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Optimal Monetary Policy Under Heterogeneous Beliefs*

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Abstract

We use a New Keynesian model that features rational and non-rational households. Assuming that both the fraction of rational households and the expectations formation process are uncertain from the perspective of the central bank, we derive robust optimal discretionary monetary policy in a simple min-max framework where the central bank plays a zero-sum game versus a fictitious, malevolent evil agent. We show that the central bank is able to improve welfare if it accounts for uncertainty while the model is being distorted. Even if the central bank accounts for the worst possible outcomes while the model is being undistorted, the central bank can still reduce the welfare loss by implementing a more aggressive targeting rule that favorably affects the inflation-output stabilization trade-off.

Keywords: Heterogeneous Expectations, Robust Monetary Policy, Policy Implementation, Uncertainty

JEL classification: E52, D84

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I Introduction

Over recent decades, New Keynesian models have become the workhorse in both monetary policy and theory, acknowledged by researchers and policy makers alike. In its basic version (see, for instance, Woodford, 2003; Walsh, 2017; Galí, 2015), the assumptions for price flexibility as well as perfect competition among producers are relaxed which - in a nutshell- implies nonneutral effects for monetary policy. In fact, there is a broad consensus that the New Keynesian framework has been both an effective and intuitive tool as it provides guidance on expectations management which is an important aspect for stabilization policy.

However, the broad strand of literature on New Keynesian models reflects that some important aspects are missing in the standard framework. Among others, abandoning the strong assumption of rational expectations has proved to provide important evidence on business cycle theory. In fact, central banks increasingly care about the importance of the development of households' expectations. This is because on the one hand, households' inflation expectations signal future inflationary risks, while on the other hand, heterogeneous expectations are regarded as potential drivers of business cycle amplification (see, for instance, King, 2012; Mokhtarzadeh and Petersen, 2021).

Understanding the nature of expectations is particularly important at the Zero Lower Bound. To the extent central banks rely on the management of expectation through policies such as forward guidance, the effectiveness of these policies rests on the assumption that the central bank knows how agents receive and process information and generate forecasts about economic variables in the future (see, for instance, Andrade et al., 2019; Hommes and Lustenhouwer, 2019; Beqiraj et al., 2019).

The failure of the rational expectations framework is well documented in survey data and laboratory experiments alike, which is ultimately based on the criticism that agents (1) are assumed to have too much information available in general and (2) can process an unlimited amount of information in particular. In particular, rational agents must know the true structure and probability distribution of the economy to form rational expectations (see, for instance, Evans et al., 2001; and Carroll, 2003). To overcome this issue, Branch and McGough (2009), Di Bartolomeo et al. (2016), Gasteiger (2014) and Massaro (2013) incorporate a fraction of non-rational households into an otherwise standard New Keynesian framework and show that both determinacy and optimal monetary policy properties are strongly affected when non-rational households are incorporated.

However, it is important to stress that incorporating heterogeneous expectations into stochastic equilibrium models ultimately stems from empirical and experimental evidence in laboratories that provide strong evidence that a significant share of agents use simple heuristics and adaptive mechanisms for forecasting (see, for instance, Branch and McGough, 2016; and Hommes, 2011). Branch (2004) uses data from the Michigan surveys of consumers and finds that belief formation across participants is both heterogeneous on the one hand and dynamic on the other. Distinguishing between vector autoregression (VAR) based, adaptive and naive expectations when forming beliefs, he finds that respondents switch their predictor depending on the size of relative mean-squared errors. Pfajfar and Žakelj (2014) study the process of inflation expectation formation by focusing on both rational and adaptive agents and find that on the one hand, they cannot reject the hypothesis of rational expectations for at least 40% of the laboratory subjects, while on the other hand 20% of the subjects are best described by adaptive learning models. In another paper, Pfajfar and Žakelj (2018) explore the formation of inflation expectations within a New Keynesian framework. Similar to their previous paper, they find that about 40% of households form rational expectations, while 35% extrapolate expectations more than one-to-one into the future. 20% of households seem to employ adaptive learning models. Chavas (2000) investigates beef price equations under different expectations regimes and finds that nearly half of the market participants seem to form naive expectations.

Even though the recent literature well documents how optimal monetary policy is affected in models with heterogeneous expectations, all above mentioned authors mostly assume that central bankers do precisely know the fraction of non-rational households and also how expectations of these non-rational households are formed. It is however likely that these parameters (the fraction of non-rational households and the expectation formation process) are unknown, as reflected in different shares of adaptive agents in the empirical (experimental) literature. Therefore, a natural question is how optimal monetary policy is affected if the central bank is uncertain about the true distribution of rational and non-rational households.

This paper tries to fill this gap by modeling uncertainty in a New Keynesian framework with heterogeneous beliefs. We use a simple min-max mechanism where the central bank plays a zero-sum game against a fictitious, malevolent 'evil agent' that draws the potentially distorted parameters from a feasible set of models such that welfare loss is maximized. The application of a min-max framework is based on two reasons. First, if the central bank conducts discretionary monetary policy (as assumed in this paper), the specific targeting rule depends on both the fraction of non-rational households as well as the adaption parameter (as a part of the expectations formation process). In simple words, the trade-off of output gap and inflation stabilization depends on the possibly mispecified parameters. A central bank that is uncertain about the distribution is therefore able to shield the economy against perturbations by accounting for uncertainty. Second, a min-max approach has proven to be a simple and intuitive tool to model uncertainty (see, for instance, Giannoni, 2002; Giannoni, 2007; Hansen and Sargent, 1993; and Tillmann, 2009). Typical outcomes in min-max equilibria imply that the central bank is better-off accounting for uncertainty by implementing worst-case beliefs into its targeting rule, i.e. it can reduce welfare losses if it accounts for uncertainty while the worst-case beliefs turn out to be warranted (see, for instance, Giannoni, 2007; and Tillmann, 2009).

In this paper, we derive a robust optimal policy plan that accounts for uncertainty.

Hereby, we refer to the same definition as in Giannoni (2007), i.e. those policy plans that perform best in a worst-case scenario, i.e. a scenario that delivers the worst possible welfare outcomes within a pre-specified set of parameter configurations.[1] We assume that the central bank does not know the true values of the unknowns, but it knows ranges (i.e. intervals) where the true values lie within. Calibrating our model to the US economy, we find that a central bank that accounts for uncertainty can substantially reduce welfare losses if it implements a robust policy plan. From the standpoint that the fraction of non-rational households is indeed unknown, our results are important insofar as they reveal that welfare losses can become large if the worst possible parameter perturbations turn out to be true. Interestingly, a central bank that implements the robust policy is able to further reduce welfare losses, relative to the case of certainty, i.e. when the true values are known. In other words, we find that the central bank can reduce the well known stabilization bias by incorporating a more 'aggressive' policy plan relative to the case of certainty. The reason for this finding is that discretionary monetary policy suffers from the stabilization bias as long as the supply shock is not white noise. That is, in the absence of commitment, inflation is too volatile. We find that adopting a robust policy approach can reduce the stabilization bias. A desire for robustness thus makes monetary policy more aggressive. As a result, inflation volatility is reduced and welfare improves compared to the rational expectations solution.

The rest of this paper is organized as follows: Section II sketches the underlying baseline model. In Section III, we introduce the min-max framework and derive the robust optimal policy plan and equilibrium outcomes. Section IV reports simulation exercises and discusses our most important results in detail. Section V argues that the implementation of robust optimal policy rules is preferable over a standard Taylor rule. Section VI concludes.

II The Model

Our underlying model substantially relies on recent work of Branch and McGough (2009) and Gasteiger (2021) who incorporate heterogeneous expectations into a simple New Keynesian model. The economy is populated by a continuum of households, firms and the central bank. Time is discrete and indexed by $t \in (0, ..., \infty)$. The IS-

[1] Gasteiger (2021) also investigates expectations mismeasurement by comparing long-run losses for three different reference models. Therein, robust optimal monetary policy is defined as being a policy plan that yields determinacy across reference models for *given* determinants that characterize expectations heterogeneity. Di Bartolomeo et al. (2016) also consider a case where the central bank is being unable to recognize how households form their beliefs and explore a robust optimal policy consisting of minimizing the maximum regret of choosing a wrong share rational households. They find that implementing a wrong share of rational households always results in higher welfare losses, even though this effect is stronger if the central bank implements a belief that is greater than the true value.

curve (1) and the New Keynesian Phillips curve (2) can be summarized as

$$x_t = \widehat{E}_t x_{t+1} - \sigma^{-1}\left(i_t - \widehat{E}_t \pi_{t+1}\right) \tag{1}$$
$$\pi_t = \beta \widehat{E}_t \pi_{t+1} + \phi x_t + e_t, \tag{2}$$

where x_t is output gap, i_t the short-term interest rate, π_t inflation and e_t a cost push shock which follows an AR(1) process specified as

$$e_t = \rho e_{t-1} + \varepsilon_t^e, \sim \mathcal{N}(0, \sigma_e^2).$$

Similar to the recent literature, we assume that aggregate expectations are a linear combination of two types different types of agents, namely rational households (indexed $\mathcal{R}$) and non-rational households (indexed $\mathcal{B}$), such that aggregate expectations for a generic variable z_t read

$$\widehat{E}_t z_{t+1} = \alpha_{\mathcal{R}} E_t^{\mathcal{R}} z_{t+1} + \alpha_{\mathcal{B}} E_t^{\mathcal{B}} z_{t+1}.$$

As is common in the specific literature, we assume that rational agents have a one-step ahead perfect foresight on economic variables, i.e. for a generic variable z_t it holds that $E_t^{\mathcal{R}} z_{t+1} = E_t z_{t+1}$, where E_t corresponds to the rational expectations operator in t. However, rational households are supposed to be not "hyperrational" in a sense that they do not know the expectation formation process of non-rational households.[2] As regards the non-rational households, we assume that expectations of non-rational households are specified as $E_t^{\mathcal{B}} z_{t+1} = \gamma z_t$.[3] That is, we depart from the assumption as in Branch and McGough (2009) and assume that non-rational households, besides the variables they seek to maximize, can observe current outcomes. In simple words, tomorrow's expectations equal today's observation, adjusted for an adaption parameter γ which is equal for all outcomes.[4] For any variable z_t this implies that, when forecasting in period t what will prevail in $t + 1$, these households simply look at z_t and extrapolate it forward. The parameter γ therefore is an adaption parameter. $\gamma < 1$ implies that non-rational households place less weight on current outcomes while $\gamma = 1$ means that today's expectation of tomorrow's outcome is equal to the current observation. $\gamma > 1$ states that non-rational households are 'extrapolative' in a sense that they overweight today's observation by extrapolating it more than one-to-one into the future. It should be stressed that in our assumption, both households, i.e. rational and non-rational households, are able to solve the underlying model, but only a fraction α is able to form rational expectations while the remainder does not.

[2]This is an important axiomatic assumption. It implies that $E_t^{\mathcal{R}} E_t^{\mathcal{B}} z_{t+1} = E_t^{\mathcal{R}} z_{t+1}$, see Branch and McGough (2009) for a detailed discussion.

[3]Albeit this assumption is admittedly crude, it does in no way affect our assumptions on aggregate expectations. However, it is likely that many people are more sophisticated in their expectation formation process.

[4]We could also assume that different values for γ prevail for different aggregate variables, i.e. (in our case) that $\gamma_x \neq \gamma_\pi$.

Since we focus on two types of households, it holds that $\alpha_{\mathcal{B}} = 1 - \alpha$, such that aggregate expectations read

$$\widehat{E}_t z_{t+1} = \alpha E_t z_{t+1} + (1-\alpha)\gamma z_t.$$

It follows that our IS-curve and New Keynesian Phillips curve read

$$x_t = \underbrace{[\alpha E_t x_{t+1} + (1-\alpha)\gamma x_t]}_{\widehat{E}_t x_{t+1}} - \sigma^{-1}\Big(i_t - \underbrace{[\alpha E_t \pi_{t+1} + (1-\alpha)\gamma \pi_t]}_{\widehat{E}_t \pi_{t+1}}\Big)$$
$$\pi_t = \beta\,[\alpha E_t \pi_{t+1} + (1-\alpha)\gamma \pi_t] + \phi x_t + e_t.$$

Importantly, both the IS-curve as well as the New Keynesian Phillips curve are microfounded and nest the standard model in the absence of rational expectations. More precisely, our model collapses to the textbook model when $\alpha = 1$.

III Uncertainty about Heterogeneous Expectations

It is assumed that the central bank does neither know the share of rational households, nor the exact value of the adaption process, i.e. there is uncertainty about the true values of α and γ. However, the central bank knows that α and γ lie within an interval of a lower bound and an upper bound, respectively, i.e.

$$\alpha \in [\alpha_l,\ \alpha_h], \qquad \gamma \in [\gamma_l,\ \gamma_h],$$

where $\alpha_h > \alpha_l > 0$ and $\gamma_h > \gamma_l > 0$.[5] This assumption enables us to model uncertainty about α and γ in a simple min-max approach. More precisely, we assume the central bank plays a zero-sum game versus an evil, malevolent agent who chooses the vector $\psi = (\alpha,\ \gamma) \in \Psi$, where $\Psi = [\psi_1, ..., \psi_m]$ is a feasible compact set $\Psi \in \mathbb{R}^m$ satisfying

$$\Psi \equiv \{\psi = (\alpha, \gamma) \mid \alpha_l \leqslant \alpha \leqslant \alpha_h,\ \gamma_l \leqslant \gamma \leqslant \gamma_h\},$$

such that welfare loss is maximized.

The game consists of four stages. In the first stage, the evil agent and the central bank as well as the entire private sector observe the cost push shock e_t. After observing the shock, in the second stage the central bank implements a policy plan $f(\psi) \in F$, where F denotes the compact set of all feasible non-inertial policy plans, i.e. $F = \{f(\psi) \mid f(\psi) \geqslant 0\}$ satisfying $F \in \mathbb{R}^n$, and designs optimal monetary policy in a discretionary fashion while still being uncertain about the true values of α and γ. Then, in the third stage, uncertainty is resolved, i.e. it turns out whether the model is distorted or not. Finally, the equilibrium outcomes are realized in the last (fourth) stage.

[5] Even if the central bank does not know the true lower and upper bounds for α and γ, we can interpret these as the desire for robustness, i.e. a measure of caution.

A. *The Robust Optimal Monetary Policy Plan*

As mentioned above, the central bank adopts a policy plan $f(\psi) \in F$ after observing the shock, but before $\psi \in \Psi$ is known. For simplicity, we assume that no commitment technology is available, such that the central bank conducts optimal monetary policy in a discretionary fashion by minimizing output and inflation volatility subject to the Phillips Curve. In particular, the central bank is assumed to be concerned about household's well-being by minimizing both output gap and inflation volatility due to monopolistic competition and sticky prices.[6] The objective function reads

$$L_t = E_t \sum_{k=0}^{\infty} \beta^k \frac{1}{2} \left[\pi_{t+k}^2 + \lambda x_{t+k}^2 \right],$$

where λ is the relative weight the monetary authority places on the output gap. Recall that the central bank is assumed to be concerned about potential parameter perturbations, that is in a min-max equilibrium it implements a policy plan $f^\star(\psi^\star)$ that takes the worst possible outcomes into account. Formally, the resulting robust policy plan $f^\star(\psi^\star)$ reads

$$f^\star(\psi^\star) = \arg\min_{f \in F} \left\{ \max_{\psi \in \Psi} E \left[\mathcal{L}_t(f(\psi)), \psi \right] \right\}.$$

It can be shown that the specific targeting rule for the discretionary case is given as

$$\pi_t = -\underbrace{\frac{\lambda}{\phi} \left[1 - \beta(1-\alpha)\gamma \right]}_{f(\psi)} x_t.$$

Assume that the central bank is faced with a positive cost-push shock which creates a trade-off between inflation and output stabilization objectives. The policymaker balances them by creating a negative output gap.

The malevolent fictitious evil agent will try to harm the central bank by maximizing the loss function with respect to the unknown parameters, i.e. α and γ. The

[6] Notice that the loss function as used here is only model consistent as long as we focus on a standard utility function in the homogeneous expectations framework, i.e. in the absence of non-rational households. That is, the loss function we assume here is actually *ad hoc* in a sense that we ignore additional terms due to the presence of heterogeneous expectations. As shown by Di Bartolomeo et al. (2016), a model-consistent loss function in the presence of heterogeneous expectations and backward-looking agents requires two additional components, implying eight additions terms in the case of two types of expectation formation, one of them being due to higher price dispersion caused by adaptive agents. The second term is based on a dispersion in consumption across households types. See Gasteiger (2021) for a detailed discussion that justifies the application of an *ad hoc* loss function as opposed to the model-consistent loss function.

maximization problem reads

$$\max_{\alpha,\,\gamma}\left\{\frac{f(\psi)^2+\lambda}{\left[f(\psi)\left[1-\beta\left(\alpha\rho+(1-\alpha)\gamma\right)\right]+\phi\right]^2}\right\}.$$

Proposition 1. If the central banker is uncertain about the true value of α and γ, a robust policy rule will incorporate $\alpha^\star = \alpha_l$ and $\gamma^\star = \gamma_h$.

Proof. (i) The first order condition with respect to α is given by

$$\frac{\partial \mathcal{L}_t}{\partial \alpha} = -\frac{2\beta f(\psi)\,(\gamma-\rho)}{\left[\phi+f(\psi)\left[1-\beta(\alpha\rho+(1-\alpha)\gamma)\right]\right]^3}\left(f(\psi)^2+\lambda\right)$$

The derivative is negative for the entire parameter space under consideration, i.e. $\frac{\partial \mathcal{L}_t}{\partial \alpha} < 0$. It follows that $\alpha^\star = \alpha_l$ maximizes $\mathcal{L}_t$.
(ii) The first order condition with respect to γ is given by

$$\frac{\partial \mathcal{L}_t}{\partial \gamma} = \frac{2\beta f(\psi)\,(1-\alpha)}{\left[\phi+f(\psi)\left[1-\beta(\alpha\rho+(1-\alpha)\gamma)\right]\right]^3}\left(f(\psi)^2+\lambda\right)$$

The derivative is positive for the entire parameter space under consideration, i.e. $\frac{\partial \mathcal{L}_t}{\partial \gamma} > 0$. It follows that $\gamma^\star = \gamma_h$ maximizes $\mathcal{L}_t$. The worst-case belief of the central bank is thus $\psi^\star = (\alpha^\star, \gamma^\star) = (\alpha_l, \gamma_h)$. ■

In what follows, we mainly focus on three possible scenarios. In the first scenario, the worst-case scneario, the central banker implements the robust policy plan and the model is distorted, such that the realized outcomes for output, inflation and the short-term interest rate are given as $x_t(f^\star(\psi^\star), \psi^\star)$, $\pi_t(f^\star(\psi^\star), \psi^\star)$ and $i_t(f^\star(\psi^\star), \psi^\star)$. In the second equilibrium, the approximating equilibrium, the central bank implements $f^\star(\psi^\star)$ but the model turns out to be undistorted. The equilibrium outcomes are therefore given as $x_t(f^\star(\psi^\star), \psi)$, $\pi_t(f^\star(\psi^\star), \psi)$ and $i_t(f^\star(\psi^\star), \psi)$.[7]

Now that we have derived the robust optimal policy plan $f^\star(\psi^\star)$, we next derive the equilibrium outcomes for both the worst-case and the approximating outcomes. In equilibrium, inflation, output gap and the interest rate will be linear functions of the supply shock e_t. The worst-case equilibrium outcomes are thus given as $\pi_t(f^\star(\psi^\star), \psi^\star) = b_\pi^{\text{min-max}} e_t$, $x_t(f^\star(\psi^\star), \psi^\star) = b_x^{\text{min-max}} e_t$ and $i_t(f^\star(\psi^\star), \psi^\star) = b_i^{\text{min-max}} e_t$,

[7] We will later investigate a scenario in which the central bank does not account for uncertainty while the model turns out to be distorted, i.e. the central bank implements $f(\psi)$ while $\psi^\star$ holds. The equilibrium outcomes therefore read $x_t(f(\psi), \psi^\star)$, $\pi_t(f(\psi), \psi^\star)$ and $i_t(f(\psi), \psi^\star)$.

where

$$
\begin{aligned}
b_\pi^{\text{min-max}} &= \frac{f^\star(\psi^\star)}{\phi + f^\star(\psi^\star)\left[1-\beta(\alpha_l\rho + (1-\alpha_l)\gamma_h\right]} \\
b_x^{\text{min-max}} &= -\frac{1}{\phi + f^\star(\psi^\star)\left[1-\beta(\alpha_l\rho + (1-\alpha_l)\gamma_h\right]} \\
b_i^{\text{min-max}} &= \frac{\sigma - (f^\star(\psi^\star)+\sigma)\left[\alpha_l\rho + (1-\alpha_l)\gamma_h\right]}{\phi + f^\star(\psi^\star)\left[1-\beta(\alpha_l\rho + (1-\alpha_l)\gamma_h\right]}.
\end{aligned}
$$

In the approximating equilibrium, the central bank still implements the robust optimal plan $f^\star(\psi^\star)$, where, however, the model turns out to be undistorted. The outcomes for the three endogenous variables are $\pi_t(f^\star(\psi^\star),\psi) = b_\pi^{\text{approx}} e_t$, $x_t(f^\star(\psi^\star),\psi) = b_x^{\text{approx}} e_t$ and $i_t(f^\star(\psi^\star),\psi) = b_i^{\text{approx}} e_t$, where the respective coefficients are

$$
\begin{aligned}
b_\pi^{\text{approx}} &= \frac{f^\star(\psi^\star)}{\phi + f^\star(\psi^\star)\left[1-\beta(\alpha\rho + (1-\alpha)\gamma\right]} \\
b_x^{\text{approx}} &= -\frac{1}{\phi + f^\star(\psi^\star)\left[1-\beta(\alpha\rho + (1-\alpha)\gamma\right]} \\
b_i^{\text{approx}} &= \frac{\sigma - (f^\star(\psi^\star)+\sigma)\left[\alpha\rho + (1-\alpha)\gamma\right]}{\phi + f^\star(\psi^\star)\left[1-\beta(\alpha\rho + (1-\alpha)\gamma\right]}.
\end{aligned}
$$

Notice that the only difference between the min-max and the approximating equilibrium are the *realized* outcomes for ψ. That is, in the worst-case equilibrium, $\psi^\star = (\alpha^\star, \gamma^\star) = (\alpha_l, \gamma_h)$ while in the approximating equilibrium it holds that $\psi = (\alpha, \gamma)$. In both cases however, the robust optimal policy plan $f^\star(\psi^\star)$ is implemented which is given as

$$
f^\star(\psi^\star) = f^\star(\alpha_l, \gamma_h) = \frac{\lambda}{\phi}\left[1 - \beta(1-\alpha_l)\gamma_h\right] \geqslant 0
$$

and is non-negative by assumption.

At this point, we can see that the stabilization between the output gap and inflation clearly depends on the set of models Ψ. To better understand how the central bank shields the economy against uncertainty, notice that the derivative of the policy plan $f(\alpha,\gamma) = \frac{\lambda}{\phi}\left[1-\beta(1-\alpha)\gamma\right]$ with respect to α and γ are given as

$$
-\frac{\partial f(\psi(\alpha))}{\partial \alpha} = -\frac{\lambda}{\phi}\gamma\beta < 0, \qquad \frac{\partial f(\psi(\gamma))}{\partial \gamma} = -\frac{\lambda}{\phi}\beta(1-\alpha) < 0.
$$

The derivates imply an important result for later purposes. If the lower bound of the interval $\alpha \in [\alpha_l,\ \alpha_h]$ increases, i.e. if the central bank considers a larger fraction of non-rational households and implements the corresponding robust policy plan, this implies that inflation can be stabilized with a smaller fall in the output gap. A similar argumentation holds for γ_h.

Confronting these results with equilibrium outcomes, Figure (1) plots inflation vari-

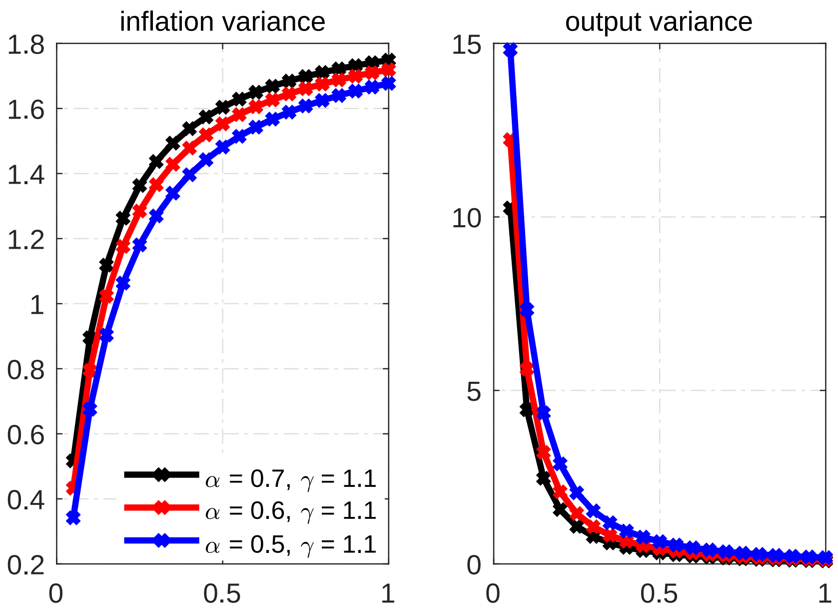

Figure 1: Inflation and output gap variance for $\lambda \in [0, 1]$ and different worst-case scenarios, in which the model is undistorted.

ances and output variances for different beliefs where the worst-case, however, does not occur, i.e. the worst-case beliefs are unwarranted.[8] More precisely, the black and the red line correspond to scenarios where the central bank implements $\alpha = 0.7$ (black line) and $\alpha = 0.6$ (red line), given $\gamma = 1.1$, whereas the blue line corresponds to the case where the central bank implements worst-case beliefs $\alpha_l = 0.5$ and $\gamma_h = 1.2$ for different values for λ between 0 and 1. Notably, the central bank can achieve a smaller inflation variance when values for α are implemented that are below the actual outcome. At the same time, however, the output gap variance increases if the desire for robustness increases, i.e. if the central bank widens its interval for α. This is an interesting result for the next section where it is shown in a simulation exercise that implementing the robust policy plan reduces welfare relative to the case of certainty, i.e. without parameter perturbations. Trivially, there will be an improvement in terms of lower welfare loss if the effect of lower inflation variance dominates the effect of higher output variance, which depends on λ, i.e. the weight in the loss function the central bank places on the output gap.

IV A Simulation Exercise

To investigate the consequences of uncertainty, we calibrate our model to the US economy and compare different scenarios for pairwise different sets of worst-case parameters $\psi^\star = (\alpha_l, \gamma_h)$.[9] More precisely, we compare inflation and output volatility as well as welfare losses for the min-max equilibrium, the approximating equilibrium

[8] The plots in (1) rely on the calibration in Table (1), i.e. it is assumed that the 'true' values for α and γ are 0.8 and 1, respectively.

[9] Section A in the appendix investigates the determinacy properties of the model. We find that the Blanchard and Kahn (1980) condition is not satisfied, saying that our system of equations does not have a unique stationary solution. Hence, we cannot rule out that stationary sunspot equilibria exist.

Parameter	β	ϕ	λ	σ	ρ	α_l	α_h	γ_l	γ_h
Value	0.99	0.15	0.25	1	0.35	[0.5; 0.7]	0.8	0.8	[1.1; 1.2]

Table 1: Baseline calibration

and the case where the model is distorted but the central bank does not account for uncertainty, i.e. it does not implement the robust policy plan.

A. *Calibration*

We calibrate out model to the US economy with the parameters as in Table (1). The values for $\beta = 0.99$ and $\lambda = 0.25$ and $\sigma = 1$ (log utility) are standard in the literature. We follow Surico (2008) and set the slope of the Phillips curve to $\phi = 0.15$.[10] Setting $\rho = 0.35$ implies a medium persistent cost-push shock, as in Woodford (2003). The bounds for α_l are set to 0.5 and 0.7 which imply shares for non-rational households of 50% and 30%, respectively. These are even larger values the share of non-rational households than found in most of the literature and thus intended to represent a grave misspecification. $\gamma_h = 1.2$ and $\gamma_h = 1.1$ are common values for the investigation of heterogeneous beliefs in New Keynesian models (see, for example, Beqiraj et al., 2019; and Gasteiger, 2021) and intended to represent the upper bounds of the interval that is known to the central bank.

B. *Results and Discussion*

Table (2) reports the results for our simulation exercises for different pairs of α_l, γ_h and ρ. We assume that the share of non-rational households in the 'true model' is 20% which implies $\alpha = 0.8$. We further assume that non-rational households in the baseline model are naive in a sense that they forecast via a simple random walk rule, i.e. we assume that the true γ takes a value of one. Comparing the first row in a case of no parameter uncertainty to the standard New Keynesian model in the absence of heterogeneous expectations, we can see that incorporating non-rational households always results in both higher inflation and output volatility as well as higher welfare losses.

In the first scenario (case B), the approximating equilibrium, we assume that the central bank implements the robust policy plan $f^\star(\psi^\star)$ while the model is being undistorted, i.e. we report inflation variance $\pi^2(f^\star(\psi^\star), \psi)$, output variance $x^2(f^\star(\psi^\star), \psi)$ and the loss function $\mathcal{L}(f^\star(\psi^\star), \psi)$. In the second scenario (case C), we assume that the central bank does not implement the robust policy plan while the model is distorted, i.e. the equilbrium outcomes are $\pi^2(f(\psi), \psi^\star)$, $x^2(f(\psi), \psi^\star)$ and the loss function $\mathcal{L}(f(\psi), \psi^\star)$. In the last scenario (case D), the min-max equilibrium, the central bank implements the robust policy plan $f^\star(\psi^\star)$ while the model is being distorted, i.e.

[10] We also tried other values for ϕ, as for example in Christiano et al. (2005) who find $\phi = 0.2$. The results are not presented but available on request.

$\psi^\star\ (\alpha_l, \gamma_h)$	$\rho = 0.35$			$\rho = 0.0$		
	π^2	x^2	$\mathcal{L}$	π^2	x^2	$\mathcal{L}$
			no parameter uncertainty (case A)			
$\alpha = 0.8,\ \gamma = 1.0$	2.464	1.379	1.405	1.196	0.670	0.682
			robust policy, undistorted model (case B)			
$\alpha_l = 0.7,\ \gamma_h = 1.1$	2.306	1.832	1.382	1.420	0.907	0.684
$\alpha_l = 0.6,\ \gamma_h = 1.1$	2.136	2.414	1.370	1.082	1.223	0.694
$\alpha_l = 0.5,\ \gamma_h = 1.1$	1.916	3.325	1.374	1.001	1.737	0.717
$\alpha_l = 0.7,\ \gamma_h = 1.2$	2.264	1.967	1.378	1.127	0.980	0.686
$\alpha_l = 0.6,\ \gamma_h = 1.2$	2.063	2.696	1.368	1.055	1.379	0.700
$\alpha_l = 0.5,\ \gamma_h = 1.2$	1.795	3.919	1.387	0.954	2.084	0.738
			non-robust policy, distorted model (case C)			
$\alpha_l = 0.7,\ \gamma_h = 1.1$	3.392	1.898	1.933	1.621	0.907	0.924
$\alpha_l = 0.6,\ \gamma_h = 1.1$	4.552	2.548	2.594	2.184	1.223	1.245
$\alpha_l = 0.5,\ \gamma_h = 1.1$	6.426	3.597	3.663	3.103	1.737	1.768
$\alpha_l = 0.7,\ \gamma_h = 1.2$	3.796	2.125	2.163	1.751	0.980	0.998
$\alpha_l = 0.6,\ \gamma_h = 1.2$	5.431	3.039	3.095	2.464	1.379	1.405
$\alpha_l = 0.5,\ \gamma_h = 1.2$	8.403	4.703	4.789	3.724	2.084	2.122
			robust policy, distorted model (case D)			
$\alpha_l = 0.7,\ \gamma_h = 1.1$	3.139	2.493	1.881	1.536	1.219	0.920
$\alpha_l = 0.6,\ \gamma_h = 1.1$	3.756	4.245	2.409	1.908	2.157	1.224
$\alpha_l = 0.5,\ \gamma_h = 1.1$	4.343	7.536	3.114	2.345	4.068	1.681
$\alpha_l = 0.7,\ \gamma_h = 1.2$	3.418	2.971	2.080	1.629	1.416	0.992
$\alpha_l = 0.6,\ \gamma_h = 1.2$	4.192	5.480	2.781	2.063	2.696	1.368
$\alpha_l = 0.5,\ \gamma_h = 1.2$	4.843	10.576	3.743	2.538	5.543	1.962
			only rational households			
$\alpha = 1.0, \gamma = n.a.$	1.809	0.651	0.986	0.842	0.303	0.459

Table 2: Simulation results for different policy scenarios, see Table (1) for the calibration.

the evil agent draws $\psi^\star$. The equilibrium outcomes in this case are $\pi^2(f^\star(\psi^\star), \psi^\star)$, $x^2(f^\star(\psi^\star), \psi^\star)$ and the loss function is $\mathcal{L}(f^\star(\psi^\star), \psi^\star)$.

A few things stand out. First, when accounting for uncertainty, the central bank can substantially reduce welfare losses when the model is distorted relative to the case when the central bank ignores possible parameter perturbations. For instance, when the true share for non-rational households turns out to be 50% (i.e. $\alpha = 0.5$) and households are extrapolative (with $\gamma = 1.2$), inflation volatility is 8.403 and output gap volatility 4.703. If the central bank however incorporates the worst-case beliefs into the targeting rule, i.e. implements α_l and γ_h, inflation volatility is substantially reduced to 4.843 while output volatility is increased to 10.576. Importantly, the central bank does not place a weight of one on the output gap in the loss function, which results in an overall reduction of losses from 4.789 to 3.743. This is one key result in this paper and holds for all worst-case configurations under consideration, i.e. that lie within the set $\psi^\star \in \Psi$.[11] Figure (2) plots this difference for different values for α_l between 0.5

[11]Note that this qualitative result, i.e. a lower welfare loss, is robust in the case where shocks are

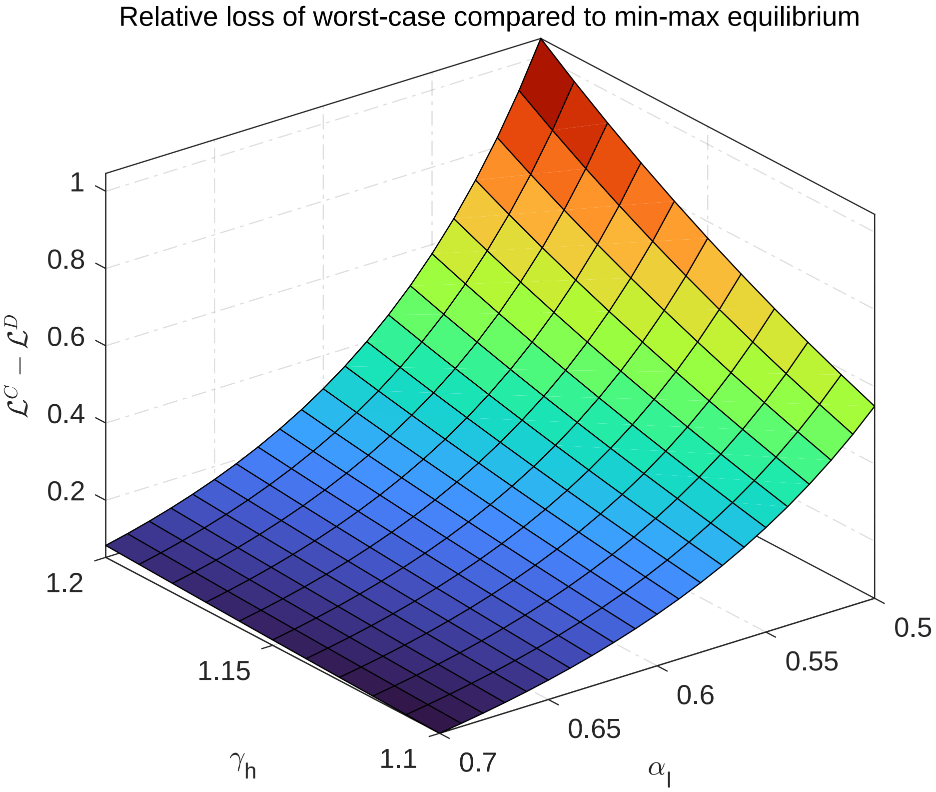

Figure 2: Difference between welfare when the central bank implements the robust policy plan and the model is and the case where there is no parameter uncertainty.

and 0.7 and for γ_h between 1.1 and 1.2. Not surprisingly, welfare always improves, meaning that the central bank can always achieve welfare gains when accounting for uncertainty, given the worst-case beliefs turn out to be warranted. Moreover, the improvement monotonically increases with respect to γ_h and decreases with respect to α_l. That is, the largest welfare improvement is achieved when the central bank is highly uncertain about the true values and these values are actually the true ones.[12]

Second, comparing the results for the case where the central bank incorporates the worst-case beliefs α_l and γ_h into the specific targeting-rule but the model is undistorted with the case of no parameter uncertainty at all, we can now confirm that there is an improvement in the stabilization between inflation and the output gap.

Interestingly, we observe the largest improvement in the case where we imposed the largest interval (that is known to the central bank) for the uncertain parameters. Notice that, identical to the former cases, this improvement is grounded on lower inflation volatility while output gap volatility is increased. If cost-push shocks are however uncorrelated, i.e. if $\rho = 0$, this improvement vanishes and welfare loss is higher than in the case of no parameter uncertainty. This result confirms our observation from Figure (1). Naturally, the question arises whether the central bank could not just implement $\alpha = 0$ and a γ that is greater than the worst-case belief to

uncorrelated.

[12]Notice that Figure (2) also confirms that the min-max equilibrium is a global Nash-equilibrium. This means that there is no pair of $\widetilde{\alpha}$ and $\widetilde{\gamma}$ that violates $\mathcal{L}(f^\star(\psi^\star), \psi^\star) > \mathcal{L}(f^\star(\widetilde{\psi}), \widetilde{\psi})$.

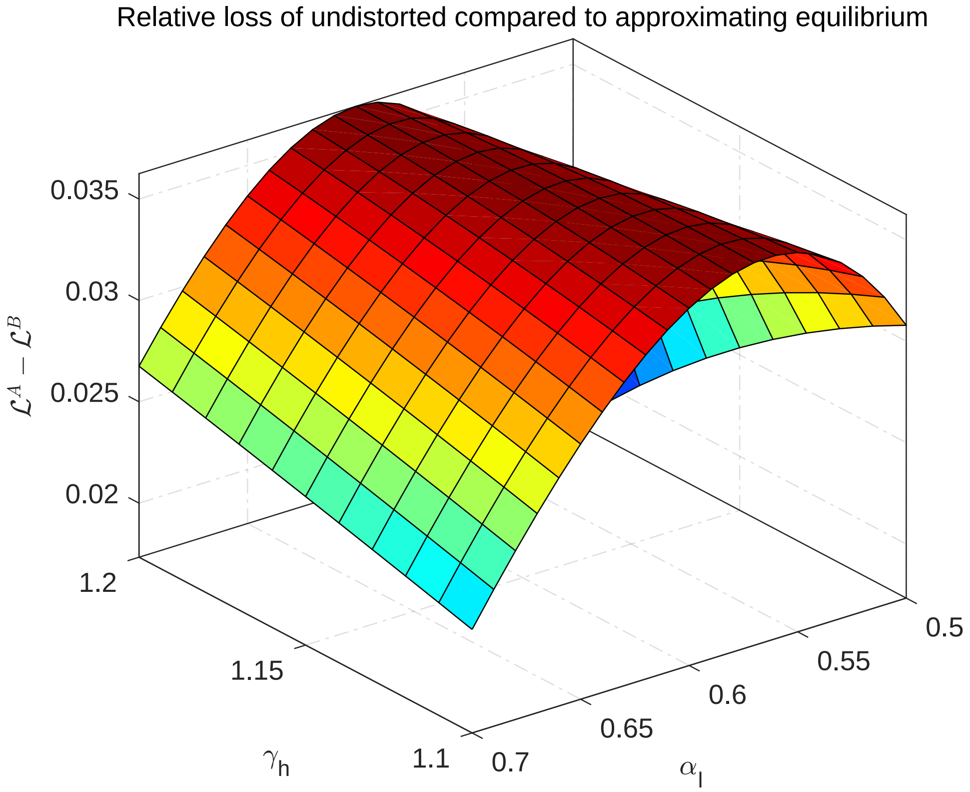

Figure 3: Difference welfare when the central bank implements the robust policy plan and the model is undistorted and the case where there is no parameter uncertainty.

further exploit the improvement described above. Therefore, Figure (3) plots the loss difference between the approximating model and the model without uncertainty, i.e. $\mathcal{L}(f^\star(\psi^\star),\psi) - \mathcal{L}(f(\psi),\psi)$. As can be seen, there seems to exist an α for every given γ that maximizes the welfare improvement which (e.g. for $\gamma = 1.2$) is about $\alpha = 0.6$.[13]

Discretionary monetary policy suffers from the stabilization bias as long as the supply shock is not white noise. The reason is that in the absence of commitment, inflation is too volatile. However, we find that adopting a robust policy approach can reduce the stabilization bias. A desire for robustness makes monetary policy more aggressive. As a result, inflation volatility is reduced and welfare improves compared to the rational expectations solution.

Overall, we conclude that, given there exists a non-negative share of non-rational households, the central bank has an incentive to incorporate the worst-possible beliefs into its targeting rule, even if the model is undistorted. Figures (4) and (6) show the immediate response of the short-term interest rate when, in both cases, the central bank implements the worst-case beliefs, i.e. $f^\star(\psi^\star)$. Not surprisingly, the responses monotonically increase in the share of worst-case beliefs for the share of non-rational households (i.e. decrease in α_l) and the adaption parameter. However, we can see that in the case where the central bank implements the worst-case belief, the immediate interest rate response is systematically lower than in the case where parameter uncer-

[13] Widening the intervals to $\alpha_l = 0$, i.e. no rational agents exist, results in a negative value (not reported).

tainty is ignored. This is based on the degree of dispersion about the true values for α and γ and the worst-case beliefs α_l and γ_h, respectively. As a result, it turns out that in the worst-case equilibrium, the response of inflation is lower than in the case where the central bank does not take perturbations into account, as can be seen in Figures (7) and (8). At the same time, the opposite is true for the output gap, as the robust policy plan results in a larger contraction of output in order to stabilize the economy, see Figures (10) and (11).

V The Gain of Discretion over a Taylor Rule

Taylor (1993) argues that US monetary policy is well described as a simple interest-rate feedback rule that responds to inflation, output, or other economic conditions. It is well known that Taylor rules also mimic optimal discretionary policies, at least under certain circumstances, i.e. they may lead to similar dynamics as in the discretionary case. In this section, we argue that a central bank that accounts for expectations heterogeneity in general and uncertainty about expectations heterogeneity in particular, as implemented in the previous section, is able to achieve gains in terms of lower welfare costs relative to the case of an implemented Taylor rule. We follow Di Bartolomeo et al. (2016) and focus on a feedback rule that responds to current inflation π_t and current output gap x_t, i.e. our reference Taylor rule takes the form

$$i_t = \vartheta_\pi \pi_t + \vartheta_x x_t,$$

where $\vartheta_\pi = 1.5$ and $\vartheta_x = 0.125$. That is, the Taylor principle, stating that the interest-rate should respond by more than one-to-one, is satisfied. Since we have no backward-looking variables in our model, it is easy to derive the equilibrium outcomes for x_t and π_t in this case and thus derive output and inflation variance

$$\mathbf{A}\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \mathbf{B}\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} + \mathbf{D}e_t,$$

where $\mathbf{A}$ and $\mathbf{B}$ contain appropriate parameters and $\mathbf{D}$ is a vector of zeros and ones.[14]

Table (3) reports simulation results with the same calibration as in Table (2). Comparing the results with those in the previous section, it stands out that welfare losses are higher in all scenarios under consideration. That is, if there is no parameter uncertainty at all, in the absence of non-rational households as well as in the case where the evil agent draws the worst-case parameter perturbations while the central bank implements the Taylor rule. Even though inflation volatility is higher than under discretion, the overall effect of lower welfare losses ultimately stems from higher output gap volatility. Furthermore, if the cost-push shock is serially uncorrelated, the central bank can achieve substantially gains with an optimal policy plan, independent

[14] Here, $\mathbf{A} = \begin{bmatrix} 1-(1-\alpha)\gamma+\sigma^{-1}\vartheta_x & \sigma^{-1}\varphi_\pi - \sigma^{-1}(1-\alpha)\gamma \\ -\phi & 1-\beta(1-\alpha)\gamma \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} \alpha & \sigma^{-1}\alpha \\ 0 & \beta\alpha \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

$\psi^\star$ (α_l, γ_h)	$\rho = 0.35$ π^2	x^2	$\mathcal{L}$	$\rho = 0.0$ π^2	x^2	$\mathcal{L}$
		no parameter uncertainty				
$\alpha = 0.8,\ \gamma = 1$	1.722	4.307	2.799	0.975	1.925	1.456
		Taylor rule, distorted model				
$\alpha_l = 0.7,\ \gamma_h = 1.1$	2.144	6.606	3.659	1.251	2.710	1.928
$\alpha_l = 0.6,\ \gamma_h = 1.1$	2.561	8.200	4.611	1.576	3.774	2.520
$\alpha_l = 0.5,\ \gamma_h = 1.1$	3.047	11.438	5.907	2.022	5.518	3.401
$\alpha_l = 0.7,\ \gamma_h = 1.2$	2.301	6.817	4.005	1.330	2.953	2.068
$\alpha_l = 0.6,\ \gamma_h = 1.2$	2.813	9.754	5.252	1.722	4.307	2.799
$\alpha_l = 0.5,\ \gamma_h = 1.2$	3.386	14.528	7.018	2.274	6.683	3.945
		only rational households				
$\alpha = 1, \gamma = n.a.$	1.303	2.869	2.020	0.694	1.235	1.003

Table 3: Simulation results for scenarios if a Taylor rule with $\vartheta_\pi = 1.5$ and $\vartheta_x = 0.125$ is implemented.

of whether the policy maker implements the robust policy plan or not. For example, if the true value for α and γ turn out to be 0.5 and 1.1 respectively, while shocks are correlated with $\rho = 0.35$, inflation variance is 3.047 and output variance 11.438 which results in a loss of 5.907. However, if the robust optimal policy plan $f^\star(\psi^\star)$ is implemented and the central bank shields the economy against uncertainty, inflation variance is 4.343, output variance is 7.536, which results in a loss of 3.114. Even if the central bank ignores possible parameter perturbations and implements the discretionary policy plan, our simulation exercise implies that welfare losses are always lower relative to the case where the Taylor rule is implemented. Summing up, we conclude that if the central bank has the opportunity to conduct a discretionary policy plan rather than following a Taylor rule, it should do so because of two reasons. First, a Taylor rule ignores expectations heterogeneity at all, that is, it only responds to changes in inflation and output while not accounting for different expectation formation processes. The policy plan (i.e. the specific targeting rule) under discretion however was shown to account for non-rational households as it implements both α and γ into the plan f. Second, and more importantly, this argument is even stronger if the central bank has a desire for robustness, i.e. if the central banker wants to account for uncertainty.

VI Conclusion

This paper uses a simple New Keynesian model that features fixed shares of rational and non-rational households, mainly as in Branch and McGough (2009), Gasteiger (2014), Gasteiger (2021) and Di Bartolomeo et al. (2016). Since laboratory experiments and expectations surveys alike imply that (i) a non-negative share of households does not form rational expectations and (ii) the share of non-rational households substantially varies across experiments/surveys, we implement model uncertainty about the determinants of heterogeneity across expectations. Given that both the fractions of rational and non-rational households as well as the expectation formation process is uncertain to the central bank, we derive robust optimal plans in a simple min-max equilibrium as pioneered in Giannoni (2002) and Giannoni (2007). Even though we implemented a very simple way to model adaptive expectations, our results have important implications. First, a central bank should account for uncertainty if it does not know the distributions of different households that are characterized by different expectation formation processes. In particular, in a simulation exercise calibrated to the US economy, we show that a central bank that accounts for uncertainty can *substantially* reduce welfare losses relative to the case where both heterogeneous expectations in general and uncertainty about the distribution of both household types in particular are uncertain.

Second, a central bank that seeks to implement a notable desire for robustness is better-off when it implements the worst-case beliefs, even if these beliefs turn out to be unwarranted. This is based on the fact that a policy rule (i.e. the targeting rule in the discretionary fashion) that incorporates the worst-case beliefs favorably affects the trade-off between output gap and inflation stabilization.

Finally, it has to be mentioned that a model that implements a more sophisticated learning process would probably better map the outcomes that are observable in laboratory experiments (see, for instance, Pfajfar and Žakelj, 2018). However, even though we abstract from backward-looking adaptive expectations formation processes, our model allows the application of a simple min-max approach that is shown to give fruitful structural insights into the mechanism behind robust optimal monetary policy when the strong paradigm of purely rational expectations is abandoned.

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Appendix

A Determinacy Properties

In order to establish determinacy properties of our model in the case where the central bank conducts optimal (robust) monetary policy, first rewrite the model in matrix form as functions of our forward-looking variables. To eliminate the short-term interest rate, recall that, in equilibrium, the interest rate is a function of the deep structural parameters and exogenous driving forces, i.e. $i_t = b_i e_t$. Rewrite the model as

$$\begin{bmatrix} \alpha & \sigma^{-1}\alpha \\ 0 & \beta\alpha \end{bmatrix} \begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1-(1-\alpha)\gamma & -\sigma^{-1}(1-\alpha)\gamma \\ -\phi & 1-\beta(1-\alpha)\gamma \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma^{-1} b_i \\ -1 \end{bmatrix} e_t$$

Define $\mathbf{\Omega} = \begin{bmatrix} \alpha & \sigma^{-1}\alpha \\ 0 & \beta\alpha \end{bmatrix}^{-1}$, the model can, without loss of generality, be rewritten as

$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \mathbf{\Omega} \begin{bmatrix} 1-(1-\alpha)\gamma & -\sigma^{-1}(1-\alpha)\gamma \\ -\phi & 1-\beta(1-\alpha)\gamma \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \mathbf{\Omega} \begin{bmatrix} \sigma^{-1} b_i \\ -1 \end{bmatrix} e_t$$
$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \mathbf{\Omega} \begin{bmatrix} \sigma^{-1} b_i \\ -1 \end{bmatrix} e_t,$$

where

$$\mathbf{A} = \mathbf{\Omega} \begin{bmatrix} 1-(1-\alpha)\gamma & -\sigma^{-1}(1-\alpha)\gamma \\ -\phi & 1-\beta(1-\alpha)\gamma \end{bmatrix}.$$

Blanchard and Kahn (1980) show that a system like the one above has a unique stationary solution if and only if the number of eigenvalues of **A** outside the unit circle is equal to the number of forward-looking variables, which is two in our case (x_{t+1} and π_{t+1}). It can be shown that the vector **z** of eigenvalues of **A** is given by

$$\mathbf{z} = \begin{bmatrix} \dfrac{\left(\phi + \sigma(1+\beta) - \sqrt{(\beta\sigma)^2 + 2\beta\sigma(\phi-\sigma) + \phi(\phi+2\sigma) + \sigma^2 - 2\beta\sigma\gamma(1-\alpha)}\right)}{2\alpha\beta\sigma} \\ \dfrac{\left(\phi + \sigma(1+\beta) + \sqrt{(\beta\sigma)^2 + 2\beta\sigma(\phi-\sigma) + \phi(\phi+2\sigma) + \sigma^2 - 2\beta\sigma\gamma(1-\alpha)}\right)}{2\alpha\beta\sigma} \end{bmatrix}.$$

It is easy to see that the only difference between both eigenvalues is the opposite sign before the square root expression. It turns out that for our entire parameter space, the first eigenvalue is always below one while the second is greater than one. Since the Blanchard and Kahn (1980) condition is not satisfied, our system of equations does not have a unique stationary solution, i.e. it is possible that stationary sunspot equilibria exist.

Notice that the same solution prevails across all different scenarios that we investigate in the main part of the paper. This is because the solutions for all three scenarios, when written as above, only affect b_i, i.e. the vector of eigenvalues $\mathbf{z}$ is unaffected by any policy rule $f(\psi(\alpha,\gamma)) \in F$.

B Additional Figures

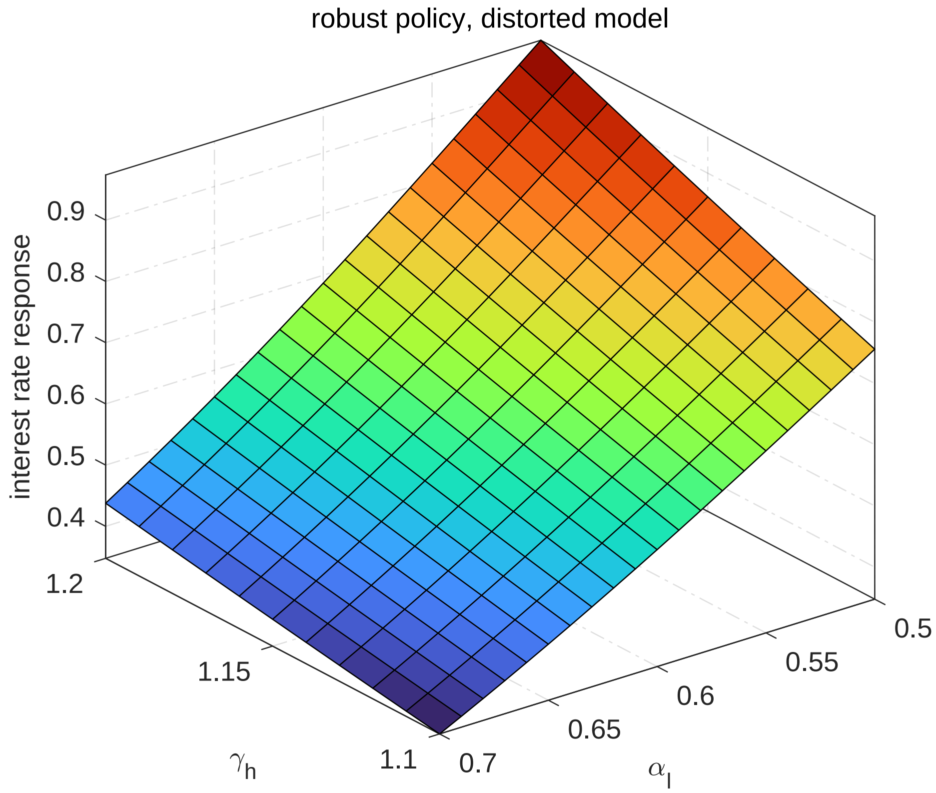

Figure 4: Interest-rate response for different worst-case combinations of α_l and γ_l.

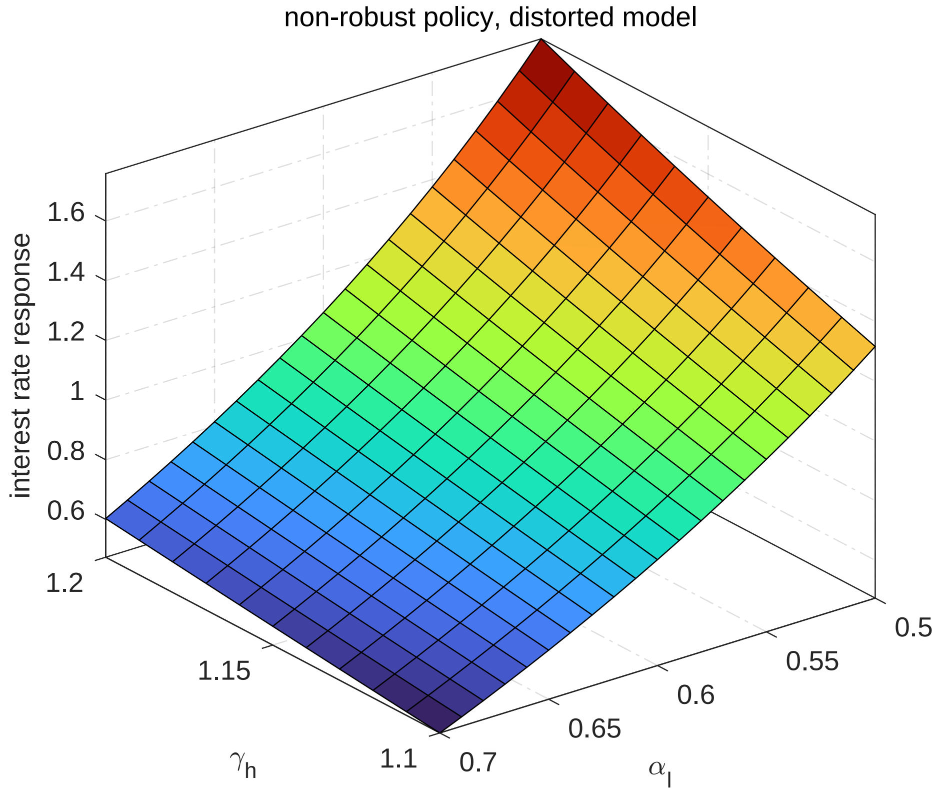

Figure 5: Interest-rate response for different worst-case combinations of α_l and γ_l.

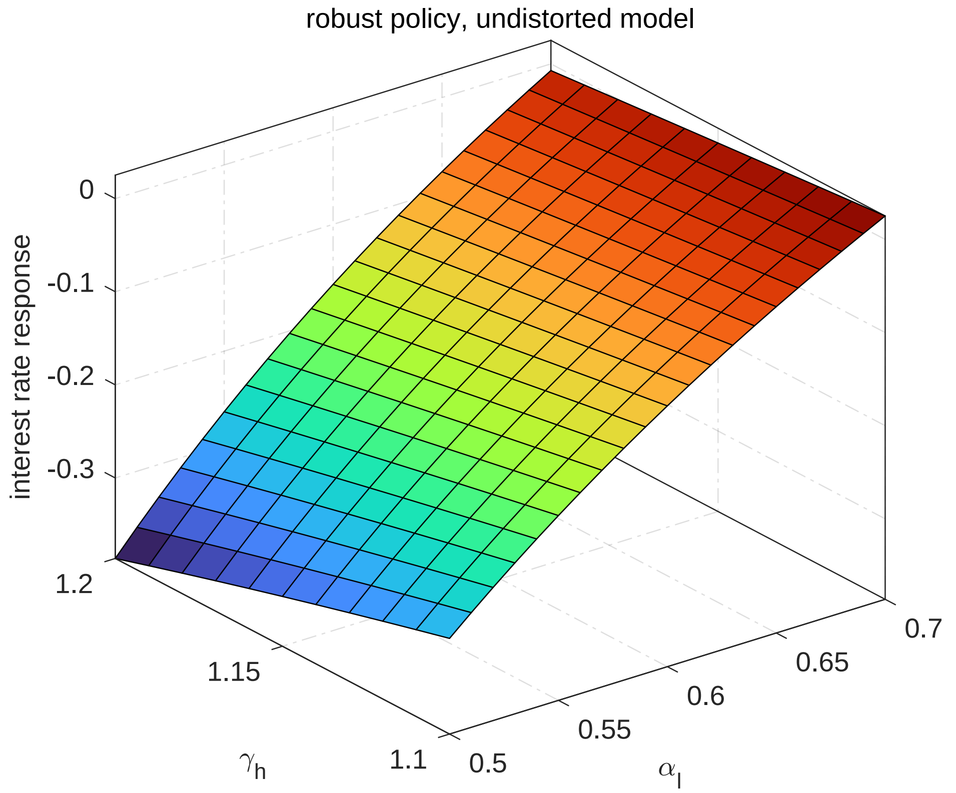

Figure 6: Interest-rate response for different worst-case combinations of α_l and γ_l.

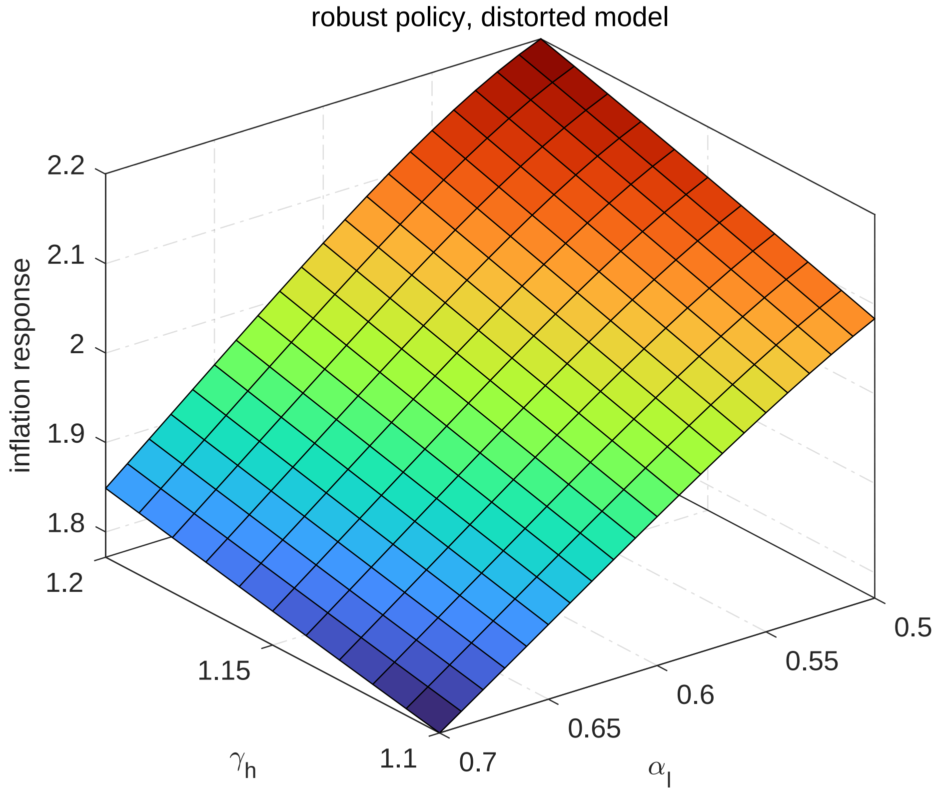

Figure 7: Inflation response for different worst-case combinations of α_l and γ_l.

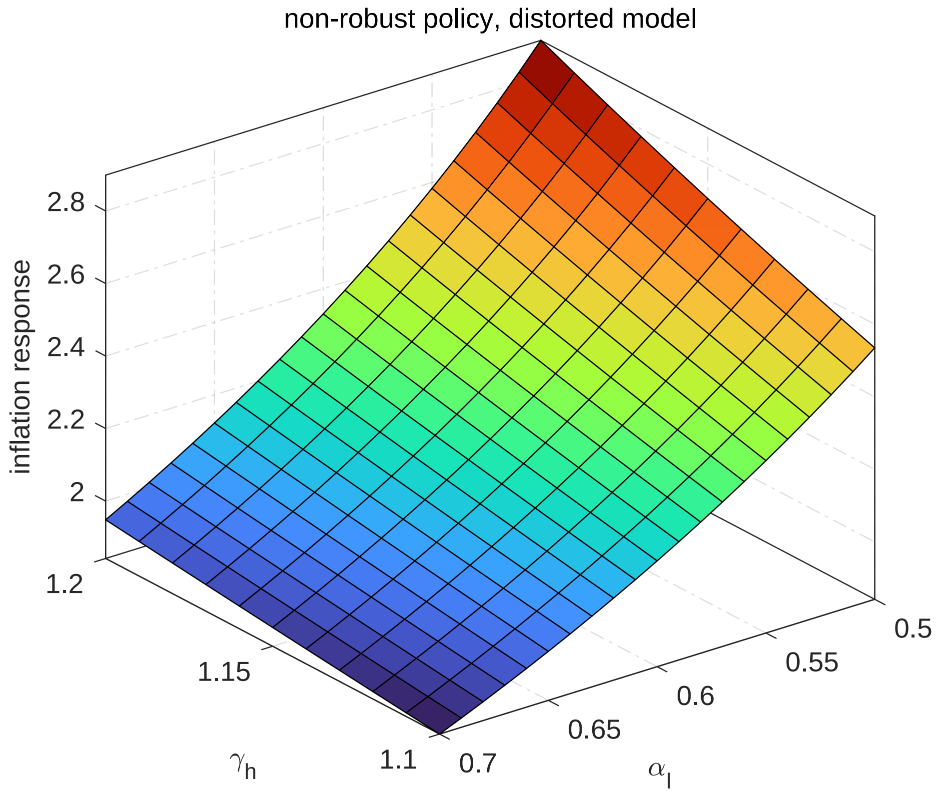

Figure 8: Inflation response for different worst-case combinations of α_l and γ_l.

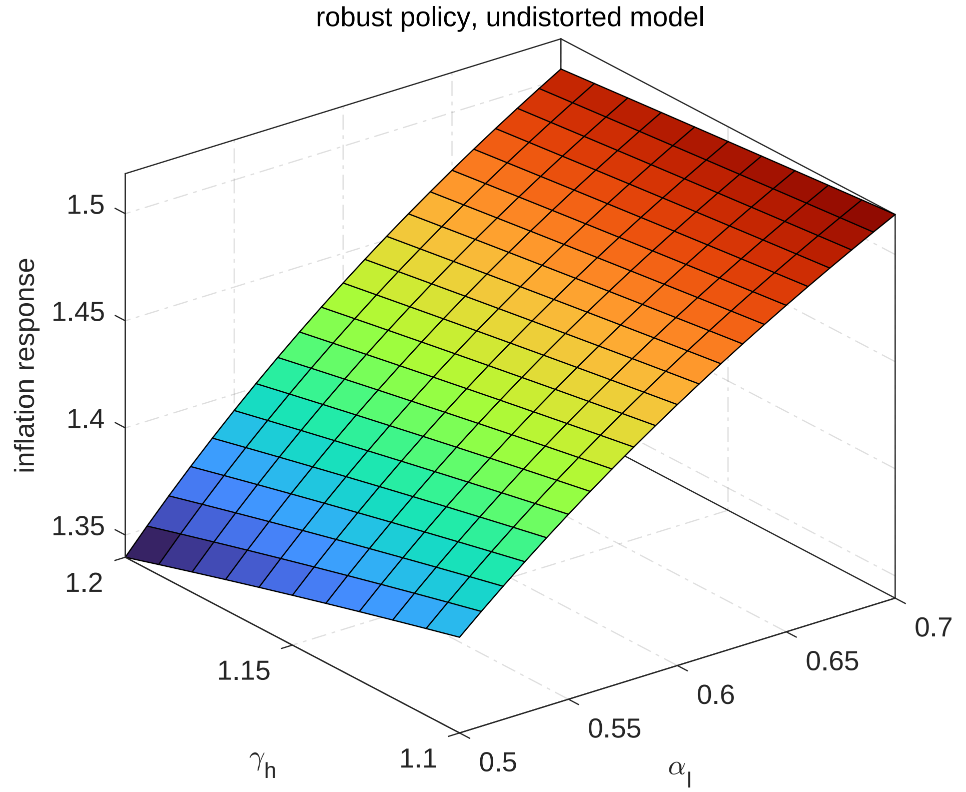

Figure 9: Inflation response for different worst-case combinations of α_l and γ_l.

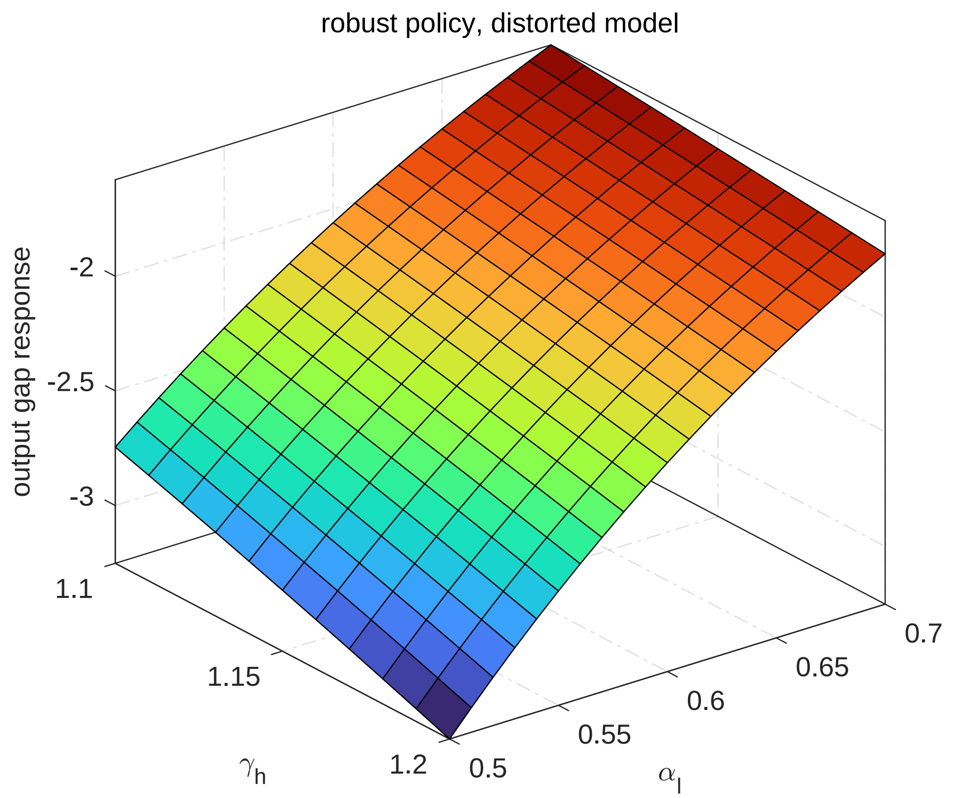

Figure 10: Output gap response for different worst-case combinations of α_l and γ_l.

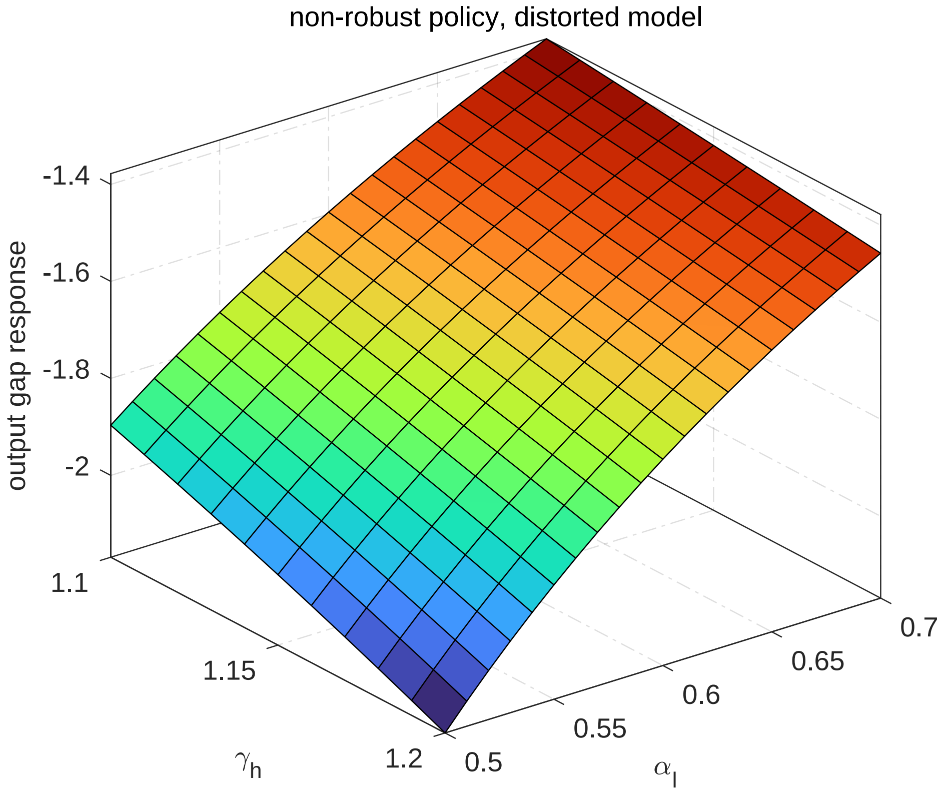

Figure 11: Output gap response for different worst-case combinations of α_l and γ_l.

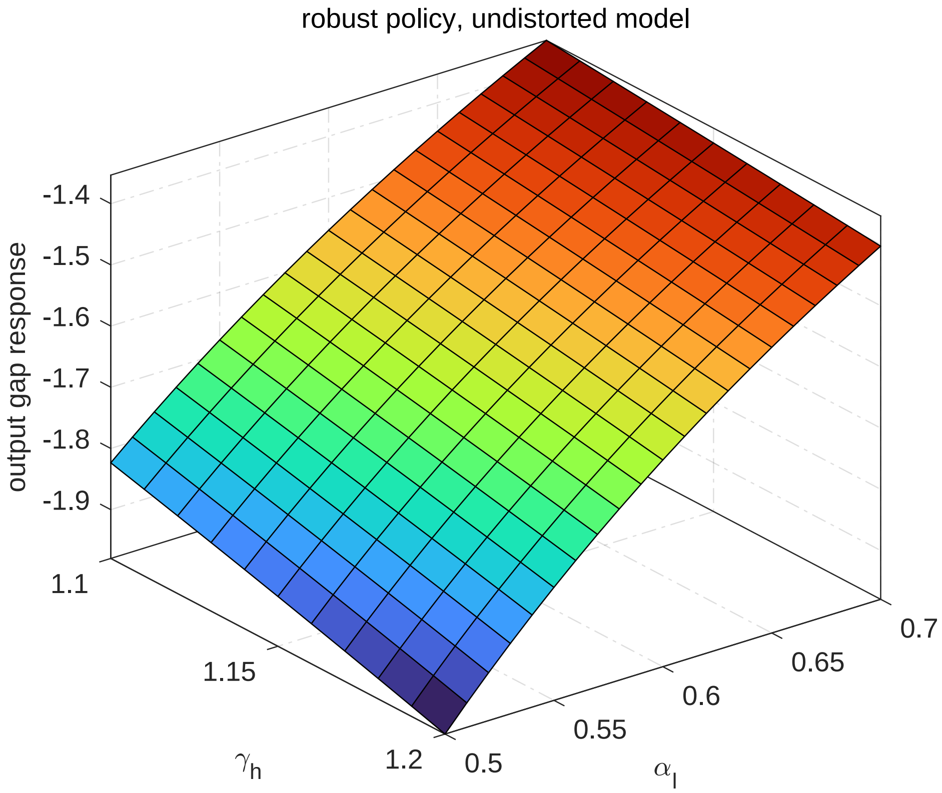

Figure 12: Output gap response for different worst-case combinations of α_l and γ_l.