard and Strulik (2017) suggest that the somatic investment channel might be of direct relevance even today, which lends further credence to the ideas developed in this paper.

3 The Basic Structure of the Model

3.1 Individuals

Consider an overlapping generations economy in which economic activity extends over infinite discrete time. In every period t, a new generation of individuals is born. Reproduction is asexual, i.e., each individual has a single parent. Individuals are identical and live for two periods (childhood and adulthood). In their childhood, individuals consume part of their parental income. In their adulthood, individuals work as hunter-gatherers and produce a single homogenous good. Adults allocate their income between quantity and quality of offspring, i.e., they choose the number of children, n_t, and the amount of somatic capital, k_{t+1}, invested in each of these. The model abstracts from adult consumption.

3.2 The Skill Intensity of the Environment

The environment of a society differs with respect to the suitability for hunting and gathering. Foraging is comparatively easy in certain areas but difficult in others. To capture this fact, we introduce a parameter $\xi_t \geq 0$ that represents the skill intensity of the environment. A skill intensity of zero represents an environment that is perfectly suited for hunting and gathering. Foraging becomes more difficult if ξ_t increases. Foraging was especially difficult in the harsh and challenging climate of Eurasia during the ice age, i.e., $\xi_t \gg 0$. The skill intensity of the environment changes over time. The climate record in Figure 1 draws a detailed picture of the climatic challenges in the northern hemisphere over large time-scales.

3.3 The Production of Final Output

Consider a population of L_t adult hunter-gatherers that support their progeny on a fixed amount X of land or resources. In every period t, the economy produces a single homogenous good, Y_t, with aggregate number of efficiency units of labor H_t and land X as inputs. The supply of land is exogenously given and constant over time. The markets of labor and the final good are perfectly competitive.

Let A_t be the level of productivity and H_t the aggregate number of efficiency units of labor in period t. The output produced is given by the neoclassical production

function

$$Y_t = (A_t X)^\alpha (H_t)^{1-\alpha}, \tag{1}$$

where $\alpha \in (0,1)$. For simplicity, the supply of land is normalized to one, $X \equiv 1$. In the absence of property rights over land, the return to land is zero. Therefore, the wage rate per efficiency unit of labor in period t, w_t, is given by the average product per efficiency unit of labor,

$$w_t = \frac{Y_t}{H_t} = \left(\frac{A_t}{H_t}\right)^\alpha. \tag{2}$$

3.4 Efficiency Units of Labor

Parental investment in the somatic capital of their children, k_{t+1}, is the single source for the number of efficiency units of labor, h_{t+1}, of a member of generation $t+1$. As somatic capital constitutes the bodily basis for skill formation, it is positively correlated with the number of efficiency units of labor, especially since formal education was absent in prehistoric times. Furthermore, it is an important feature of this model that somatic investment has to be evaluated in the skill context of the environment. That is, the negative effect of a high skill intensity of the environment on the number of efficiency units of labor can be compensated by higher somatic investment.

We assume that a minimum level of somatic investment, $\tilde{k}$, is required for participation in the labor market. For $k_{t+1} \geq \tilde{k}$, let the number of efficiency units of labor of a member of generation $t+1$, h_{t+1}, be a strictly concave function of the level of somatic investment in childhood, k_{t+1}, and a strictly convex function of the skill intensity of the environment during childhood, ξ_t, i.e.,

$$h_{t+1} = h(k_{t+1}, \xi_t) = \begin{cases} = 0 & \text{if} \quad k_{t+1} < \tilde{k} \\ > 0 & \text{if} \quad k_{t+1} \geq \tilde{k}, \end{cases} \tag{3}$$

with $h_k > 0$, $h_{kk} < 0$, $h_\xi < 0$, $h_{\xi\xi} > 0$ for $k_t \geq \tilde{k}$. If the skill intensity of the environment increases, the number of efficiency units of labor decreases to zero in the absence of further somatic investment: $\lim_{\xi_t \to \infty}(k_{t+1}, \xi_t) = 0$. Moreover, there is no upper bound on the number of efficiency units of labor if somatic investment increases: $\lim_{k_{t+1} \to \infty}(k_{t+1}, \xi_t) = \infty$.

We assume that somatic investment complements the skill intensity of the environment,

(A1) $h_{k\xi}(k_{t+1}, \xi_t) > 0$ for $k_{t+1} \geq \tilde{k}$,

i.e., the adverse effect of a high skill intensity of the environment can be overcompensated by higher somatic investment in each child. Furthermore, we assume that the elasticity, $\eta_{h_k,k}$, of the effect of somatic investment on the number of efficiency units of labor, $h_k(k_{t+1}, \xi_t)$, with respect to somatic investment, k_{t+1}, is smaller than one in absolute value. Hence, an increase in somatic investment generates less than a proportional decrease in the effect of somatic investment on the number of efficiency units of labor,

(A2) $\eta_{h_k,k} \equiv \left| \frac{h_{kk}(k_{t+1}, \xi_t) k_t}{h_k(k_{t+1}, \xi_t)} \right| < 1$ for $k_{t+1} \geq \tilde{k}$,

which assures that the factor demand for somatic investment is elastic.

3.5 Reproductive Fitness

Following the insight of Kaplan (1996), it is a first key assumption of this model that the human fertility system is naturally selected to generate preferences regarding quantity and quality of offspring that are evolutionary optimal in variable environments. Hence, an individual's optimization problem is formulated using reproductive fitness as objective function.[9] That way, the model is deeply rooted in human biology. As will become apparent in Section 3.7, the resulting optimal behavior regarding quantity and quality of offspring is identical to using unified growth preferences as in Galor and Weil (2000).

An individual's level of somatic capital is determined by parental investment during childhood. Hence, an adult member of generation t decides on somatic investment in each child, k_{t+1}. Since the model abstracts from adult consumption, the number of children is given by

$$n_t = \frac{w_t h_t}{k_{t+1}}, \tag{4}$$

where $w_t h_t$ is the adult's income from hunting–gathering activities.

Individuals do not make particular numerical choices of quantity and quality. Rather, preferences are assumed to maximize reproductive fitness, W_t, which is

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[9]Reproductive fitness is typically measured in terms of a quantity, r, called intrinsic rate of increase of a population. r depends on factors like age–specific birth rates, age–specific death rates, and the age distribution, which are not representable in an overlapping–generations setup (Kaplan, 1996: 97).

measured by a dynastic setup in terms of the number of descendants at a distant future. In its simplest form, reproductive fitness of an adult member of generation t is given by

$$W_t = \prod_{j=0}^{\omega} n_{t+j}, \tag{5}$$

where n_{t+j} is the number of children in generation j at time $t + j$. The entire product is equal to the expected number of living descendants after ω generations have passed, with $\omega \gg 0$.

3.6 Optimal Somatic Investment

An adult member of generation t decides on somatic investment in each child, k_{t+1}. Substituting (4) into (5) and taking the logarithm of both sides, the optimization problem of a member of generation t is

$$k_{t+1} = \arg\max \left\{ \sum_{j=0}^{\omega} \left(\ln w_{t+j} + \ln h(k_{t+j}, \xi_{t-1+j}) - \ln k_{t+1+j} \right) \right\} \quad \text{s.t.} \quad k_{t+1} \geq \tilde{k} \tag{6}$$

Optimizing this expression implies that the implicit functional relationship between optimal somatic investment, k_{t+1}, and the skill intensity of the environment, ξ_t, is given by

$$G(k_{t+1}, \xi_t) \equiv \frac{h_k(k_{t+1}, \xi_t)}{h(k_{t+1}, \xi_t)} - \frac{1}{k_{t+1}} \begin{cases} = 0 & \text{if} \quad k_{t+1} > \tilde{k} \\ \leq 0 & \text{if} \quad k_{t+1} = \tilde{k}. \end{cases} \tag{7}$$

Without loss of generality, let $\tilde{k}$ be the optimal level of somatic investment if the skill intensity of the environment is zero, and $\tilde{h}$ the corresponding number of efficiency units of labor,

(A3) $G(\tilde{k}, 0) = 0, \quad \tilde{h} \equiv h(\tilde{k}, 0)$.

Lemma 1. *Under (A1) and (A3), the optimal level of somatic investment is a unique single–valued function of the skill intensity of the environment,*

$$k_{t+1} = k(\xi_t),$$

where $k(0) = \tilde{k}$ and $k(\xi_t) > \tilde{k}$ for all $\xi_t > 0$. Optimal somatic investment increases with the skill intensity of the environment,

$$k'(\xi_t) > 0.$$

Proof. Follows from the properties of (7) and Implicit Function Theorem together with (A1) and (A3). See the proof of Corollary 1 for an explicit calculation of $k'(\xi_t)$. □

Corollary 1. *Under (A1), (A2), and (A3), the optimal number of efficiency units of labor is a unique single–valued function of the skill intensity of the environment,*

$$h_{t+1} = h(k(\xi_t), \xi_t) \equiv h(\xi_t),$$

where $h(0) = \tilde{h}$ and $h(\xi_t) > \tilde{h}$ for all $\xi_t > 0$. The optimal number of efficiency units of labor increases with the skill intensity of the environment,

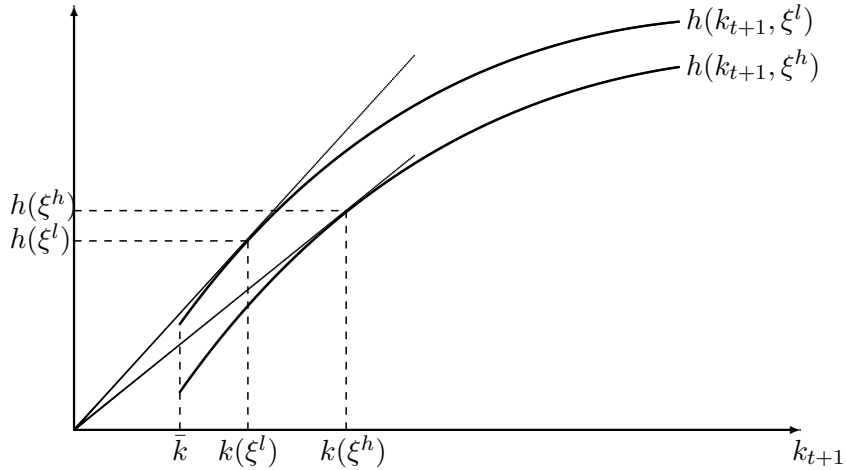$$h'(\xi_t) > 0.$$

Proof. See Appendix A. □

Figure 4: Optimal Somatic Investment for Two Skill intensities, $\xi^h > \xi^l$.

Equation (7) is depicted in Figure 4. Optimal somatic investment is given by the unique tangency point between the efficiency units of labor function, $h(k_{t+1}, \xi_t)$, and a ray from the origin. An increase in the skill intensity of the environment from ξ^l to ξ^h shifts the efficiency units of labor function downwards. As long as (A1) is satisfied, optimal somatic investment rises from $k(\xi^l)$ to $k(\xi^h)$. Furthermore, as long as (A2) is satisfied, the adverse effect of the shift on the number of efficiency units

of labor is overcompensated by the increase in optimal somatic investment. That is, the optimal number of efficiency units of labor rises from $h(\xi^l)$ to $h(\xi^h)$.

Clearly, the theory suggests that an increase in the skill intensity of the environment triggers an adaptive response in individuals to increase somatic investment in offspring. As a consequence, a child's number of efficiency units of labor increases as well. This captures the fundamental idea that foraging skills become more important for hunting and gathering in a challenging environment. The effect of an increase in the skill intensity of the environment on optimal somatic investment, $k(\xi_t)$, and on the optimal number of efficiency units of labor, $h(\xi_t)$, is quantified in the following Corollary:

Corollary 2. *Let $\varepsilon_{k,\xi}$ and $\varepsilon_{h,\xi}$ be the elasticities of optimal somatic investment, $k(\xi_t)$, and the optimal number of efficiency units of labor, $h(\xi_t)$, with respect to the skill intensity of the environment,*

$$\varepsilon_{k,\xi} \equiv \frac{k'(\xi_t)\xi_t}{k(\xi_t)}, \qquad\qquad \varepsilon_{h,\xi} \equiv \frac{h'(\xi_t)\xi_t}{h(\xi_t)}.$$

It then follows that $\varepsilon_{k,\xi} > \varepsilon_{h,\xi} > 0$. Furthermore, let ε be the relative difference between both elasticities. It immediately follows that

$$\varepsilon \equiv \frac{\varepsilon_{k,\xi} - \varepsilon_{h,\xi}}{\varepsilon_{k,\xi}} < 1.$$

Proof. See Appendix A. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

Hence, if the skill intensity of the environment increases both the optimal level of somatic investment and the optimal number of efficiency units of labor increase as well. The rise in optimal somatic investment, however, is relatively larger, as depicted in Figure 4.

3.7 A Deep–Rooted Foundation for Unified Growth Preferences

It is a key feature of this model that an individual's optimization problem is formulated using reproductive fitness as objective function. That way, the model is deeply rooted in human evolutionary biology. The following proposition, however, shows that the optimal behavior regarding quantity and quality of children is the same if the optimization problem is formulated using a utility function known from Unified Growth Theory (Galor, 2011).

Proposition 1. *Adult members of generation t that choose the quantity and quality according to*

$$k_{t+1} = \arg\max \left\{ n_t w_{t+1} h_{t+1} \right\} \qquad s.t. \qquad k_{t+1} \geq \tilde{k}$$

maximize reproductive fitness W_t. Abstracting from adult consumption, c_t, this objective function is known from Unified Growth Theory (Galor and Weil, 2000: 812),

$$u^t = (c_t)^{1-\gamma}(n_t w_{t+1} h_{t+1})^{\gamma} \qquad s.t. \qquad c_t \geq \tilde{c}.$$

Consequently, individuals that choose the quantity and quality of their children so as to maximize their aggregate income behave optimally in evolutionary terms.

Proof. See Appendix A. □

This is an important first result. Abstracting from adult consumption, humans seem to be naturally selected to maximize the aggregate income of their children. Hence, the theory provides a deep–rooted foundation for the utility function used in Unified Growth Theory.

4 The Time Paths of the Macroeconomic Variables

4.1 The Dynamics of Population Size

Consider a population of L_t adult hunter-gatherers. The aggregate number of efficiency units of labor at time t is given by the linear relationship

$$H_t = h_t L_t. \tag{8}$$

It is convenient to use the term population size for both expressions, H_t and L_t. The population size in period $t + 1$ is determined by the size of the population in the previous generation and the number of children per adult, n_t. Hence, it follows from (2), (4), and (8) that the population size evolves over time according to

$$H_{t+1} = h_{t+1} n_t L_t = \frac{h(\xi_t)}{k(\xi_t)} A_t^{\alpha} H_t^{1-\alpha} \equiv H(H_t, A_t, \xi_t), \tag{9}$$

where the initial size of the population, H_0, is exogenously given.

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4.2   The Dynamics of Knowledge

Although Paleolithic societies did not support a class of specialized knowledge creators, the term research is used in this paper to indicate improvements of the stock of knowledge, κ_t. Innovations in the Upper Paleolithic between 50 and 10 kya have been things like bone awls, bone points, stone lamps, heated shelters, eyed needles, tailored clothing, storage pits, domesticated dogs, fishhooks, bow and arrow, and so on. Klein (2009, p. 672–682) provides a brief overview of Upper Paleolithic technology.

In a Malthusian framework, the process of knowledge creation is typically viewed as driven by population size (Kremer, 1993). We follow this view and assume that the creation of new knowledge depends on the size of the population, H_t. However, as discussed in Section 2, there exists a significant correlation between body and brain size on the one hand and brain size and intelligence on the other hand. Hence, it is not only the quantity, but also by the quality of individuals that matters for the creation of new knowledge. To reflect this, we assume that an individual's capacity to innovate increases with parental investment during childhood, i.e., the process of knowledge creation is complemented by an individual's endowment with somatic capital.

Together, the advancement in the stock of knowledge between periods t and $t+1$, $\Delta\kappa_t$, is a function of the quantity and the quality of individuals, H_t and $k(\xi_t)$. In particular, we assume the following functional form

$$\Delta\kappa_t = \kappa_{t+1} - \kappa_t = k(\xi_t)^{1-\lambda} H_t^{\lambda}, \tag{10}$$

where the initial stock of knowledge, $\kappa_0 > 0$, is given, and

(A4) $0 < \lambda < \frac{\alpha}{\alpha+\varepsilon}$.

This is a simple Cobb–Douglas function that captures the idea that both–the quantity and the quality of individuals–have an effect on the process of knowledge creation. Note that $\lambda < 1$ follows with $\varepsilon > 0$ from Corollary 2.

4.3   The Dynamics of Productivity

The development of the level of productivity, A_t, over time is governed by two opposing processes: On the one hand, productivity is influenced positively by the creation

of new knowledge. On the other hand, productivity is influenced negatively by the skill intensity of the environment, which reflects the idea that harsh environmental conditions adversely affect the production of final output both directly and indirectly: directly via a negative externality on productivity itself, and indirectly via a negative effect on the number of efficiency units of labor that fitness maximizing individuals take into account.

Thus, the level of productivity is enhanced by the advancement in the stock of knowledge, $\Delta\kappa_t$, and diminished at a rate $\delta(\xi_t)$ that captures the erosion in productivity due to climatic challenges. In particular, we assume the following functional form for the erosion in productivity,

$$\delta(\xi_t) = e^{-\xi_t} \tag{11}$$

which simplifies the exposition of the model.[10] Together, the evolution of the productivity parameter between periods t and $t+1$, as follows from (10) and (11), is determined by

$$A_{t+1} = e^{-\xi_t}k(\xi_t)^{1-\lambda}H_t^\lambda A_t \equiv A(H_t, A_t, \xi_t), \tag{12}$$

where the initial level of productivity, $A_0 > 0$, is given. Furthermore, let g_t denote the growth rate of productivity between periods t and $t+1$. It follows directly from (12) that

$$g_t = \frac{A_{t+1} - A_t}{A_t} = e^{-\xi_t}k(\xi_t)^{1-\lambda}H_t^\lambda - 1 \equiv g(H_t, \xi_t) \tag{13}$$

which can be positive or negative in general.

4.4 The Transition to Agriculture

This paper promotes the idea that a region–specific rise in somatic investment has significantly accelerated pre–transition growth rates and therefore expedited the

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[10]The concrete functional form is not relevant for the outcome of the model. Rather, it is possible to work with the general function, $\delta(\xi_t)$, as long as the following additional assumptions,

$$\delta(0) = 1, \qquad \lim_{\xi_t \to \infty} \delta(\xi_t) = 0, \qquad \delta'(\xi_t) < 0, \qquad \lim_{\xi_t \to \infty} \left| \frac{\delta'(\xi_t)\xi_t}{\delta(\xi_t)} \right| \geq \varepsilon_{k,\xi},$$

are fulfilled. Furthermore, the key results of this paper hold independently of the erosion assumption. However, erosion of productivity allows the derivation of an optimal growth rate of productivity with respect to the skill intensity of the environment, which is a realistic and desirable feature of the model.

transition to agriculture in areas with a high skill intensity of the environment. To show this formally, it is assumed that a certain level of productivity,

(A5) $A^N > A_0,$

is necessary for the Neolithic transition. A hunter–gatherer society that exhibits higher growth rates will reach this level of productivity earlier in time.

Note that this assumption is very simplistic. It is illusory to expect to identify some precise moment in history when foragers decided to take their first steps as farmers. According to Barker (2006: 31), the transition from hunting to farming must be understood in terms of gradually evolving relationships between people, animals, and plants. Nevertheless, this simplification is helpful to clearly state the key issue of this paper.

5   The Dynamical System

The development of the economy is characterized by the evolution of the level of productivity, the population size, and the skill intensity of the environment. It is determined by a sequence $\{A_t, H_t, \xi_t\}_{t=0}^{\infty}$ that satisfies the following discrete dynamical system in every period t,

$$
\begin{cases}
H_{t+1} = H(H_t, A_t, \xi_t) \\
A_{t+1} = A(H_t, A_t, \xi_t)
\end{cases}
\tag{14}
$$

where the evolution of the skill intensity of the environment $\{\xi_t\}_{t=0}^{\infty}$ is exogenously given. A trajectory according to Figure 1 seems appropriate with respect to the time span of the model. However, keeping the dynamic character of long–term climate at the back of one's mind, it is sufficient to treat the skill intensity of the environment as fixed and do a comparative static analysis. Hence, the dynamical system is analyzed in three steps. First, we assume a fixed level of the skill intensity of the environment and a fixed level of productivity, and we characterize the evolution of population size towards a conditional steady state equilibrium. Second, we relax the assumption of fixed productivity and study global dynamics for a constant skill intensity of the environment. Third, we consider an increase in the skill intensity of the environment and derive the consequences for the timing of the Neolithic transition.
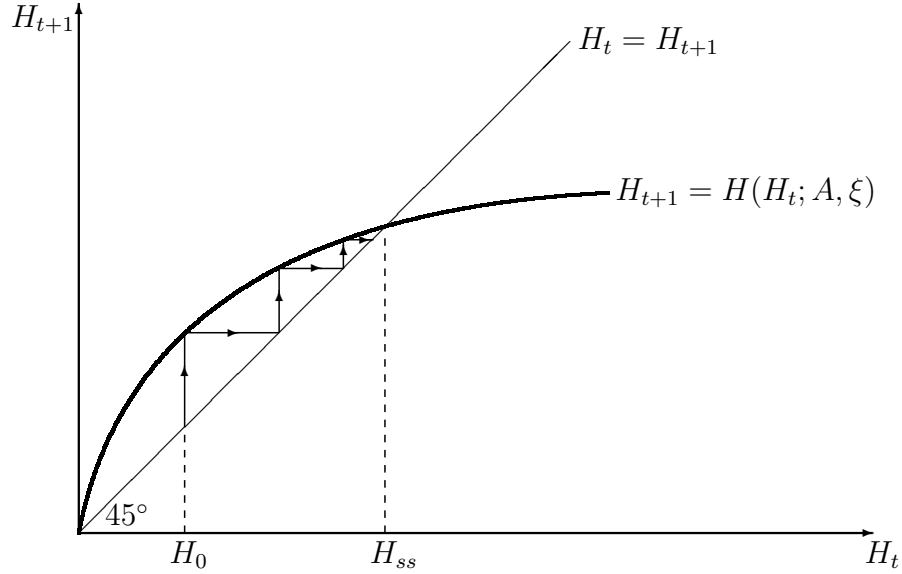
Figure 5: The Conditional Dynamical System for a Fixed Level of Productivity, A.

5.1 The Conditional Steady State Equilibrium of Population Size

Initially, suppose a fixed level of the skill intensity of the environment, ξ, and a fixed level of productivity, A. The conditional evolution of population size is characterized by a sequence $\{H_t\}_{t=0}^{\infty}$ that satisfies the following nonlinear difference equation in every period t:

$$H_{t+1} = \frac{h(\xi)}{k(\xi)} A^{\alpha} H_t^{1-\alpha}. \tag{15}$$

This dynamical system is depicted in Figure 5. It is characterized by a unique globally stable steady state equilibrium, H_{ss}. Given H_0, the state variable H_t evolves towards the steady state equilibrium, i.e., $\lim_{t\to\infty} H_t = H_{ss}$. The position of the steady state equilibrium depends on the level of productivity, A, and the skill intensity of the environment, ξ.

Lemma 2. *Let $(A, \xi) > 0$ be fixed. There exists a unique globally stable steady state equilibrium, H_{ss}, of the conditional dynamical system (15). H_{ss} is a function of the level of productivity, A, and the skill intensity of the environment, ξ,*

$$H_{ss} = A \left(\frac{h(\xi)}{k(\xi)} \right)^{\frac{1}{\alpha}} \equiv H_{ss}(A, \xi),$$

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where $\frac{\partial H_{ss}(A,\xi)}{\partial A} > 0$ *and* $\frac{\partial H_{ss}(A,\xi)}{\partial \xi} < 0.$

Proof. The functional form follows from (15) with $H_{t+1} = H_t$. The derivatives can be calculated with the help of Corollary 2. Existence and uniqueness follow from $\frac{\partial H_{t+1}}{\partial H_t}\big|_{H_{ss}} = 1 - \alpha < 1.$ $\qquad \square$

A rise in the skill intensity of the environment reduces the equilibrium value of the aggregate number of efficiency units of labor. The intuition for this result is simple. A rise in the skill intensity leads to higher somatic investment per child. Individuals get less children of higher quality. The increase in quality, however, cannot compensate the loss in quantity, since the number of efficiency units of labor is a strictly concave function of somatic investment, $h_k > 0$ and $h_{kk} < 0$. Thus, the aggregate number of efficiency units of labor decreases.

In the following, we will refer to a decrease in the aggregate number of efficiency units of labor as a decrease in population size. This is a only a small loss in verbal precision, but it helps to get a better intuition for the results of this paper.

Noting (13) and Lemma 2, the conditional steady–state equilibrium is characterized by the following (fictitious) growth rate of productivity,

$$g(H_{ss}(\xi), \xi) = e^{-\xi} k(\xi)^{1-\lambda} \left(\frac{h(\xi)}{k(\xi)} \right)^{\frac{\lambda}{\alpha}} A^\lambda - 1 \equiv g_{ss}(A, \xi) \qquad (16)$$

which can be positive or negative in general. In order to permit a positive growth rate of productivity at least for small values of the skill intensity of the environment, it is assumed that

(A6) $\qquad g_{ss}(A_0, 0) = \tilde{k}^{1-\lambda} \left(\frac{\tilde{h}}{\tilde{k}} \right)^{\frac{\lambda}{\alpha}} A_0^\lambda - 1 > 0.$

The properties of the growth rate of productivity at the conditional steady state are derived in the following lemma.

Lemma 3. 1. *Under (A4), the first derivatives of $g_{ss}(A, \xi)$ are given by*

$$\frac{\partial g_{ss}(A, \xi)}{\partial A} > 0 \qquad and \qquad \frac{\partial g_{ss}(A, \xi)}{\partial \xi} \gtreqqless 0 \qquad if \ and \ only \ if \qquad \xi \lesseqqgtr \xi^*,$$

where $\xi^* \equiv \varepsilon_{k,\xi} \left(1 - \frac{\alpha + \varepsilon}{\alpha} \lambda \right)$ with $0 < \xi^* < \varepsilon_{k,\xi}$.

2. *Under (A4) and (A6), it follows that $\lim_{\xi \to \infty} g_{ss}(A, \xi) = -1$. Moreover, there exists a unique $\hat{\xi} = \hat{\xi}(A) > \xi^*$ with $\hat{\xi}'(A) > 0$ so that*

$$g_{ss}(A, \xi) \gtreqqless 0 \qquad if \ and \ only \ if \qquad \xi \lesseqqgtr \hat{\xi}.$$

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Proof. See Appendix A. □

The growth rate of productivity at the conditional steady–state equilibrium, $g_{ss}(A, \xi)$, has a unique root, $\hat{\xi}$, in the domain (ξ^*, ∞). It is positive for all values below the root and negative for all values above the root. As depicted in Figure 6, it has a hump–shaped relationship with the degree of environmental skill intensity. An increase in the level of productivity, A, shifts the function upwards and the unique root moves to the right. Thus, the model predicts that societies with an advanced level of technology are able to inhabit environments that are characterized by a harsher climate, which is very plausible. Note further that the growth rate is monotonically increasing for $\xi < \xi^*$. In this range, a rise in the skill intensity of the environment leads to higher growth rates of productivity, which is the basis for the key result of this paper.
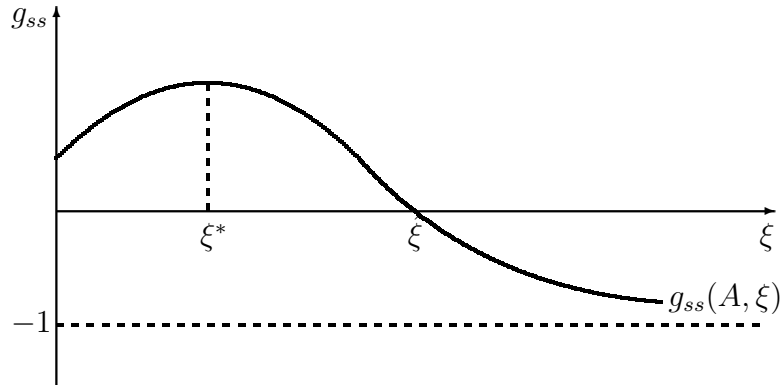

Figure 6: The Effect of the Skill intensity on the Growth Rate of Productivity at the Conditional Steady State Equilbrium.

5.2 The Common Evolution of Population and Productivity

In this section, we relax the assumption of a fixed level of productivity and study global dynamics for a constant skill intensity of the environment, ξ. The development of the economy is characterized by a sequence $\{H_t, A_t\}_{t=0}^{\infty}$ that satisfies the following two–dimensional system in every period t:

$$
\begin{cases}
H_{t+1} = \frac{h(\xi)}{k(\xi)} A_t^{\alpha} H_t^{1-\alpha} \\
A_{t+1} = e^{-\xi} k(\xi)^{1-\lambda} H_t^{\lambda} A_t.
\end{cases}
\tag{17}
$$

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If for a given initial level of productivity, $A_0 > 0$, and the initial population size is large enough, $H_0 \gg 0$, this dynamical system is dominated by hyperbolic growth in the population size, H_t, and the level of productivity, A_t. To see this, note that the growth rate of productivity increases with the level of population and that the growth rate of population increases with the level of productivity. Thus, feedback effects between population and productivity lead to increasing growth rates over time. Hyperbolic growth is faster than exponential growth, where the growth rate is constant.
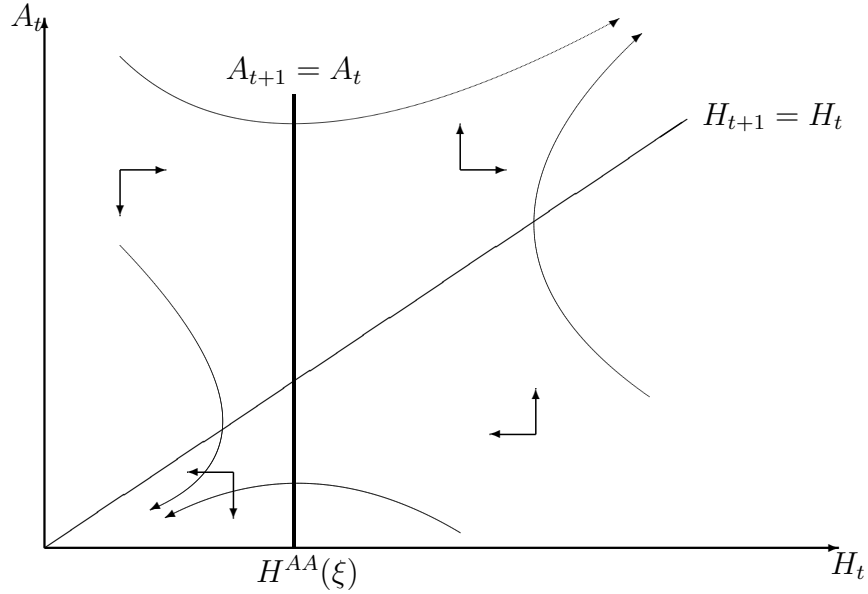

Figure 7: The Common Evolution of Population and Productivity in a Phase Diagram.

The phase diagram in Figure 7 illustrates the common evolution of population and productivity. It contains an *HH locus*, which denotes all pairs (H_t, A_t) such that the population size is constant,

$$HH \equiv \left\{ (H_t, A_t) : H_{t+1} - H_t = \frac{h(\xi)}{k(\xi)} A_t^\alpha H_t^{1-\alpha} - H_t = 0 \right\}, \tag{18}$$

and an *AA locus*, which denotes all pairs (H_t, A_t) such that the level of productivity is constant,

$$AA \equiv \left\{ (H_t, A_t) : A_{t+1} - A_t = e^{-\xi} k(\xi)^{1-\lambda} H_t^\lambda A_t - A_t = 0 \right\}. \tag{19}$$

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Lemma 4. *If $(H_t, A_t) \in HH$, it follows that*

$$A_t = \left(\frac{k(\xi)}{h(\xi)} \right)^{\frac{1}{\alpha}} H_t \equiv A^{HH}(H_t, \xi)$$

where $\frac{\partial A^{HH}(H_t,\xi)}{\partial H_t} > 0$, $\frac{\partial^2 A^{HH}(H_t,\xi)}{(\partial H_t)^2} = 0$, and $\frac{\partial A^{HH}(H_t,\xi)}{\partial \xi} > 0$. Furthermore,

$$H_{t+1} - H_t \gtreqless 0 \qquad \text{if and only if} \qquad A_t \gtreqless A^{HH}(H_t, \xi).$$

Proof. The functional form follows with $H_{t+1} = H_t$ from (9). The derivatives are readily given with Corollary 2. The second part follows from (2) and (4) since $n_t = 1$ along the HH locus. $\qquad\square$

Hence, the HH locus is an upward sloping ray from in the origin in the (H_t, A_t) space. The population increases over time in the region above the locus since a population size smaller than the steady state value increases the wage rate and allows fertility to be above replacement level. In contrast, the population decreases over time in the region below the locus since a population size larger than the steady state value reduces the wage rate such that fertility drops below replacement level. Furthermore, if the skill intensity of the environment increases, the HH locus rotates upward in the (H_t, A_t) space.

Lemma 5. *If $(H_t, A_t) \in AA$, it follows that*

$$H_t = \left(\frac{e^\xi}{k(\xi)^{1-\lambda}} \right)^{\frac{1}{\lambda}} \equiv H^{AA}(\xi),$$

where $H^{AA}(\xi) > 0$ and $\frac{\partial H^{AA}(\xi)}{\partial \xi} \gtreqless 0$ if and only if $\xi \gtreqless (1-\lambda)\varepsilon_{k,\xi}$. Furthermore,

$$A_{t+1} - A_t \gtreqless 0 \qquad \text{if and only if} \qquad H_t \gtreqless H^{AA}(\xi).$$

Proof. Follows immediately from the definition of the AA locus in (19). $\qquad\square$

Hence, the AA locus is a vertical line in the (H_t, A_t) space. The level of productivity increases over time in the region to the right of the locus. It decreases over time in the region to the left of the locus. For $\xi > (1-\lambda)\varepsilon_{k,\xi}$, the AA locus shifts to the right in the (H_t, A_t) space if the skill intensity of the environment increases. For $\xi < (1-\lambda)\varepsilon_{k,\xi}$, if shifts to the left. As the arrows indicate, the economy either enters the upper right area of the phase diagram in Figure 7 and

winds up to a situation of ever increasing population and productivity, or enters the lower left area and collapses to the origin. Which situation occurs, depends on the initial values of A_0 and H_0. If for a given initial level of productivity, $A_0 > 0$, the initial population size is large enough, $H_0 \gg 0$, the economy approximately follows a hyperbolic growth pattern. Furthermore, if the skill intensity of the environment is high, $\xi > (1 - \lambda)\varepsilon_{k,\xi}$, the population size required for a growing economy is much higher, as the AA locus shifts to the right.

Proposition 2. *Let the skill intensity of the environment be fixed, $\xi \geq 0$. If for a given level of productivity, $A_0 > 0$, the initial population size is large enough, $H_0 \gg 0$, the dynamical system (14) exhibits hyperbolic growth in population and productivity. That is, H_t and A_t grow proportionally to their level,*

$$\frac{H_{t+1} - H_t}{H_t} \sim H_t \qquad and \qquad \frac{A_{t+1} - A_t}{A_t} \sim A_t.$$

Proof. See Appendix A. $\qquad\qquad\square$

This prediction of the model is entirely in line with empirical data about the prehistoric growth of the world population (Kremer, 1993). Specifically, the model approximately behaves like a set of two difference equations that Korotayev, Malkov and Khaltourina (2006, p. 24) propose for the simulation of world population growth. They show in a simulation that the model replicates the population estimates of Kremer (1993) in a reasonable way. In this case, the productivity parameter, A, can directly be interpreted as maximum population capacity.

5.3 Skill Intensity and Comparative Economic Development

In the following, we relax the assumption of a fixed skill intensity of the environment and analyze the effect of an increase in the skill intensity on comparative economic development. The aim is to show that an increase in the skill intensity of environment induces higher somatic investment in offspring, leading to a higher growth rate of productivity, and an earlier transition to agriculture–at least as long as the skill intensity is not too large. The analysis is divided into two parts. First, we consider the effect of a rise in the skill intensity on the conditional dynamical system (15). This can be understood as short-run dynamics. Second, we extend the analysis to the long–run and show that the pattern of hyperbolic growth reinforces the
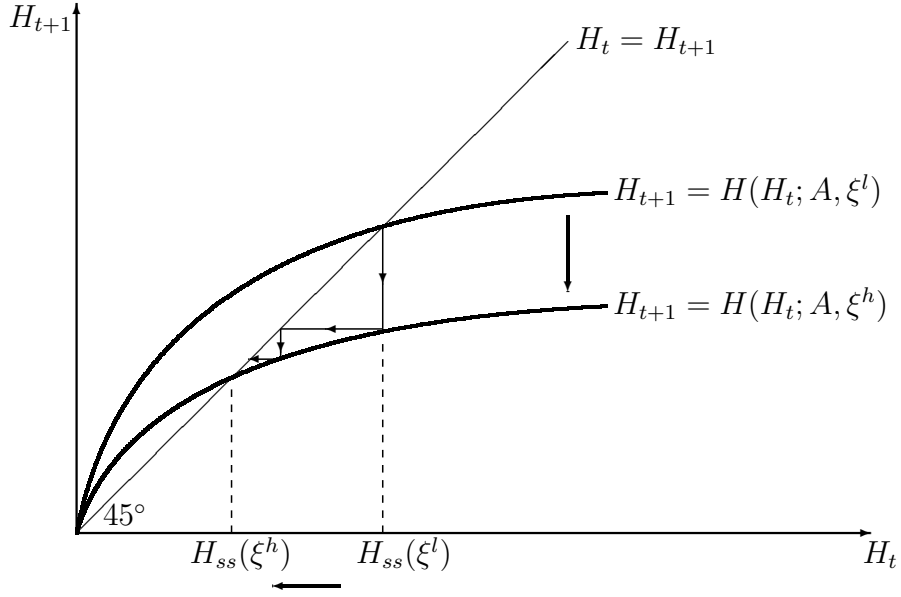
Figure 8: The Effect of an Increase in the Skill intensity for a Fixed Level of Productivity, A.

short–run effects, permanently rising the growth of productivity and accelerating the Neolithic transition.

Proposition 3. *Let $A > 0$ be fixed. Under assumptions (A4) and (A6), an increase in the skill intensity of the environment, as long as the skill intensity is not too large, has a beneficial effect on somatic investment per child and on the growth rate of productivity at the conditional steady–state equilibrium, but a detrimental effect on the steady–state level of population size, i.e., for all $\xi < \xi^*$,*

$$\frac{\partial k(\xi)}{\partial \xi} > 0, \qquad \frac{\partial g_{ss}(A,\xi)}{\partial \xi} > 0, \qquad \frac{\partial H_{ss}(A,\xi)}{\partial \xi} < 0.$$

Proof. Follows immediately from Lemma 1, Lemma 2, and Lemma 3. □

Hence, if the skill intensity of the environment rises from ξ^l to ξ^h with $\xi^l < \xi^h \leq \xi^*$ at time t, there is a jump up in somatic investment to $k_{t+1} = k(\xi^h) > k(\xi^l)$, as depicted in Figure 4. Adult individuals choose less children of higher quality. Consequently, as depicted in Figure 8, the population size decreases monotonically to the new steady state equilibrium in subsequent periods, $\lim_{t\to\infty} H_t \downarrow H_{ss}(A,\xi^h) < H_{ss}(A,\xi^l)$. Moreover, the rise in somatic investment leads to a jump up in the growth

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rate of productivity at the conditional steady state equilibrium which monotonically decreases to the new steady state value in subsequent periods due to the loss in population size, $\lim_{t\to\infty} g_t \downarrow g_{ss}(A, \xi^h) > g_{ss}(A, \xi^l)$.

Therefore, if the level of productivity is fixed, an economy that is characterized by a relatively high skill intensity of the environment is associated with a relatively superior conditional steady state equilibrium in terms of somatic investment per child and productivity growth. Intuitively, as long as the skill intensity of the environment is not too large, the beneficial role of increased skill intensity on the level of productivity via the creation of new knowledge outweighs its detrimental effect via the erosion of productivity.

A rise in the skill intensity, however, also has an effect on the timing of the Neolithic transition. This effect is summarized in the following proposition.

Proposition 4. *Let A_0 be smaller than A^N. Under assumptions (A4), (A5) and (A6), an increase in the skill intensity of the environment from ξ^l to ξ^h at time t, as long as the skill intensity is not too large, has a beneficial effect on the sequence of growth rates, $\{g_{t+j}\}_{j=0}^{\infty}$, and thus on the timing of the Neolithic transition. That is, for all $\xi^l < \xi^h \leq \xi^*$ and $j = 0, 1, 2, \ldots,$*

$$g(H_{t+j}, \xi^h) > g(H_{t+j}, \xi^l).$$

Proof. See Appendix A. □

Intuitively, the economy is characterized by hyperbolic growth in population and technology as long as the economy is located in the upper right area of the phase diagram in Figure 7. The growth rate of population size is therefore positive. Short–run dynamics, on the other hand, imply a negative rate of population growth for several subsequent periods. The net rate of population growth after a rise in the skill intensity can therefore be positive or negative in general. It is clear from Proposition 3, however, that the growth rate of productivity would increase even if the population size decreases instantaneously to the new (lower) steady state value. Consequently, the growth rate of productivity increases even more. Clearly, (A4) is only a sufficient condition for accelerated productivity growth after an increase in the skill intensity. It is not a necessary condition as it can be relaxed to some degree.

Following Proposition 4, the growth rate of productivity is monotonically increasing in the skill intensity of the environment at least up to the critical level,

ξ^*. This is directly associated with an earlier Neolithic transition to agriculture. In contrast, if the skill intensity of the environment is too large, an increase in the skill intensity raises the creation of new knowledge but its eroding effect on productivity increases even more such that the net effect on the growth rate of productivity is negative. This can be associated with a delayed Neolithic transition. However, if the growth rate of productivity falls below zero, an increase in the skill intensity of the environment will lead to a collapse of the economy. Intuitively, the right–shift of the AA locus in Figure 7 is large enough such that the economy enters the lower left area of the phase diagram and collapses to the origin.

It is the central idea of this paper that the dispersal anatomically modern humans out of Africa into western Asia and eastern Europe between 45 and 40 kya and to western Europe between 40 and 36 kya can be modeled by an increase in the skill intensity of the environment. Living in the cold and harsh climate of Eurasia at that time triggered an behavioral adaptation in human populations. Adult hunter–gatherers increased somatic investment in offspring at the expense of fertility. Thus, population growth slowed down or decreased at first. Higher somatic investment in offspring, however, increased the innovation capability of individuals and permanently accelerated the rate of productivity growth, at least as long as the increase in the skill intensity of the environment was not too large. Eventually, the Neolithic transition to agriculture was significantly accelerated.

Corollary 3. *Consider two societies indexed by $i \in \{A, E\}$. Suppose that society E migrates out of Africa into Eurasia, which is characterized by a higher skill intensity of the environment, $\xi^A < \xi^N < \xi^*$, where ξ^i is the skill intensity of the environment of society i. Society E will then be characterized by an earlier Neolithic transition to agriculture than society A.*

Proof. Follows immediately from Proposition 4. □

6   Conclusion

The present paper develops a model of long–run growth in population and technology that explains the ecogeographic patterning in the timing of the Neolithic transition by introducing somatic capital as a hitherto neglected variable. The model has three key features. First, it uses somatic capital as a variable that replicates the fundamental trade–off between quantity and quality of offspring. Somatic investment in

offspring is modeled to maximize reproductive fitness and to respond adaptively to variable environmental conditions. Thus, the theory is deeply rooted in the evolutionary history of our species, which has been shaped by biological constraints, at least until very recent times.

Second, the model uses somatic capital as a variable that increases the innovation capability of individuals. This link seems plausible, since common measures of embodied energy investment like body size or stature are significantly correlated with brain size or intelligence. Consequently, the model advances the concept of human capital one step further to the physical level. While ideas and human capital accumulation have been identified as key drivers of economic growth, the role of somatic capital as bodily foundation of human capital accumulation has not been considered yet.

Third, the model proposes climate–induced variation in human somatic capital as an indirect link between climate and the timing of the Neolithic transition. While it is generally accepted today that climate has played a key role in the transition from foraging to farming, the impact of this factor is usually argued to be direct, i.e., by changing biogeographic endowments or by triggering population pressure during climatic downturns. The indirect channel proposed in this paper is interesting because it arguably had an influence on both, the timing of the transition and the rapid growth thereafter. That way, the theory suggests climate–induced differential somatic investment as a deep–rooted determinant of comparative economic development.

The last point is also interesting with respect to the empirical relationships of Olsson and Hibbs (2005), Putterman (2008), and Putterman and Weil (2010). The empirical findings between the timing of the Neolithic Revolution and modern per–capita incomes could be spurious, if some omitted variable caused both an early switch to agriculture and rapid subsequent growth. The indirect mechanism in this model suggests that somatic capital might be such a variable provided that the pattern of differential somatic investment hat some persistence beyond the Neolithic transition. This could be the case as Ruff (1994) demonstrates an ecogeographic patterning in body size for living humans as well. This is, however, beyond the scope of the model.

With respect to the object function, the paper shows that using reproductive fitness leads to identical behavior regarding quantity and quality of offspring to using

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unified growth preferences. Apparently, humans seem to be naturally selected to maximize the aggregate income of their children. Hence, the theory provides a deep-rooted foundation for the utility function commonly used in Unified Growth Theory (Galor, 2011). Interestingly, our evolutionary–shaped preferences regarding quantity and quality of offspring don't seem to be fitness-maximizing in the modern era of sustained economic growth (Kaplan, 1996). As Galor and Moav (2002) demonstrate, the demographic transition reverses the evolutionary advantage to offspring quantity, which is contrary to the prevalent pattern of high parental and societal investment.

Of course, the model presented in this paper is overly simplistic in several respects. One desirable extension would be to allow adult consumption. A form of adult consumption that integrates well into the life–history perspective of the model is energy expenditure to maintain health. This would replicate the fundamental trade–off between current and future reproduction (Kaplan, 1996). Moreover, it would be interesting to extend the model beyond the adoption of agriculture. The interaction of somatic and human capital in the run–up to the Industrial Revolution might be a promising field of future research.

In his remarkable book, Diamond (1997 p. 52) claimed that "[...] an observer transported back in time 11,000 b.c. could not have predicted on which continent human societies would develop most quickly, but could have made a strong case for any of the continents." The present paper takes up a different view. It argues that the crucial date in history was the Out–of–Africa expansion about 50 kya. At the beginning of the Holocene, human societies already differed remarkably with respect to patterns of somatic investment in offspring. Thus, an observer transported back in time 11 kya could have made a strong case for the continent Eurasia. Living with the ice age for thousands of years had significantly changed the patterns of parental investment in response to environmental and climatic challenges.

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Appendix A. Proofs

Proof of Corollary 1. It follows from (A1), the properties of (7), and the Implicit Function Theorem that the derivate of evolutionary optimal somatic investment is positive and given by

$$k'(\xi_t) = -\frac{h_{k\xi}(k_{t+1},\xi_t)k_{t+1} - h_\xi(k_{t+1},\xi_t)}{h_{kk}(k_{t+1},\xi_t)k_{t+1}} > 0.$$

Using this equation to calculate the derivate of the optimal number of efficiency units of labor with respect to the skill intensity of the environment ξ_t yields

$$
\begin{aligned}
h'(\xi_t) &= \frac{\partial h(k(\xi_t),\xi_t)}{\partial \xi_t} = h_k(k_{t+1},\xi_t)k'(\xi_t) + h_\xi(k_{t+1},\xi_t) \\
&= -h_k(k_{t+1},\xi_t)\frac{h_{k\xi}(k_{t+1},\xi_t)k_{t+1} - h_\xi(k_{t+1},\xi_t)}{h_{kk}(k_{t+1},\xi_t)k_{t+1}} + h_\xi(k_{t+1},\xi_t) \\
&= -h_k(k_{t+1},\xi_t)\frac{h_{k\xi}(k_{t+1},\xi_t)}{h_{kk}(k_{t+1},\xi_t)} + h_\xi(k_{t+1},\xi_t)\left[1 + \frac{h_k(k_{t+1},\xi_t)}{h_{kk}(k_{t+1},\xi_t)k_{t+1}}\right] \\
&= -h_k(k_{t+1},\xi_t)\frac{h_{k\xi}(k_{t+1},\xi_t)}{h_{kk}(k_{t+1},\xi_t)} + h_\xi(k_{t+1},\xi_t)\left[1 - \frac{1}{\eta_{h_k,k}}\right] > 0,
\end{aligned}
$$

where the positive sign follows from the properties of (3), the positivity of $h_{k\xi}(k_{t+1},\xi_t)$ from (A1), and the fact that $\eta_{h_k,k}$ is smaller than one in absolute value from (A2). Finally, $h(\xi_t) > 0$ for $\xi_t \geq 0$ follows from the properties of (3) and (A3). $\qquad\square$

Proof of Corollary 2. To simplify the proof, we define the following elasticities:

$$\eta_{h_k,\xi} \equiv \frac{h_{k\xi}(k_{t+1},\xi_t)\xi_t}{h_k(k_{t+1},\xi_t)} > 0, \quad \eta_{h,\xi} \equiv \left|\frac{h_\xi(k_{t+1},\xi_t)\xi_t}{h_{t+1}}\right| < 1, \quad \eta_{h,k} \equiv \frac{h_k(k_{t+1},\xi_t)k_{t+1}}{h_{t+1}} < 1.$$

They are positive by definition and the latter two are smaller than one in absolute value. Then, using $k'(\xi_t)$ from the proof of Corollary 1, the elasticity $\varepsilon_{k,\xi}$ is given by

$$
\begin{aligned}
\varepsilon_{k,\xi} &= \frac{k'(\xi_t)\xi_t}{k_{t+1}} = -\frac{h_{k\xi}(k_{t+1},\xi_t)k_{t+1} - h_\xi(k_{t+1},\xi_t)}{h_{kk}(k_{t+1},\xi_t)k_{t+1}}\frac{\xi_t}{k_{t+1}} \\
&= -\frac{h_k(k_{t+1},\xi_t)}{h_{kk}(k_{t+1},\xi_t)k_{t+1}}\left[\frac{h_{k\xi}(k_{t+1},\xi_t)\xi_t}{h_k(k_{t+1},\xi_t)} - \frac{h_\xi(k_{t+1},\xi_t)\xi_t}{h_{t+1}}\frac{h_{t+1}}{h_k(k_{t+1},\xi_t)k_{t+1}}\right] \\
&= \frac{1}{\eta_{h_k,k}}\left[\eta_{h_k,\xi} + \frac{\eta_{h,\xi}}{\eta_{h,k}}\right] > 0,
\end{aligned}
$$

which is positive. Furthermore, using $h'(\xi_t)$ from the proof of Corollary 1, the elasticity $\varepsilon_{h,\xi}$ is given by

$$
\begin{aligned}
\varepsilon_{h,\xi} &= \frac{h'(\xi_t)\xi_t}{h_{t+1}} = \left[h_k(k_{t+1},\xi_t)\frac{h_{k\xi}(k_{t+1},\xi_t)}{h_{kk}(k_{t+1},\xi_t)} + h_\xi(k_{t+1},\xi_t)\left(1 - \frac{1}{\eta_{h_k k}}\right) \right]\frac{\xi_t}{h_{t+1}} \\
&= \frac{h_k(k_{t+1},\xi_t)}{h_{kk}(k_{t+1},\xi_t)k_{t+1}}\frac{h_{k\xi}(k_{t+1},\xi_t)\xi_t}{h_k(k_{t+1},\xi_t)}\frac{h_k(k_{t+1},\xi_t)k_{t+1}}{h_{t+1}} + \frac{h_\xi(k_{t+1},\xi_t)\xi_t}{h_{t+1}}\left(1 - \frac{1}{\eta_{h_k,k}}\right) \\
&= \frac{\eta_{h_k,\xi}\eta_{h,k}}{\eta_{h_k,k}} - \eta_{h,\xi}\left(1 - \frac{1}{\eta_{h_k,k}}\right) \\
&= \eta_{h,k}\left[\frac{1}{\eta_{h_k,k}}\left(\eta_{h_k,\xi} + \frac{\eta_{h,\xi}}{\eta_{h,k}}\right)\right] - \eta_{h,\xi} \\
&= \eta_{h,k}\varepsilon_{k,\xi} - \eta_{k,\xi} < \varepsilon_{k,\xi},
\end{aligned}
$$

which is positive and smaller than $\varepsilon_{k,\xi}$ since $\eta_{h,k}$ is smaller than one. $\qquad\square$

Proof of Proposition 1. Since w_t, h_t, and w_{t+1} do not depend on somatic investment, it is clear that the optimization problem as given in (6) can be rewritten as follows:

$$
\begin{aligned}
k_{t+1} &= \arg\max\left\{ \sum_{j=0}^{\omega} \left(\ln w_{t+j} + \ln h(k_{t+j},\xi_{t-1+j}) - \ln k_{t+1+j} \right) \right\} \quad\text{s.t.}\quad k_{t+1} \geq \tilde{k} \\
&= \arg\max\left\{ -\ln k_{t+1} + \ln h(k_{t+1},\xi_t) \right\} \quad\text{s.t.}\quad k_{t+1} \geq \tilde{k} \\
&= \arg\max\left\{ \ln w_t + \ln h_t - \ln k_{t+1} + \ln w_{t+1} + \ln h_{t+1} \right\} \quad\text{s.t.}\quad k_{t+1} \geq \tilde{k} \\
&= \arg\max\left\{ \ln(n_t w_{t+1} h_{t+1}) \right\} \quad\text{s.t.}\quad k_{t+1} \geq \tilde{k},
\end{aligned}
$$

where the last line follows with $n_t = \frac{w_t h_t}{k_{t+1}}$ from (4). $\qquad\square$

Proof of Lemma 3. Let $\kappa_{ss}(A,\xi)$ be the the knowledge production function (10) at the conditional steady state equilibrium,

$$
\kappa_{ss}(A,\xi) \equiv k(\xi)^{1-\lambda} H_{ss}(A,\xi)^{\lambda},
$$

and ε_κ the elasticity of $\kappa_{ss}(A,\xi)$ with respect to ξ.

1. Under (A4), calculating the first derivative of $\kappa_{ss}(A,\xi)$ with respect to ξ yields

$$
\frac{\partial \kappa_{ss}(A,\xi)}{\partial \xi} = \left[(1-\lambda)\frac{k'(\xi)}{k(\xi)} + \frac{\lambda}{\alpha}\left(\frac{h'(\xi)}{h(\xi)} - \frac{k'(\xi)}{k(\xi)} \right) \right]\kappa_{ss}(A,\xi).
$$

Noting Corollary 2, the elasticity of $\kappa_{ss}(A, \xi)$ with respect to ξ is then readily given as

$$\varepsilon_\kappa = \frac{\partial \kappa_{ss}(A, \xi)}{\partial \xi} \frac{\xi}{\kappa_{ss}(A, \xi)} = (1 - \lambda)\varepsilon_{k,\xi} + \frac{\lambda}{\alpha}(\varepsilon_{h,\xi} - \varepsilon_{k,\xi}) = \varepsilon_{k,\xi}\left(1 - \frac{\alpha + \varepsilon}{\alpha}\lambda\right),$$

where $0 < \varepsilon_\kappa < \varepsilon_{k,\xi}$ follows from $0 < \lambda < \frac{\alpha}{\alpha + \varepsilon}$.

With $g_{ss}(A, \xi) = e^{-\xi}\kappa_{ss}(A, \xi) - 1$, calculating the first derivate of $g_{ss}(A, \xi)$ with respect to ξ yields

$$\frac{\partial g_{ss}(A, \xi)}{\partial \xi} = (\varepsilon_\kappa - \xi)\xi^{-1}e^{-\xi}\kappa_{ss}(A, \xi),$$

which is positive for $\xi < \varepsilon_\kappa$ and negative for $\xi > \varepsilon_\kappa$. Therefore, $g_{ss}(A, \xi)$ has a global maximum at $\xi^* = \varepsilon_\kappa$. Furthermore, under (A4), the first derivate of $g_{ss}(A, \xi)$ with respect to A yields

$$\frac{\partial g_{ss}(A, \xi)}{\partial A} = \frac{\lambda}{A}e^{-\xi}\kappa_{ss}(A, \xi),$$

which is unambiguously positive.

2. It follows from (A6) and $\frac{\partial g_{ss}(A, \xi)}{\partial A} > 0$ that $g_{ss}(A, 0) > 0$. Moreover,

$$\lim_{\xi \to \infty} g_{ss}(A, \xi) = \lim_{\xi \to \infty} \frac{\kappa_{ss}(A, \xi)}{e^\xi} - 1 = -1,$$

since ε_κ is restricted from above by $\varepsilon_{k,\xi}$, whereas the elasticity of e^ξ with respect to ξ, $\frac{e^\xi \xi}{e^\xi} = \xi$, grows without bound as $\xi \to \infty$. Therefore, the existence of a unique root with $\hat{\xi} > \xi^*$ follows immediately from the continuity of $g_{ss}(A, \xi)$. Furthermore, it is clear from $\frac{\partial g_{ss}(A, \xi)}{\partial A} > 0$ that the unique root increases with the level of productivity, A. Thus, the unique root is a single-valued function of the level of productivity, $\hat{\xi} = \hat{\xi}(A)$, with $\hat{\xi}'(A, \xi) > 0$.

$\square$

Proof of Proposition 2. Let $\tau > 0$ be the point in time when the economy enters the upper right area in Figure 7. It follows that $\hat{A}_t \equiv \frac{A_{t+1} - A_t}{A_t} > 0$ and $\hat{H}_t \equiv \frac{H_{t+1} - H_t}{H_t} > 0$ for all $t > \tau$. Define for fixed ξ the function

$$\psi(H_t, A_t) \equiv \frac{h(\xi)}{k(\xi)}\left(\frac{A_t}{H_t}\right)^\alpha - 1 = \hat{H}_t,$$

40

where $\psi_H = -\frac{\alpha}{H_t}(\psi + 1) < 0$ and $\psi_A = \frac{\alpha}{A_t}(\psi + 1) > 0$. Calculating the first order Taylor approximation to ψ near the point (H_τ, A_τ) yields

$$\hat{H}_t \approx \psi(H_\tau, A_\tau) + \psi_H(H_\tau, A_\tau)(H_t - H_\tau) + \psi_A(H_\tau, A_\tau)(A_t - A_\tau)$$
$$= \psi(H_\tau, A_\tau) + \psi_H(H_\tau, A_\tau)H_t + \psi_A(H_\tau, A_\tau)A_t.$$

It follows that the growth rate of population size is approximately given by

$$\hat{H}_t \approx aA_t - bH_t,$$

where $a \equiv \psi_A(H_\tau, A_\tau)$, $b \equiv -\psi_H(H_\tau, A_\tau)$. Furthermore, define for fixed ξ the function

$$\phi(H_t) \equiv e^{-\xi}k(\xi)^{1-\lambda}H_t^\lambda - 1,$$

where $\phi_H = \frac{\lambda}{H_t}(\phi + 1) > 0$. Again, calculating the first order Taylor approximation to ϕ near H_τ yields

$$\hat{A}_t \approx \phi(H_\tau) + \phi_H(H_\tau)(H_t - H_\tau),$$

such that the growth rate of productivity is approximately given by

$$\hat{A}_t \approx cH_t,$$

where $c \equiv \phi_H(H_\tau)$. The difference between both growth rates, $\hat{A}_t - \hat{H}_t = (b+c)H_t - aA_t$, converges to a steady state value of zero, so that $\hat{A}_t = \hat{H}_t$. Hence, $H_t = \frac{a}{b+c}A_t$, and it follows that H_t and A_t approximately grow hyperbolically, $\hat{H}_t \approx cH_t$ and $\hat{A}_t \approx \frac{ac}{b+c}A_t$. Note that Korotayev, Malkov and Khaltourina (2006, P. 24) propose exactly the difference equations

$$\hat{H}_t \approx aA_t - bH_t \qquad \text{and} \qquad \hat{A}_t \approx cH_t$$

as a model for world population growth. They show in a simulation that the model replicates the population estimates of Kremer (1993) in a quite reasonable way. The parameters have been calibrated as $a = b = 1$ and $c = 0.05135$. In this case, the productivity parameter, A, can directly be interpreted as carrying capacity of the environment, i.e., maximum population size. $\square$

Proof of Proposition 4. The instantaneous effect at time t can easily be verified via differentiation of (13), i.e.,

$$\frac{\partial g(H_t, \xi)}{\partial \xi} = ((1 - \lambda)\varepsilon_{k,\xi} - \xi)\,\xi^{-1}e^{-\xi}k(\xi)^{1-\lambda}H_t^\lambda > 0,$$

where the positivity follows with $(1 - \lambda)\varepsilon_{k,\xi} > \xi^* > \xi$ from Lemma 3. The sequence of subsequent growth rates, $\{g_{t+j}\}_{j=0}^{\infty}$, can be understood as a superposition of long-run dynamics from Proposition 2 and short-run dynamics from Proposition 3. Firstly, as long as the skill intensity of the environment is not too large, Proposition 3 guarantees that the growth rate of productivity increases with the skill intensity in every period $t + j$, even if the population size H_t decreases for several subsequent periods. Secondly, the property of hyperbolic growth from Proposition 2 reinforces the initial effect for all periods, such that the positive short-run effect is enlarged. It follows that for all $\xi^l < \xi^h \leq \xi^*$ and for all $j \in \mathbb{N}$,

$$g(H_{t+j}, \xi^h) > g(H_{t+j}, \xi^l).$$

Thus, the timing of the Neolithic transition is significantly accelerated. $\square$

Appendix B: Robustness

Table 2: Latitude, Pre–1500 Body Mass, and the Timing of the Neolithic Transition.

	Pre-1500 body mass	Neolithic transition timing (ancestry adjusted)			Neolithic transition timing (unadjusted)		
	OLS (1)	OLS (2)	OLS (3)	2SLS (4)	OLS (5)	OLS (6)	2SLS (7)
Pre-1500 body mass		203.138** (0.017)	207.656* (0.069)	194.811** (0.033)	239.865** (0.017)	157.463 (0.303)	321.048*** (0.000)
Absolute latitude	0.196*** (0.004)		-2.518 (0.929)			30.423 (0.310)	
Biogeograpic conditions	-0.168 (0.804)	407.405 (0.106)	422.362 (0.140)	420.205* (0.078)			
Cont. FE	No	No	No	No	No	No	No
Obs.	22	22	22	22	22	22	22
R^2	0.505	0.509	0.509	0.509	0.236	0.263	0.209
Anderson-Rubin (*p-value*)				0.105			0.001
Kleibergen-Paap (*F-statistic*)				11.000			20.000

Notes: This table establishes a significant positive effect of absolute latitude on pre–1500 body mass in a 22 country sample, while controlling for the Diamond channel, i.e., biogeographic conditions (column 1). Furthermore, it documents a significant positive effect of pre–1500 body mass on the timing of the Neolithic transition, again controlling for biogeographic conditions (columns 3 to 5). Columns 6 and 8 show that the effect of pre–1500 body mass on the timing of the Neolithic is robust if the response variable is not ancestry adjusted. The associated R^2, however, is much lower then. Heteroskedasticity-robust standard errors are reported in parentheses.

***Significant at the 1 percent level.

**Significant at the 5 percent level.

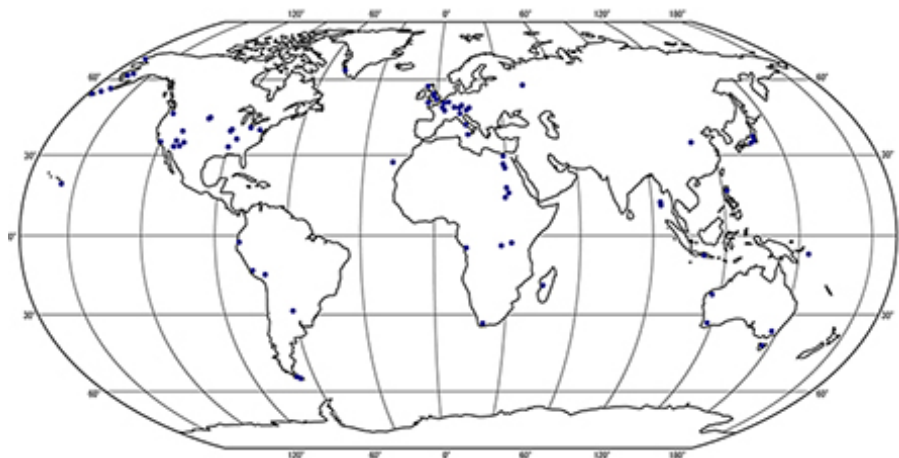*Significant at the 10 percent level.

Figure 9: The Approximate Location of Individuals Included in the Goldman Data Set.

Appendix C. Data Definitions and Sources

Table 3: Descriptive Statistics.

	Obs	Mean	Std. Dev.	Min	Max
Neolithic transition timing	23	4830.435	2123.983	400	9000
Neolithic transition timing (ancestry adjusted)	23	5540.263	1742.603	2033	9000
Pre-1500 body mass	23	54.06	4.305	45.3	60.2
Absolute latitude	23	29.304	17.716	1	60
Biogeographic conditions	22	.096	1.392	-1.181	2.014

Neolithic transition timing The number of years elapsed, until the year 2000 CE, since the majority of the population within a country's modern national borders began practicing sedentary agriculture as the primary mode of subsistence. This measure is reported by Putterman (2008).

Neolithic transition timing (ancestry adjusted) The cross-country weighted average of the timing of the Neolithic transition, where the weight associated with a given country in the calculation represents the fraction of the year 2000 CE population (of the country for which the measure is being computed) that

can trace its ancestral origin to the given country in the year 1500 CE. More precisely, the ancestry adjusted years since the Neolithic transition are constructed as follows: Suppose a country today comprises citizens with ancestry from n countries. Let the Neolithic transition timing of country i be x_i for $i = 1, \ldots, n$. Let further the share of citizens that originate from country i be λ_i. Then, the ancestry adjusted Neolithic transition timing is given by $\sum_1^n \lambda_i x_i$. The ancestry weights are obtained from the *World Migration Matrix, 1500–2000* of Putterman and Weil (2010).

Pre–1500 body mass The data on pre–1500 body mass is taken from the Goldman data set available at http://web.utk.edu/ãuerbach/GOLD.htm (Auerbach and Ruff, 2004). The data set consists of osteometric measurements taken from 1538 human skeletons dating from throughout the Holocene (from as early as 3500 BCE to as late as 1500 CE). Figure 9 shows the approximate location of individuals included in the Goldman data set. In the data set, individual body mass is calculated from the data on femoral head anterior–posterior breadth using a formula developed by Ruff et al. (1991). Individual measurements are assigned to a country. We use average female body mass as an indicator for pre–1500 body mass, as the male data for China, Greenland, India, and South Africa seems to be subject to measurement error.

Biogeographic conditions The data on biogeographic conditions refers to the first principle component of the number of domesticable species of plants and animals, respectively, that were prehistorically native to the continent or landmass to which a country belongs. These variables are obtained from the data set of Olsson and Hibbs (2005).