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Robust Monetary Policy Under Uncertainty About the Lower Bound*

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Abstract

Central banks face uncertainty about the true location of the effective lower bound (ELB) on nominal interest rates. We model optimal discretionary monetary policy during a liquidity trap when the central bank designs policy that is robust with respect to the location of the ELB. If the central bank fears the worst-case location of the ELB, monetary conditions will be more expansionary before the liquidity trap occurs.

Keywords: optimal monetary policy, discretion, robust control, uncertainty, liquidity trap

JEL classification: E31, E32, E58

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1 Introduction

During the recent Great Recession, central banks in many advanced economies cut interest rates aggressively until they believed to have reached the effective lower bound (ELB) on nominal interest rates. Figure (1) presents selected negative policy rates since 2012. The exact location of the ELB is unknown as the ELB is not necessarily binding at an interest rate of zero. The figure suggests that over time the central banks' and the public's beliefs about the lower bound shifted further into negative territory. In the December 2008 meeting of the Federal Open Market Committee, at which the federal funds target rate was reduced to its lowest level, Charles Plosser, the President of the Federal Reserve Bank of Philadelphia, reflected on this uncertainty:

> "Whether that lowest rate is 100 basis points, 75 basis points, 50 basis points, or 25 basis points is very hard to say. However, given the law of unintended consequences and our lack of experience at the lower bound in this country, I do not want to go all the way to zero."[1]

Cœuré (2016) summarizes the different notions of the lower bound. He differentiates between a "physical lower bound", at which the nominal interest rate equals the opportunity cost of holding cash, and an "economic lower bound", at which the detrimental effect on the banking sector outweighs the expansionary effects of a further policy easing. As the location of the ELB is not known, it needs to be estimated (see Wu and Xia, 2016, among others), and these estimates are surrounded by a considerable degree of uncertainty. In addition, the ELB is time-varying (see Kortela, 2016, and Wu and Xia, 2017).[2]

In this paper, we study optimal monetary policy under discretion in a deterministic model that is standard in the literature (see, for example, Eggertsson and Woodford, 2003). We consider "ELB risk" (Hills et al., 2018), i.e. a drop in the natural real interest rate potentially generates a liquidity trap, in which the ELB becomes binding. We model uncertainty about the ELB through means of a robust-control (Hansen and Sargent, 2008) approach. The central bank designs monetary policy that is robust with respect to the exact position of the ELB as reflected in the quote from Charles Plosser. When agents and the central bank fear the worst-case realization of the ELB, the drop in inflation and output *before* the liquidity trap is

[1] See https://www.federalreserve.gov/monetarypolicy/files/FOMC20081216meeting.pdf.

[2] Witmer and Yang (2016) document the discussion inside the Bank of Canada about the location of the ELB. While the ELB was initially located at 25bp (basis points), later estimates suggest the ELB to lie at -50bp.

aggravated. As a result, the central bank will be more expansionary in the period before the liquidity trap. As in Evans et al. (2015), uncertainty about the ELB affects policy through the "expectations channel".[3]

2 The model

We draw on the standard deterministic framework used by Eggertsson and Woodford (2003), Barthélemy and Mengus (2018) and Bilbiie (2018), among others. The economy is described by the textbook New Keynesian model of equations (1) and (2). The Phillips curve, equation (1), relates inflation π_t to expected future inflation and the current output gap, y_t. This relationship can be derived from a standard intertemporal sticky-price model with monopolistic competition.[4] The second equation, the log-linearized intertemporal Euler equation, reflects the negative effect of the gap between the real interest rate, $i_t - E_t\pi_{t+1}$, and the natural real rate, r_t^n, on the output gap

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t \tag{1}$$

$$y_t = E_t y_{t+1} - \sigma^{-1} \left(i_t - E_t \pi_{t+1} - r_t^n \right). \tag{2}$$

The parameters β, κ and σ are the discount factor, the slope of the Phillips curve, which is inversely related to the degree of price stickiness, and the inverse of the intertemporal substitution elasticity. All these coefficients are strictly positive. In contrast to the textbook model, the nominal interest rate, which is the policy instrument of the central bank, is constrained by zero, i.e.

$$i_t \geq 0. \tag{3}$$

For simplicity, we thus assume the true effective lower bound is at zero. The natural real rate of interest is the only exogenous variable of the model and is eventually driving inflation, output and the nominal rate. The model has three key periods. The first period is the period before the lower bound becomes binding. We will use the subscript *pre* to characterize the results for this period, in which the natural rate r_t^n equals the steady state real rate $r^* > 0$. The third period, indexed by *post*, is an absorbing state in which $r_t^n = r^*$. In the second period, however, the natural rate falls to $r < 0$ with probability γ and remains at r^* with probability $1 - \gamma$. The

[3] Gust et al. (2017) consider uncertainty at the ELB about the state of the economy, but not about the location of the ELB.

[4] See Walsh (2017).

solutions for this period will be indexed by LB, though strictly speaking the ELB will be reached only with a probability γ. As will become clear shortly, the drop in the natural rate makes the lower bound on the nominal interest rate a binding constraint such that the economy is in a liquidity trap.
The central bank operates under discretion. Hence, policy takes expectations as given. The central bank minimizes the following quadratic loss function,

$$L = \frac{1}{2} E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\pi_s^2 + \lambda y_s^2\right],$$

where λ is the relative weight of output stabilization.

2.1 Certainty about the level of the lower bound

Let us assume agents and the central bank know the true lower bound on the short rate, which we set to zero, $i_t \geq 0$. The central bank's problem can be written as a Lagrangian

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\left[\pi_t^2 + \lambda y_t^2\right] + \mu^\pi \left[\pi_t - \beta E_t \pi_{t+1} - \kappa y_t\right] \\ &\quad + \mu^y \left[y_t - E_t y_{t+1} + \sigma^{-1}\left(i_t - E_t \pi_{t+1} - r_t^n\right)\right] \\ &\quad + \mu^i \left[i_t\right], \end{aligned} \tag{4}$$

where μ^π, μ^y and μ^i are the Lagrange multipliers associated with the Phillips curve, the Euler equation and the lower bound constraint, respectively.
The first-order conditions can be combined to the following targeting rule

$$y_t = -\frac{\kappa}{\lambda}\pi_t + \frac{\sigma}{\lambda}\mu^i. \tag{5}$$

In the absence of a binding lower bound, i.e. for $\mu^i = 0$, we obtain the textbook solution for the targeting rule under discretion, according to which the central implements a lower output gap in order to offset higher inflation. We will now derive the solution for each period proceeding backwards.

After the liquidity trap. The natural rate equals r^*. The central bank will set its policy rate to equal to r^* such that the real interest rate gap is closed and inflation and output are zero. Hence, we obtain

$$r_{post}^n = r^* = i_{post} \tag{6}$$

$$\pi_{post} = y_{post} = 0 \tag{7}$$

During the liquidity trap. The natural rate is $r_{LB}^n = r < 0$. The central bank lowers is policy rate but is constrained by the lower bound, $i_{LB} = 0$. As the policy is no longer set optimally, the targeting rule is not binding. Since expected inflation and output for all subsequent periods are zero, the solutions are

$$\pi_{LB} = \kappa\sigma^{-1}r \tag{8}$$
$$y_{LB} = \sigma^{-1}r \tag{9}$$

The lower the natural rate is (as long as the nominal rate is bounded and $0 > r_t^n$), the higher the real interest rate and the lower output and inflation. Since $r < 0$, the outcome is $\pi_{LB} < 0$ and $y_{LB} < 0$.

Before the liquidity trap. The lower bound is not binding and policy is set according to the standard targeting rule, i.e. $y_t = -\frac{\kappa}{\lambda}\pi_t$. As agents and the central bank attach a probability γ to the lower bound-state in the following period, the expectations for inflation and output are

$$E_t\pi_{t+1} = \gamma\pi_{LB} + (1-\gamma)\,\pi_t \tag{10}$$
$$E_t y_{t+1} = -\gamma\frac{1}{\kappa}\pi_{LB} - (1-\gamma)\,\frac{\kappa}{\lambda}\pi_t \tag{11}$$

Since agents are forward looking, the possibility of hitting the lower bound in the next period affects output and inflation today. Putting these equations and the targeting rule into the model equations provides us with the solution for inflation, π_{pre}, and output, y_{pre}, as a function of inflation in the lower bound-state in the subsequent period

$$\pi_{pre} = \frac{\lambda\beta\gamma}{\lambda\left[1-\beta\left(1-\gamma\right)\right]+\kappa^2}\pi_{LB} \tag{12}$$
$$y_{pre} = -\frac{\kappa\beta\gamma}{\lambda\left[1-\beta\left(1-\gamma\right)\right]+\kappa^2}\pi_{LB}. \tag{13}$$

Using the solution for $\pi_{LB} < 0$, we find that

$$\pi_{pre} = \frac{\lambda\beta\gamma}{\lambda\left[1-\beta\left(1-\gamma\right)\right]+\kappa^2}\kappa\sigma^{-1}r < 0 \tag{14}$$
$$y_{pre} = -\frac{\kappa\beta\gamma}{\lambda\left[1-\beta\left(1-\gamma\right)\right]+\kappa^2}\kappa\sigma^{-1}r > 0. \tag{15}$$

For $\gamma = 0$, i.e. for a zero probability of reaching the lower bound, inflation and output are perfectly stabilized. With a positive probability of reaching the lower bound tomorrow, the economy exhibits deflation and a positive output gap. Since households expect a deflation in case the lower bound binds in the next period, the real rate increases and inflation falls. The central bank lowers the policy rate and generates a positive output gap. The solution for the interest rate as a function of inflation at the lower bound can be found by solving the Euler equation for the interest rate

$$i_{pre} = \gamma \left[1 + \frac{\sigma}{\kappa} + \frac{\sigma\kappa\beta}{\lambda\left[1-\beta\left(1-\gamma\right)\right]+\kappa^2}\right]\pi_{LB} + r^* \tag{16}$$

Since $\pi_{LB} < 0$, $i_{pre} < r^*$ and the real interest rate is expansionary. The drop in inflation and output due to the possibility of reaching the lower bound in the following period forces the central bank to cut the policy rate, which also reduces the real interest rate.

2.2 Solution under robust control

We formalize the precautionary motive of the central bank through a game against a fictitious evil agent. The evil agent wants to maximize the loss function by setting a distortion, while the central bank wants to minimize the loss function. In the resulting minmax-equilibrium, the central bank chooses the policy that is optimal given the optimal choice of the misspecification by the evil agent and vice versa.

Let as assume the evil agent sets a distortion z_{LB}, which in our case is the level of the lower bound

$$i_t \geq z_{LB}.$$

The evil agent is constrained by his budget constraint requiring $z_{LB}^2 \leq \omega$. The parameter ω measures the amount of misspecification the evil agent has available with the standard rational expectations solution for optimal monetary policy corresponding to $\omega = 0$, such that the evil agent's budget is empty. The Lagrangian reads

$$\begin{aligned}\min_{\pi_t, y_t}\max_{z_t}\mathcal{L} \;&=\; \frac{1}{2}\left[\pi_t^2 + \lambda y_t^2\right] + \mu^\pi\left[\pi_t - \beta E_t\pi_{t+1} - \kappa y_t\right] \\ &\quad + \mu^y\left[y_t - E_t y_{t+1} + \sigma^{-1}\left(i_t - E_t\pi_{t+1} - r_t^n\right)\right] \\ &\quad + \mu^i\left[i_t + z_t\right] + \theta z_t^2,\end{aligned} \tag{17}$$

where θ is the Lagrange multiplier associated with the budget constraint of the evil agent, which is inversely related to ω. Hence, for θ approaching infinity, the model collapses to the case under certainty.

Combining the first-order conditions leads to a targeting rule under robustness, in which the misspecification of the lower bound enters

$$\kappa\pi_t + \lambda y_t = -\sigma\theta z_t. \tag{18}$$

We can now solve the model for all three periods.

After the liquidity trap. Assume that z is small enough such that it is not binding in period 3. The solution is

$$r^n_{post} = r^* = i_{post} \tag{19}$$

$$\pi_{post} = y_{post} = 0, \tag{20}$$

which corresponds to the solution under certainty derived before.

During the liquidity trap. The natural rate falls to $r^n_t = r < 0$ and the lower bound binds at $i_{LB} = z_{LB}$, which is chosen by the evil agent. The solution is characterized by the following three conditions

$$\begin{aligned} \pi^{robust}_{LB} &= \kappa y_{LB} \\ y^{robust}_{LB} &= -\sigma^{-1}\left(z_{LB} - r\right) \\ \kappa\pi^{robust}_{LB} + \lambda y^{robust}_{LB} &= -\sigma\theta z_{LB}. \end{aligned}$$

Note that at the ELB, the targeting rule is not binding for the central bank. However, the evil agent's choice is guided by the desire to maximize the central bank's loss. Hence, the evil agent will take into account what the central bank would do in the absence of the lower bound. The three equations can be combined to find the solution for inflation, output and the lower-bound distortion

$$\pi^{robust}_{LB} = -\kappa\sigma^{-1}\frac{\sigma^2\theta}{\kappa^2 + \lambda - \sigma^2\theta}r \tag{21}$$

$$y^{robust}_{LB} = -\sigma^{-1}\frac{\sigma^2\theta}{\kappa^2 + \lambda - \sigma^2\theta}r \tag{22}$$

$$z_{LB} = \frac{\kappa^2 + \lambda}{\kappa^2 + \lambda - \sigma^2\theta}r \tag{23}$$

Note that if $\kappa^2 + \lambda < \sigma^2\theta$, z is a negative function function of r. The lower the natural rate, the higher the level of the nominal interest rate at which the lower bound becomes binding. The higher the fear of a misperceived lower bound, i.e. the lower θ, the larger z becomes for a given natural rate. For θ approaching infinity, the distortion approaches zero.

Before the liquidity trap. As in the RE case, the standard targeting rule holds as the lower bound is not binding. The expectations about inflation and output are given by (10) and (11). We can combine these with the structure of the model economy to obtain

$$\pi_{pre}^{robust} = \frac{\lambda\beta\gamma}{\lambda\left[1-\beta\left(1-\gamma\right)\right]}\pi_{LB}^{robust} \tag{24}$$

$$y_{pre}^{robust} = -\frac{\kappa\beta\gamma}{\lambda\left[1-\beta\left(1-\gamma\right)\right]}\pi_{LB}^{robust} \tag{25}$$

$$i_{pre}^{robust} = \gamma\left[1+\frac{\sigma}{\kappa}+\frac{\sigma\kappa\beta}{\lambda\left[1-\beta\left(1-\gamma\right)\right]+\kappa^2}\right]\pi_{LB}^{robust}+r^* \tag{26}$$

For $\gamma = 0$, i.e. for a zero probability of reaching the ELB next period, inflation and output are zero and the nominal interest rate equal the natural rate. Since $\partial\pi_{LB}^{robust}/\partial\theta < 0$, a lower θ leads to more expansionary monetary policy before the lower bound period. Hence, the solution is a case for a breakdown of Brainard's (1969) principle of cautionary policy in the face of uncertainty. In our case, policy eases more aggressively if uncertainty increases.
Table (1) reports the parameterization of this model. All parameter values are standard in the literature. In the following, we report interest rates and inflation rates in annualized form.

Table 1: Parameter values

β	κ	σ	γ	λ	θ	r^*	r
0.99	0.10	1.00	0.50	0.25	1.30	2.00	-2.00

Figure (2) illustrates the solutions for each variable. To ease the interpretation, we plot the behavior of the economy over five periods. The first and the fifth period just reflect the steady state. In the third, the natural rate drops to -2%.
Under certainty about the location of the ELB, the central bank sets the the nominal rate to zero. The resulting real interest rate is slightly larger than the steady rate thus remains at the steady state level since expected inflation in period four remain at 2%. Hence, policy is contractionary at the ELB in $t = 3$, such that inflation is negative and the economy is in a recession. The expected deflation in $t = 3$ translates into deflation in $t = 2$. The central bank lowers the nominal rate in $t = 2$, which is causing a small drop in the real rate and an expansion of the economy despite the drop in expected inflation.

Under uncertainty and a robust control approach to optimal policy, we can differentiate between the worst case equilibrium and the approximating equilibrium (Leitemo and Söderström, 2008). The former refers to the equilibrium in which the central bank implements a robust policy and the worst-case misspecification eventually realizes. In the latter, the central follows the robust policy but the fear of misspecification is unwarranted as the undistorted outcome realizes in $t = 3$.
In the worst case equilibrium, see Figure (2), the ELB becomes binding at a positive level of the nominal rate, thus leading to an increase in the real rate and a widening of the real interest rate gap compared to the equilibrium under certainty. The resulting policy stance is more contractionary than under certainty, such that the fall in inflation and output in $t = 3$ is stronger. This translates into a stronger deflation in the period proceeding the lower bound. Monetary policy reduces the nominal rate more aggressively in period two than under certainty.
In the approximating outcome, see Figure (3), the real interest rate gap in period three is identical to the one under certainty. Since monetary policy is able to reduce the nominal rate to zero because the true lower bound is known in $t = 3$, the drop in inflation and output in period 3 is smaller than in the worst case. There is no difference between the approximating equilibrium and the worst case outcome in period two, i.e. before it is revealed whether robust policy is warranted or not.
To summarize the results, we find that under a robust-control approach to uncertainty, policy is more expansionary before the liquidity trap because expectations of hitting the lower bound in the future lead to lower inflation and output. This fall in inflation and output is higher if uncertainty increases. Thus, an upward shift in the believed location of the ELB, which is the optimal response under a robust control approach to uncertainty, is contractionary.

3 Conclusions

The ELB is unobservable and time-varying. Hence, central banks face uncertainty about how strongly they can reduce nominal interest rates in a liquidity trap. This paper models this kind of uncertainty in a standard New Keynesian model of optimal discretionary monetary policy. We show that under robust control, i.e. when the central bank designs policy that is robust with respect to the worst-case position of the ELB, monetary policy is more expansionary in the run-up to the liquidity trap. This result has been derived in a deterministic model under discretionary policy. A natural extension is to study forward guidance under worst-case beliefs about the nature, the persistence or the probability of lower-bound episodes.

References

[1] Barthélemy, J. and E. Mengus (2018): "The signaling effect of raising inflation", *unpublished*, HEC Paris.

[2] Bilbiie, F. (2018): "Optimal forward guidance", *unpublished*, University of Lausanne.

[3] Brainard, W. (1967): "Uncertainty and the effectiveness of policy", *American Economic Review* 57, 411-425.

[4] Cœuré, B. (2016): "Assessing the implications of negative interest rates", speech at the Yale Financial Crisis Forum, 28 July 2016, https://www.bis.org/review/r160805b.pdf.

[5] Eggertsson, G. B. and M. Woodford (2003): "The zero bound on interest rates and optimal monetary policy," *Brookings Papers on Economic Activity* 34(1), 139-235.

[6] Evans, C., J. Fisher, F. Gourio and S. Krane (2015): "Risk management for monetary policy near the zero lower bound", *Brookings Papers on Economic Activity*, spring 2015, 141-219.

[7] Grisse, C., S. Krogstrup and S. Schumacher (2017): "Lower-bound beliefs and long-term interest rates", *International Journal of Central Banking* 13, 165-202.

[8] Gust, C., B. K. Johannsen and J. D. Lopez-Salido (2017): "Monetary policy, incomplete information, and the zero lower bound", *IMF Economic Review* 65, 37-70.

[9] Hansen, L. P. and T. J. Sargent (2008): *Robustness*, Princeton: Princeton University Press.

[10] Hills, T., T. Nakata and S. Schmidt (2018): "Effective lower bound risk", *unpublished*, Federal Reserve Board.

[11] Kortela, T. (2016): "A shadow rate model with time-varying lower bound of interest rates", *Discussion Paper* No. 19-2016, Bank of Finland.

[12] Leitemo, K. and U. Söderström (2008): "Robust monetary policy in the New-Keynesian framework", *Macroeconomic Dynamics* 12, 126-135.

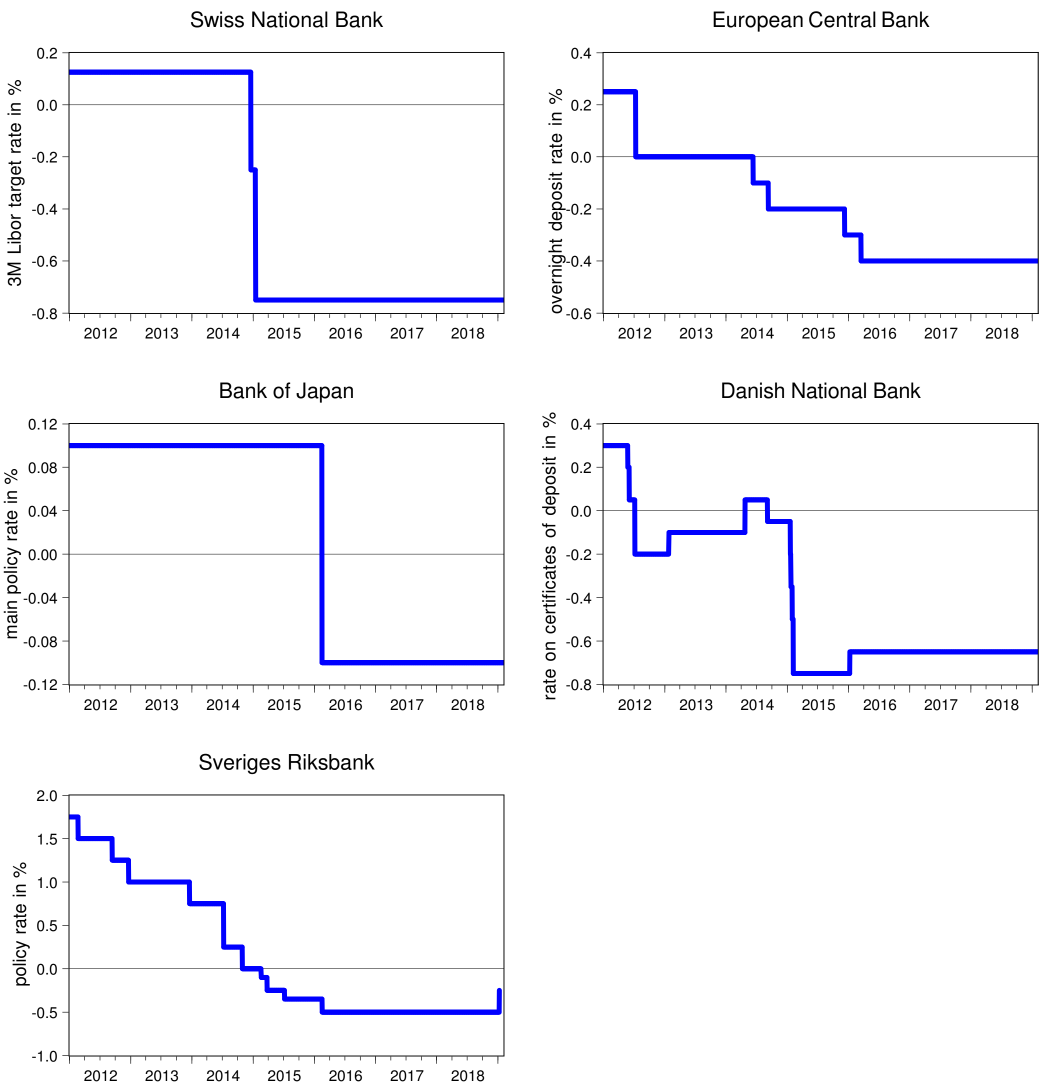
[13] Walsh, C. E. (2017): *Monetary Theory and Policy*, 4th edition, Cambridge: MIT Press.

[14] Witmer, J. and J. Yang (2016): "Estimating Canada's effective lower bound", *Bank of Canada Review*, spring 2016, 3-14.

[15] Wu, J. C. and F. D. Xia (2016): "Measuring the macroeconomic impact of monetary policy at the zero lower bound", *Journal of Money, Credit and Banking* 48, 253-291.

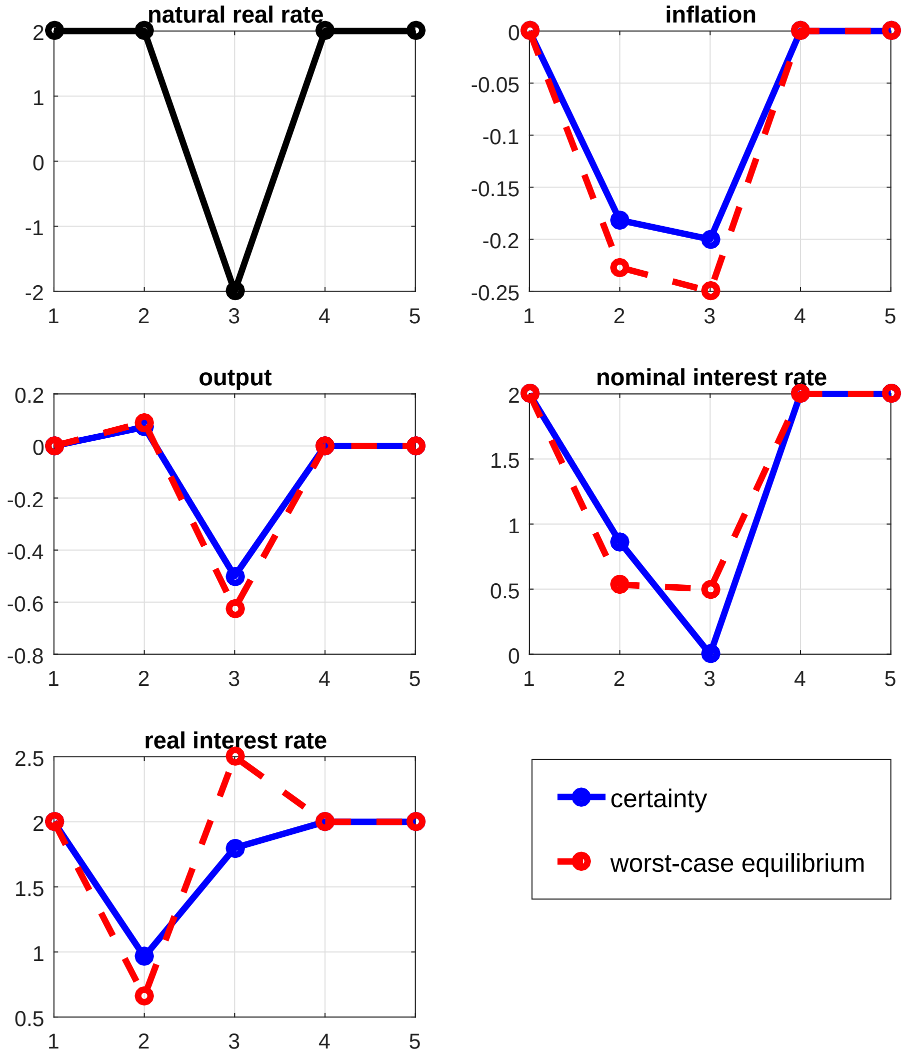
[16] Wu, J. C. and F. D. Xia (2017): "Time-varying lower bound of interest rates in Europe", *unpublished*.

Figure 1: Main policy rates below zero



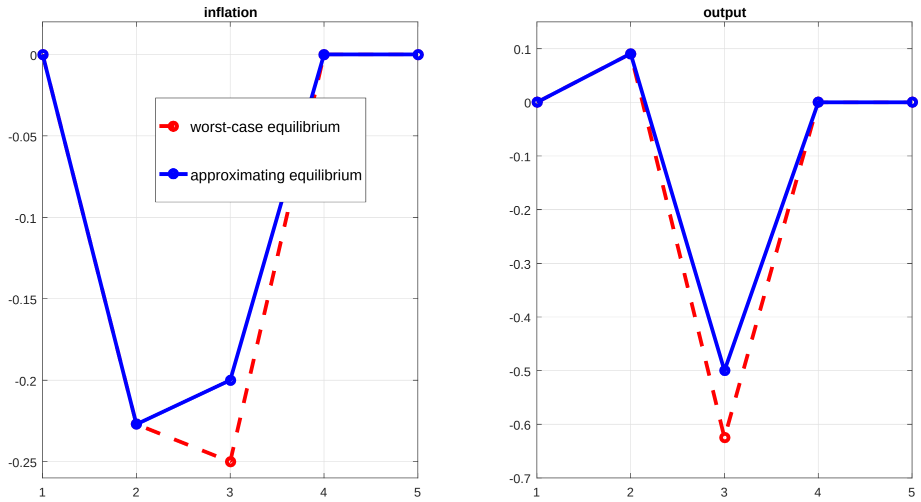
Notes: The graph shows selected policy rates below zero since 2012. The graph is an updated version of Figure (2) in Grisse et al. (2017), where the Datastream mnemonics can be found.

Figure 2: Paths of the variables over time



Notes: The graphs show the solutions for the variables over time based on the calibration shown in Table (1). All magnitudes are in percentage points. The inflation rate and the nominal and real rates are shown in annualized form. To ease the interpretation, we added a first and a fifth period in which the economy is in its steady state. Thus, period 3 is the period in which the ELB becomes binding.

Figure 3: Paths of inflation and output in the worst-case and the approximating equilibrium



Notes: The graphs show the solutions for the variables over time based on the calibration shown in Table (1). All magnitudes are in percentage points. The inflation rate is shown in annualized form. To ease the interpretation, we added a first and a fifth period in which the economy is in its steady state. Thus, period 3 is the period in which the ELB becomes binding.