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And yet They Help. An Analytical Model of how Subsidies for Modernizing Landlords Overcome Inefficient Tenancy Law

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And yet they help

An analytical model of how subsidies for modernizing landlords overcome inefficient tenancy law

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Abstract

In the rental residential building stock, the landlord-tenant-dilemma is a well-known barrier to investments in energy efficiency and exacerbated where rent control limits the possibility to raise rents to finance landlords' investments. Some jurisdictions, like Germany, allow landlords to extraordinarily increase rents in proportion to a modernization's costs. In addition, the government grants subsidies to home-owners investing in energy efficiency. However, landlords must deduct these subsidies from modernization costs that may be levied on tenants. In this paper we model the interaction of these two policies. We find that the modernization surcharge itself is inefficient regarding landlords' and tenants' welfare. Non-deductible subsidies help incentivizing otherwise unprofitable modernizations, thereby improving the modernization width at the cost of tenants' welfare, but at low levels they do not enlarge otherwise profitable modernizations. Deductible subsidies prove to be beneficial for landlords and achieve increases in landlords' optimal modernization extent, improving modernization depth. Deductible subsidies can still incentivize investment where none is profitable without, albeit less effectively. When large enough to overcome the inefficient incentives of the modernization surcharge, deductible subsidies can also guarantee both landlords and tenants to gain welfare as well as increasing overall Social Welfare.

Highlights

- The landlord-tenant-dilemma causes energy inefficiency in rental housing.
- We theoretically model how subsidies and tenancy law interact.
- Deductible subsidies counterintuitively incentivize larger modernizations.
- Sufficiently large subsidies can benefit both landlords and tenants while increasing modernization width and depth.

Keywords

- energy efficiency
- landlord-tenant-dilemma
- residential building sector
- subsidies
- tenancy law

JEL-Classification

- H23 Externalities • Redistributive Effects • Environmental Taxes and Subsidies
- K25 Real Estate Law
- Q48 Government Policy

Declaration of interests

none

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- Leo Reutter: Conceptualization, Methodology, Formal analysis, Writing - Original Draft, Writing - Review & Editing, Visualization, Project administration, Funding acquisition
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1 Introduction

Causing ca. 11 % of Germany's greenhouse gas emissions with only minor reductions for the last decade (German Environment Agency, 2022), heat production for residential buildings is an obvious target for climate protection policies. The German Climate Protection Act of 2019 with its 2021 amendment aims at net climate neutrality by 2045, encompassing amongst others the residential building sector. After Russia's 2022 invasion of Ukraine, improving energy efficiency to reduce dependency on foreign fossil fuel imports has gained additional political relevance on top of climate policy. More than half of Germany's dwellings are rental homes. As a consequence, energy policies and tenancy law interact, with potential conflicts of aims.

For newly built homes, regulatory law such as the Buildings Energy Act (*Gebäudeenergiegesetz* GEG) could directly achieve the emission goals. Existing buildings, in contrast, need to abide by most regulatory law only if they undergo modernization. Therefore, an increase in energy efficiency up to and beyond the level prescribed by regulatory law requires a homeowner's decision to actually modernize. Owner-occupiers tend to modernize only when they are triggered to do so. This may occur, for example, when they perceive economic benefits from energy cost savings, when their home needs repair anyway, or when they recently have acquired it (Gossen and Nischan, 2014, pp. 20–21; Weiß et al., 2018, pp. 12–15). Landlords, however, do not share the same economic motivation for modernization, as energy consumption is usually paid for by tenants. Hence costs of investment in residential energy efficiency are paid for by landlords, while the benefits from such investments accrue to tenants.

For more than 25 years, the economic literature has been agreeing to analyze investment in energy-efficiency in terms of a principle-agent-problem (e.g. Jaffe and Stavins, 1994, p. 98) with landlords acting as agents for tenants. When tenants pay energy bills, as is usually the case in Germany, and landlords pay for energy efficiency, their incentives diverge (Ástmarsson et al., 2013, p. 357; Bird and Hernández, 2012, p. 508; Charlier, 2015, p. 465). Empirically landlords underinvest in energy efficiency as compared to owner-occupied homes, generally attributed to the landlord-tenant dilemma (Charlier, 2015; Gillingham et al., 2012; Murtishaw and Sathaye, 2006; Petrov and Ryan, 2021). Adan and Fuerst (2015) combine theory and empirical data to scrutinize incentives for English landlords to modernize their buildings. While there is very little structured and reliable information on the modernization investment in the German residential building sector (März, 2018), the available data indicate that ceteris paribus landlords invest less than owner-occupiers (Renz and Hacke, 2016; Testorf et al., 2010). This can partially be attributed to rental properties achieving lower green premiums on the market compared to owner-occupied properties (Hahn et al., 2018; Hyland et al., 2013; Kholodilin et al., 2017; Taruttis and Weber, 2022). Another important reason lies in tenancy law exerting rent control on existing rental contracts. In Germany like many other countries, landlords may usually only raise rents within strictly defined limits, generally exacerbating the landlord-tenant-dilemma.

Similar to some other jurisdictions (e.g. Austria, Denmark, Poland; Federal Institute for Research on Building, Urban Affairs and Spatial Development, 2016, p. 50), German tenancy law aims to overcome the landlord-tenant-dilemma by re-allocating some of the investment costs from landlords to their tenants by offering opportunities for extraordinary rent increases. This modernization surcharge is the basis of our micro-economic analysis. Further legal discussion on the German system can be found e.g. in Bürger et al. (2013), Klinski (2010), and Rehkugler et al. (2014). As another policy to incentivize energy efficiency in the building sector, subsidies are offered to increase the profitability of socially desirable investments (Brown et al., 2019). The German government even chose substantial subsidies as an immediate action program in 2021 to compensate for missing the legally binding sector emissions reduction targets in 2020 (Federal Ministry for Economic Affairs and Energy and Federal Ministry of the Interior, for construction and homeland, 2021). However, using subsidies to fund modernizations reduces the costs that may be shifted onto tenants in an extraordinary rent increase. This is lamented to decrease the appeal and the efficacy of subsidies in the rental market (DMB and DUH, 2020; Henger et al., 2021). Recently, Ahlrichs and Rockstuhl (2022) used an analytical model to estimate a modernization surcharge that is fair to landlords, tenants, and other agents and how subsidies or carbon pricing affect that fair modernization surcharge. However, their policy analysis is limited by simply assuming that subsidies "increase the attractiveness" of building retrofits (Ahlrichs and Rockstuhl, 2022, p. 5). We scrutinize that notion in detail by explicitly analyzing landlords' profits without and with subsidies.

Inspired by German tenancy law, our paper investigates the general case of incentives and outcomes of the modernization surcharge and how it interacts with subsidies. The aim of our research is to provide clear qualitative insights into the incentives for landlords' investments and the ensuing welfare effects of the modernization surcharge, whether and how subsidies change these incentives, and whether subsidies should or should not be deducted from the investment costs for the modernization surcharge. We differentiate between the effect of subsidies on the modernization depth, that is whether subsidies incentivize landlords to pursue a more ambitious modernization if a modernization is already profitable without subsidies, and on the modernization width, that is whether subsidies contribute to more modernizations being undertaken that would be non-profitable without subsidization.

We show that the modernization surcharge itself is inefficient regarding landlords' and tenants' welfare and that socially desirable improvements in energy efficiency are a mere by-product from landlords' perspective. Furthermore, we find non-deductible subsidies to help enabling otherwise unprofitable modernizations but otherwise having no effect on the investment unless subsidies are large enough that landlords choose to maximize first and foremost subsidization instead of rental income. Only if non-deductible subsidies are sufficiently large do they increase landlords' profits as well as weakly increase (instead of weakly decrease) incumbent tenants' welfare. Moreover, we find that, counterintuitively, the obligation to deduct subsidies from the costs that may be shifted onto tenants actually leads to an increase in modernization investment if a modernization is profitable already without subsidies. Furthermore, deductible subsidies can still render unprofitable investments worthwhile for landlords, albeit less effectively than non-deductible subsidies. When large enough to overcome the inefficient incentives of the modernization surcharge, deductible subsidies can also guarantee both landlords and tenants to gain welfare as well as increasing overall Social Welfare with the government possibly facing costs. When subsidies are sufficiently high to incentivize larger investments despite a legal cap on rent increases, the obligation to deduct subsidies from the costs that may be shifted onto tenants becomes irrelevant.

The paper is structured as follows: we first set up our microeconomic model of landlords' profits depending on their investment, as well as market conditions and general legal limits, without, with non-deductible and with deductible subsidies (Section 2). We use our model to determine landlords' optimal investment (Section 3) before analyzing the ensuing welfare effects (Section 4). We confirm that the results are general enough to accommodate the intricacies of, for example, the German tenancy law (Section 5). We acknowledge the limitations of our results (Section 6) and conclude with some policy recommendations (Section 7).

2 The Model

In this section we develop our model for profits that landlords may accrue from modernizing the apartment to increase energy efficiency. We assume profit-maximizing landlords to capture the interaction of two inherently economic incentive mechanisms. Tenancy law, regulating the available rent revenue when there is rent control, directly aims at influencing landlords' profits to incentivize certain behavior and so does public funding for energy efficiency in the form of subsidies. We analyze these policies by their combined impact on landlords' profits. Assuming profit-maximization as a driving motive is certainly adequate for professional corporate landlords owning ca. 2.9 million dwellings in Germany, i.e. 14 per cent of the rental building stock (Destatis, 2016). But also private amateur landlords, while not single-mindedly pursuing profits (März, 2018), have shown that economic concerns also do play a significant role in their decision-making process (Ambrose, 2015; Cischinsky et al., 2015; Hope and Booth, 2014; Schätzl, 2007).

2.1 Landlords' profits without subsidies

Let $\pi_n(c)$ be landlords' additional profits from modernization relative to the monetary investment in energy efficiency c without any available subsidies. We assume that tenancy law affects these profits by placing legal limits on the ability to increase incumbent tenants' rents. Profits also depend on incumbent tenants' and future tenants' willingnesses to pay (henceforth: WTP). Obviously, they also depend on landlords' monetary costs for performing a modernization. We only consider the net present value of future cash flows like the additional rent revenue or possible interest payments to ease the algebraic analysis.

To capture the fact that landlords expect current tenants to vacate the apartment in question only at some time in the future we model landlords' profits as the weighted sum of incumbent and future tenants' WTP — possibly restricted by legal constraints — minus the cost of modernization, with $\mu \in (0,1)$ being the weight on incumbent tenants' WTP and $1-\mu$ the weight on future tenants' WTP. This distinction is important as there are tenancy law regimes such as in Germany or Austria, that place strict legal bounds on possible rent increases in ongoing rental contracts while there is less regulation on rent levels of newly formed contracts. Often the incumbent tenants' rent level is legally limited but performing a modernization allows landlords to extraordinarily raise the rent. This modernization surcharge depends on the size of the investment but must not exceed some absolute limit $L > 0$. We model the rent increase by $a\, c$, where $a > 0$ is the maximum proportion of the investment costs that may be levied on incumbent tenants. Note that if $a < 1$ no investment would prove profitable for landlords and an according design of the modernization surcharge would simply yield inactivity. The additional revenue gained from incumbent tenants after modernization not only depends on what is legally permissible but also on their WTP for energy efficiency which we model to encompass two factors. First, we assume that due to general rent control incumbent tenants gain a quasi-rent $Q > 0$ from living in an apartment where rent levels are capped. Over time, tenants derive some utility from staying in that specific apartment, e.g. by developing emotional connections to the apartment or by avoiding the various costs of moving home. In other words, because of the legal limits, tenants' WTP increases more throughout the tenure than the rent payments. If there was no rent control and landlords and tenants freely negotiated rent levels, landlords, assuming perfect information about tenants' WTP, could always increase the rent levels just to the point where incumbent tenants are indifferent between staying or moving. In that case, landlords would accrue the quasi-rent. Second, based on empirical research in many regions using choice experiments (e.g. Banfi et al., 2008; Carroll et al., 2016; Collins and Curtis, 2018) or hedonic pricing analyses (e.g. Hahn et al., 2018; Hyland et al., 2013; Kholodilin et al., 2017; Taruttis and Weber, 2022), we assume that tenants indeed value improved energy efficiency. These studies often express the WTP relative to the efficiency class according to the energy performance certificate or relative to the monetary energy savings where Collins and Curtis (2018) have shown that the marginal WTP per unit energy saved is decreasing. For our model setup, however, we need a WTP relative to landlords' investment. As the additional energy savings per investment are also decreasing (Jakob, 2006), we have to account for two nested concave functions translating landlords' investment to tenants' WTP. Coupled with Kholodilin et al.'s (2017) observation, which has recently been confirmed by Taruttis and Weber (2022), that tenants generally undervalue energy cost savings, we simply assume tenants' WTP to be proportional to landlords' investment by a factor of $\gamma \in (0,1)$. As γ is less than unity, modernizations produce less value than their costs within the rental market, explaining the empirically low investment activity. We will discuss this simplification in Section 6. The size of the modernization surcharge for incumbent tenants is ultimately determined by the lesser of what they are willing to pay and what is legally allowed.

Mainly in the political discussion, writers (e.g. Cischinsky et al., 2015, p. 166, Gaßner et al., 2019, p. 54) claim that the increased value following an investment in energy efficiency demands separate consideration in the landlords' profit function. This however misses the very basic economic fact that this value is nothing else than the net present value of all rent premiums resulting from future tenants' increased WTP. Our profit function therefore excludes any separate term representing the 'increased value of the building'.

Landlords' profits are then defined by:

$$\pi_n(c) \equiv \underbrace{\mu \min\left(\overbrace{a\,c}^{r}, \overbrace{L}^{a}, \overbrace{Q+\gamma\,c}^{w}\right)}_{\text{add'l revenue from incumbent tenants}} + \underbrace{[1-\mu]\,\gamma\,c}_{\text{add'l revenue from future tenants}} - \underbrace{c}_{\text{costs}} \tag{1}$$

Note that from now on we utilize indices to refer to specific terms in the model. We use a lower index $x \in \{n, \delta, 0, 1\varphi, \Phi\}$ to refer to the relevant subsidy regime: $x = n$ means that no subsidies are considered. $x = s$ means that some not further-specified subsidies are considered. $x = 0$ means that the subsidies are non-deductible whereas $x = \delta$ means that at least a share $\delta \in (0,1]$ has to be deducted from the modernization surcharge. $x = \varphi$ means that the subsidy is bounded by the subsidy quota φ and likewise $x = \Phi$ means that the subsidy is bounded by the maximum available subsidy Φ. Similarly, the upper index $y \in \{r, a, w\}$ indicates if one or more arguments of the min-function representing the additional revenue that landlords can and may extract from incumbent tenants after a modernization is binding. $y = r$ means that the relative legal limit is binding, $y = a$ refers to the absolute legal limit and $y = w$ to incumbent tenants' WTP. If two of these arguments are equal and less than the other one, we use a double index, e.g. π_n^{ra} and c_n^{ra} for the profits and the underlying investment, when the two legal bounds to rent extraction coincide and no subsidies are considered.

2.2 Landlords' profits with subsidies

Many governments incentivize investment in energy efficiency by (Pigouvian) subsidies, which we denote by s. Such subsidies are often restricted to the amount applied for, to a fraction $\varphi \in (0,1)$ of the investment costs and to some maximum subsidy amount $\Phi > 0$. We abbreviate the maximum subsidy that may be applied for by $\bar{s}(c) = \min(\varphi\, c, \Phi)$. For owner-occupiers, such subsidization is easy to implement legally as there is only one suitable recipient of the funding. In tenancy relations, the subsidy often interacts with rent control in a complex way. In Germany, for example, the subsidy awarded to the landlord has to be deducted from the costs that may be shifted onto incumbent tenants to avoid double financing. In our model, this changes landlords' profit function with subsidies $\pi_s(c, s)$ to the following:

$$\pi_s(c,s) \equiv \underbrace{\mu \min(a\,[c - \delta \cdot s], L, Q + \gamma\, c)}_{\text{add'l revenue from incumbent tenants}} + \underbrace{[1-\mu]\,\gamma\,c}_{\text{add'l revenue from future tenants}} - \underbrace{c}_{\text{costs}} + \underbrace{s}_{\text{subsidies}} \tag{2}$$

where $\delta \in [0,1]$ denotes the degree to which the subsidy is deducted from the modernization costs that may be levied on the tenant.

At a first glance, having to deduct subsidies completely from the modernization costs ($\delta = 1$) negates any additional value for landlords from subsidies. After all, why should they apply for subsidies if only to reduce incumbent tenants' payments equivalently? However, as we will show in the remainder of this paper, subsidies still offer additional value for landlords and the obligation to deduct them from the modernization surcharge incentivizes larger investments and may offer improvements benefitting both landlords and tenants and for social welfare overall. Comparing this regulation to the case where no such obligation exists even shows that the obligation has superior welfare effects and better incentivizes larger investment. To derive these results, we next turn to landlords' choices of investment and subsidy levels.

3 Landlords' choice of modernization costs

In this section we derive the profit maximizing level of modernization costs first without subsidies, then with non-deductible subsidies and finally with deductible subsidies. While the latter form of subsidies actually exists in Germany, the former is called for, for instance, by the German tenant association (DMB and DUH, 2020) and by the now co-ruling Green Party (Gaßner et al., 2019).

3.1 No Subsidies

Without subsidies, the profit function is given by equation (1) and thus continuous and piecewise linear in c. Table 1 presents the resulting profits for different values of the min-function and defines the values of c at which kinks may occur in $\pi_n(c)$.

Table 1: Landlords' profits in the absence of subsidies

Add'l revenue from inc. tenants $= \min(a\,c, L, Q+\gamma\,c)$ given by:	Profits $\pi_n(c)$
$a\,c$	$\pi_n^r \equiv [\mu\,a + [1-\mu]\,\gamma - 1]\,c$
$a\,c = L \Rightarrow c = c_n^{ra} \equiv \frac{L}{a}$	$\pi_n^{ra} \equiv \pi_n^r = \pi_n^a$
L	$\pi_n^a \equiv \mu\,L + \big[[1-\mu]\,\gamma - 1\big]\,c$
$L = Q + \gamma\,c \Rightarrow c = c_n^{aw} \equiv \frac{L-Q}{\gamma}$	$\pi_n^{aw} \equiv \pi_n^a = \pi_n^w$
$Q + \gamma\,c$	$\pi_n^w \equiv \mu\,Q + [\,\gamma - 1]\,c$
$a\,c = Q + \gamma\,c \Rightarrow c = \widetilde{c_n^{rw}} \equiv \frac{Q}{a-\gamma}$	$\pi_n^{rw} \equiv \pi_n^r = \pi_n^w$

The kink at $\widetilde{c_n^{rw}}$ obviously is relevant only if $\widetilde{c_n^{rw}} > 0$. Since this kink will turn out to be relevant only when smaller than c_n^{ra}, we redefine the values of c which mark the relevant kinks as follows:

Definition 1: $c_n^{ra} \equiv \frac{L}{a}$, $c_n^{aw} \equiv \frac{L-Q}{\gamma}$, and $c_n^{rw} \equiv \Gamma\left(\frac{Q}{a-\gamma}\right)$ with $\Gamma(y) \equiv \begin{cases} y & \text{if } y \in (0,\infty) \\ +\infty & \text{otherwise} \end{cases}$.

Since the slope of $\pi_n(c)$ may obviously only be positive if $\min(a\,c, L, Q+\gamma\,c) = a\,c$ and is negative otherwise, optimal investments are either zero or equal to $\min(c_n^{ra}, c_n^{rw})$. The latter is the case if

$$\mu\,a + [1-\mu]\,\gamma - 1 > 0 \qquad (3)$$

and the former otherwise. We call the case where the relative legal limit a, incumbent tenants' weight μ and future tenants' WTP γ are sufficiently large to satisfy condition (3) the *profitable case* whereas the opposing case is called the *unprofitable case*. Note that for the remainder of this paper we assume that whenever indifferent between two investments, landlords choose the larger. We thus get:

Lemma 1: Without subsidies, the optimal investment is given by

$$c_n^* = \begin{cases} 0 & \text{if } \mu\,a + [1-\mu]\,\gamma - 1 < 0 \\ \min(c_n^{ra}, c_n^{rw}) & \text{if } \mu\,a + [1-\mu]\,\gamma - 1 \geq 0 \end{cases}.$$

The proof, albeit simple, is relegated to the appendix together with the proofs of all other lemmata.

Intuitively, this means that the modernization surcharge for incumbent tenants has to be sufficiently large to counterbalance their foreseeable replacement by future tenants whose WTP, while positive, is insufficient to recover the investment costs. A modernization surcharge proportional to investment costs levied on incumbent tenants thus likely fails to incentivize landlords if the latter expect a quick tenant turnover.

Figure 1 shows landlords' profits as a function of investment costs without subsidies and marks the critical investment costs that may maximize profits.

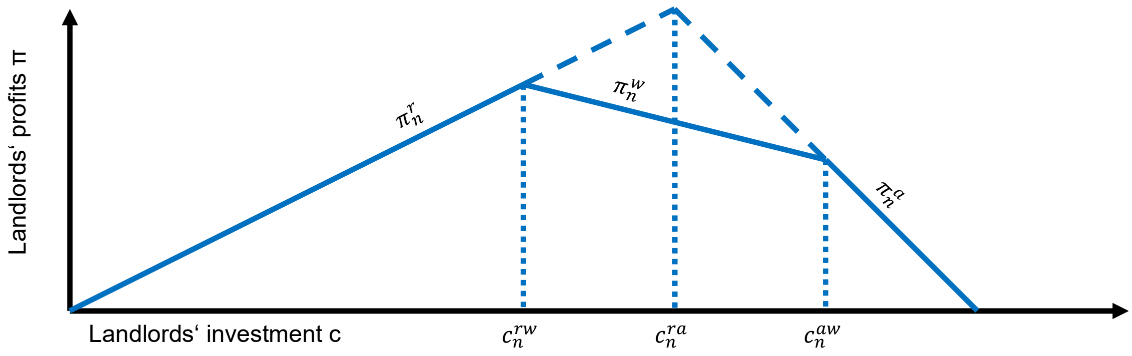

Figure 1: Landlords' profits $\pi_n(c)$ as a function of investment costs c without subsidies in blue. Hand-drawn figure qualitatively equivalent to the following parametrization: $\alpha = 1.7$, $\gamma = 0.7$, $L = 380$, $Q = 130$, $\mu = 0.35$. [Please print in color]

3.2 Non-deductible subsidies

Allowing for subsidies changes the profit function to equation (2). If subsidies are not to be deducted ($\delta = 0$), landlords will obviously choose subsidies as large as legally permissible, i.e. $s^*(c) = \bar{s}(c) = \min(\varphi\ c, \Phi)$. The profit function is thus given by $\pi_0(c) = \pi_n(c) + \min(\varphi\ c, \Phi)$. As $\pi_n(c)$ and $\min(\varphi\ c, \Phi)$ are continuous and piecewise linear, the same is true for $\pi_0(c)$. Graphically, the profit function gets tilted upwards when $c \leq \Phi/\varphi$ and shifted upward when $c \geq \Phi/\varphi$ (Figure 2).

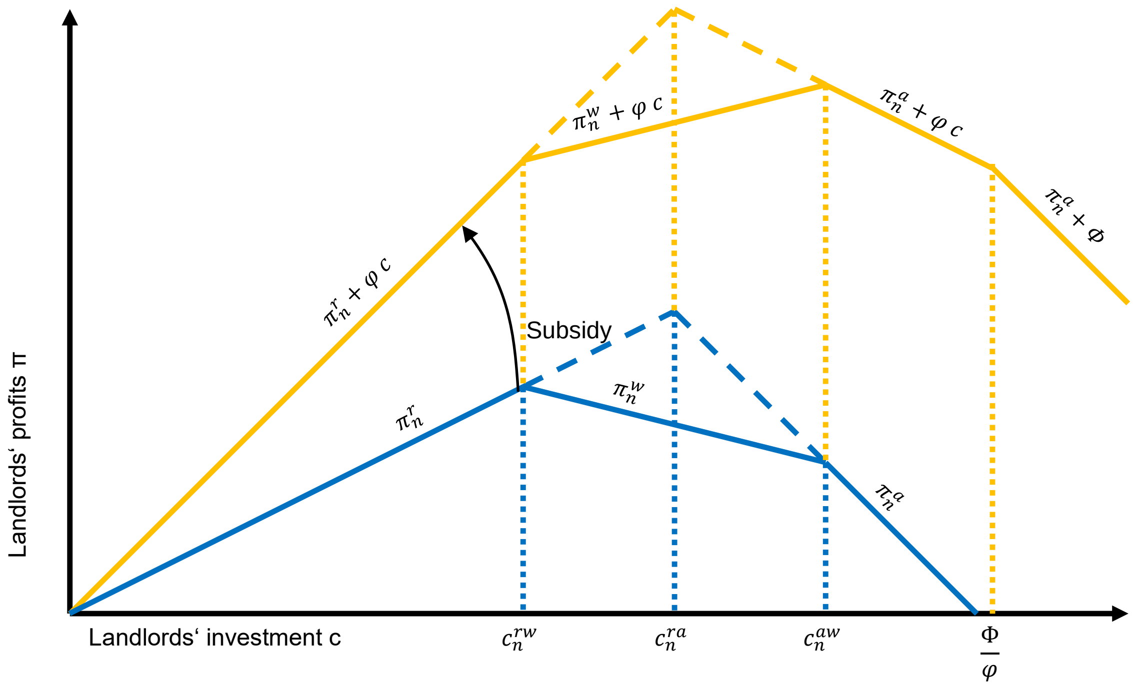

Figure 2: Landlords' profits $\pi_n(c)$ (without subsidies, in blue) and $\pi_0(c)$ (with non-deductible subsidies, in orange) as a function of investment costs c. Hand-drawn figure qualitatively equivalent to the following parametrization: $\alpha = 1.7$, $\gamma = 0.7$, $L = 380$, $Q = 130$, $\mu = 0.35$, $\varphi = 0.25$, $\Phi = 100$. [Please print in color]

If φ is small enough to leave the sign of the slopes of $\pi_0(c)$ the same as of $\pi_n(c)$ for all linear pieces of the profit function, or if $\mu\, a + [1-\mu]\,\gamma - 1 \geq 0$ and Φ is small enough to ensure $\Phi/\varphi \leq \min(c_n^{ra}, c_n^{rw})$, the subsidy fails to change the profit-maximizing modernization expenses. Otherwise, optimal modernization costs will increase. To express this in our next lemma, we use the following

Definition 2: $\underline{\varphi}_0 \equiv 1 - \mu\, a - (1-\mu)\gamma$, $\bar{\varphi} \equiv 1 - \gamma$, and $\bar{\bar{\varphi}} \equiv 1 - (1-\mu)\gamma$.

If φ exceeds these critical values, the slopes of $\pi_0^r(c)$, $\pi_0^w(c)$, and, respectively $\pi_0^a(c)$ turn positive. We draw the reader's attention to the facts that $\bar{\bar{\varphi}} > \max\left(\bar{\varphi}, \underline{\varphi}_0\right)$ and that $c_n^{rw} > 0$ implies $c_n^{ra} > c_n^{rw} \Leftrightarrow c_n^{aw} > c_n^{ra}$ and $\gamma < a$ and thus $\bar{\varphi} > \underline{\varphi}_0$. These inequalities on the critical values of φ imply that the conditions in the following lemma are mutually exclusive.

Lemma 2: With non-deductible subsidies, the optimal modernization costs is given by:

$$c_0^* = \begin{cases} 0 & \text{if } \varphi < \underline{\varphi}_0 \\ \Phi/\varphi & \text{if } \begin{cases} \varphi \geq \max\left(\underline{\varphi}_0, \Phi/\min(c_n^{ra}, c_n^{rw})\right) + H\left(-\underline{\varphi}_0\right) \\ \text{or } \max\left(\underline{\varphi}_0, \bar{\varphi}, \Phi/c_n^{aw}\right) \leq \varphi < \Phi/c_n^{rw} \\ \text{or} \max\left(\underline{\varphi}_0, \bar{\bar{\varphi}}\right) \leq \varphi < \Phi/\min(c_n^{ra}, c_n^{rw}) \end{cases} \\ c_n^{aw} & \text{if } c_n^{ra} > c_n^{rw} > 0 \text{ and } \varphi \in \left[\max\left(\underline{\varphi}_0, \bar{\varphi}\right), \min(\bar{\bar{\varphi}}, \Phi/c_n^{aw})\right) \\ \min(c_n^{ra}, c_n^{rw}) & \text{otherwise} \end{cases} \tag{4}$$

where $H(\cdot)$ is the Heaviside function[1]. Intuitively, the first line represents the case where subsidies are not large enough to make unprofitable investments profitable. The second line is the case where the subsidy quota is large enough to turn the slope of $\pi_0(c)$ non-negative for all $c < \Phi/\varphi$ beyond which marginal profits remain negative. Note that the first of the three alternative conditions describes the case where only the slope of π_δ^r is tilted to the positive (adding $H\left(-\underline{\varphi}_0\right)$ excludes the cases where this slope already is positive). The second condition describes the case where the slope of π_δ^w is tilted to the positive up to $c = \Phi/\varphi$ lying between c_n^{rw} and c_n^{ra}. It may only be satisfied if $c_n^{ra} > c_n^{rw}$, i.e. when tenants' WTP limits the modernization surcharge rather than the absolute legal limit. Similarly, the third condition refers to the case where the slope of π_0^a turns positive, that is when the subsidy quota is sufficiently large to incentivize extending the modernization although no additional revenue can be gained from incumbent tenants. The second line thus describes the cases where landlords' investment decision is rather driven by maximizing subsidy income within the confines of the modernization surcharge than by maximizing the modernization surcharge with consideration of subsidies. The third line describes the case where π_0^w is tilted to the positive up to $c = c_n^{aw} < \Phi/\varphi$, that is when it is optimal to invest as much as to match tenants' WTP with the absolute legal limit. The last line describes the remaining cases, where the subsidy quota is large enough to make investments profitable, but small enough to maintain the negative signs of the slopes of π_0^a and, when relevant, π_0^w even for $c < \Phi/\varphi$.

Figure 3 provides a graphical representation of Lemma 2. It highlights regions depending on φ and Φ exist in which a certain investment is optimal for landlords. Figure 3 is divided into four panels to account for all of the cases laid out in Lemma 2. Column A shows the profitable case $\left(\underline{\varphi}_0 < 0\right)$ and column B the unprofitable case whereas the rows differentiate whether the incumbent tenants' WTP binds the landlords' decision (row 1, $c_n^{rw} < c_n^{ra}$) or the absolute legal limit is binding (row 2, $c_n^{ra} < c_n^{rw}$).

[1] $H(z) \equiv \begin{cases} 1 \; if \; z > 0 \\ 0 \; if \; z \leq 0 \end{cases}$

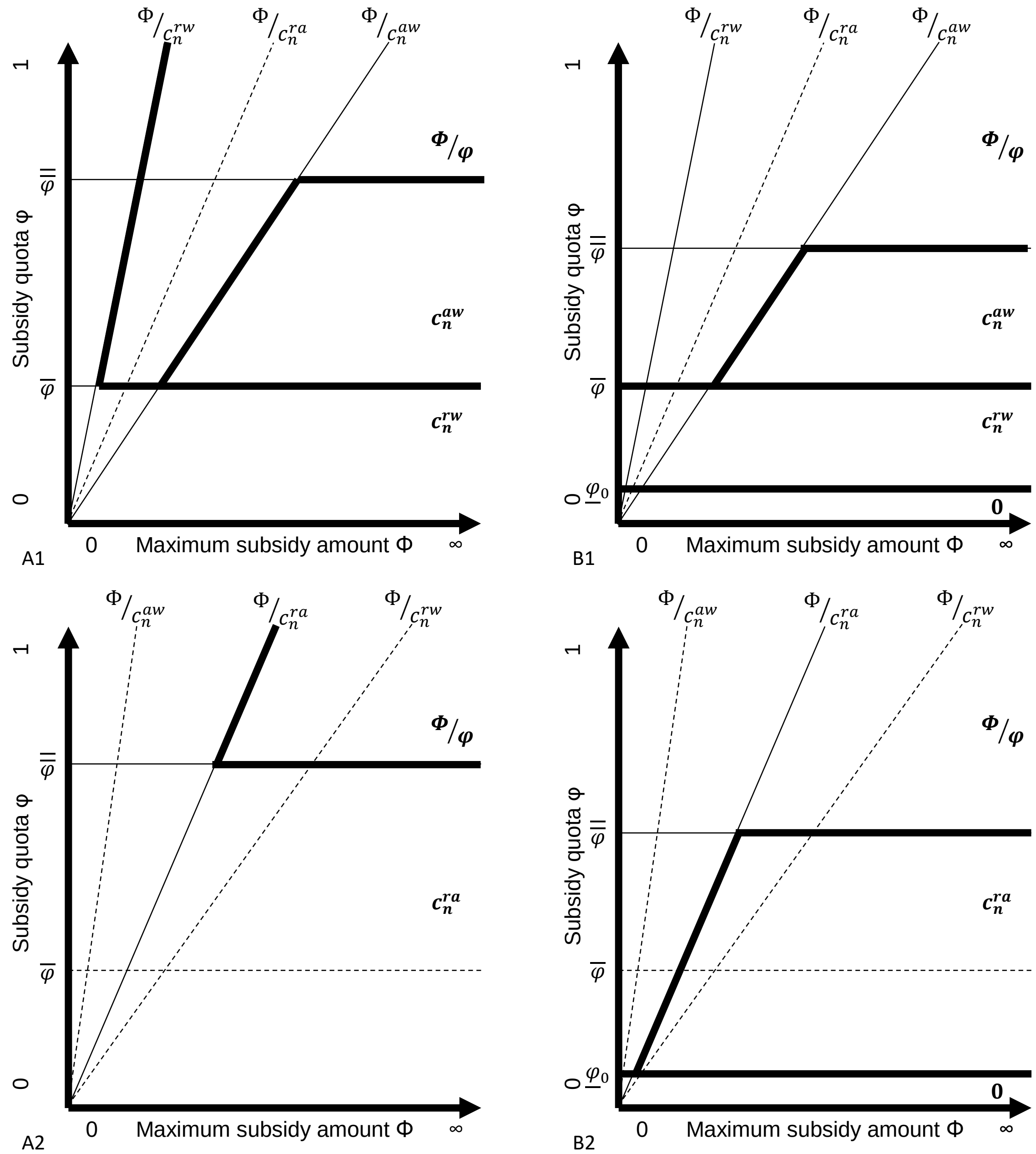

Figure 3: Landlords' optimal investment c_0^* depending on the available subsidies without the obligation to deduct subsidies from the modernization surcharge. Column A shows the profitable case with $\mu\, a + [1-\mu]\,\gamma - 1 \geq 0$. Column B shows the unprofitable case. Row 1 shows the respective subcase 1 where incumbent tenants WTP is binding with $c_n^{rw} = Q/(a-\gamma) < L/a = c_n^{ra}$. Row 2 shows the respective subcase 2 where the legal limit on the modernization surcharge is binding. Bold lines indicate binding borders, dashed lines are for orientation only. Hand-drawn figure qualitatively equivalent to the following parametrization: $a = 1.7$, $\gamma = 0.7$, $L = 380$, Column A: $\mu = 0.35$, Column B: $\mu = 0.2$, Row 1: $Q = 130$, Row 2: $Q = 380$.

From Lemma 2, we get an obvious

> Corollary 1: With non-deductible subsidies, the optimal modernization costs are larger than without subsidies ($c_0^* > c_n^*$), when one of the conditions in lines two or three of equation (4) are satisfied. Otherwise, optimal modernization costs are the same with and without subsidies ($c_0^* = c_n^*$).

3.3 Deductible subsidies

We now allow for deductibility of subsidies from the modernization costs that may be levied to the tenant ($\delta \in (0,1]$). Obviously, this leaves the profit function $\pi_s(c,s)$ unaffected when $\pi_s(c,s) =$

$\pi_n^a(c) + s$ or $\pi_s(c, s) = \pi_n^w(c) + s$. Only when modernization costs are small enough to make the relative legal bound to the modernization surcharge, that the landlord may levy on the tenant, binding, the deduction becomes relevant. In terms of Figure 1, increasing δ turns the first branch of the profit function clockwise around the origin and leaves the other branches unaffected as shown in Figure 4.

Obviously, deduction of subsidies from the basis of the modernization surcharge reduces profitability of subsidies. In the remainder of this section, we will first show, that this effect is never strong enough to induce the landlord to apply for less than the maximum subsidy she is eligible for, and then deduce the profit maximizing investment costs.

We define the landlord's optimal choice of the subsidy application as $s^*(c) \in [0, \min(\varphi\, c, \Phi)]$ for any given level of investment c and the optimal choice of the investment level c_δ^* as the level that maximizes the landlord's profits with optimal subsidies $c_\delta^* \equiv \arg\max_c \pi_s(c, s^*(c))$.

While for a given level of investment the profit-maximizing subsidy for landlords $s^*(c)$ may be less than what the government grants at most, they will never forgo any possible subsidies with the profit-maximizing investment:

> Lemma 3: For every strictly positive optimal investment, $c_\delta^* > 0$, landlords choose maximum subsidies, $s^*(c_\delta^*) = \min(\varphi\, c_\delta^*, \Phi)$.

The intuition is that with less than maximal subsidies, it is always possible to increase both subsidies and investment costs in a proportion that guarantees that the min-function in the profit function (2) does not decline and the rest of the profit function strictly increases, whence a combination on investment costs and less than maximal subsidies cannot be a profit maximum.

This contradicts Henger et al. (2021, pp. 7–10) who argue that deductible subsidies do not pose any additional incentive to increase the investment. We posit that their analysis is incomplete. They compare landlords' profits for a given modernization investment without and with deductible subsidies and find that landlords lose from deductible subsidies, prompting the conclusion that deductible subsidies are not attractive for landlords. However, as shown with Lemma 3, deductible subsidies increase landlords' profits when landlords appropriately extend their investment.

Due to Lemma 3, maximizing $\pi_\delta(c) \equiv \pi_\delta(c, \min(\varphi\, c, \Phi))$is equivalent to maximizing $\pi_\delta(c, s)$. Strictly positive levels of δ alter the slope of $\pi_\delta(c)$ only when the relative legal limit of the modernization surcharge is relevant. If subsidies have to be deducted we thus refer to landlords' profits as $\pi_\delta(c)$. As shown in Figure 4, $\pi_n^r + \varphi\, c$ gets tilted clockwise to $\pi_n^r + (1 - \mu\, a\, \delta)\varphi\, c$ and, respectively, $\pi_n^r + \Phi$ gets shifted down to $\pi_n^r + (1 - \mu\, a\, \delta)\Phi$. Note that this tilting and shifting may cause those lines to lie below π_n^r, however due to Lemma 3 the maximum profits are indeed larger with subsidies because the relevant kink in the profit function is shifted sufficiently far to the right. $\pi_n^a + \min(\varphi\, c, \Phi)$ and $\pi_n^w + \min(\varphi\, c, \Phi)$ are unaffected by the obligation to deduct the subsidies. As a consequence, we replace c_n^{ra} and c_n^{rw} by the following:

> Definition 3: $c_\delta^{ra} \equiv \min(c_\varphi^{ra}, c_\Phi^{ra})$ with $c_\varphi^{ra} \equiv \frac{L}{(1-\delta\varphi)a}$ and $c_\Phi^{ra} \equiv \frac{L}{a} + \delta\Phi$, and $c_\delta^{rw} \equiv \min(c_\varphi^{rw}, c_\Phi^{rw})$ with $c_\varphi^{rw} \equiv \Gamma\left(\frac{Q}{(1-\delta\varphi)\, a - \gamma}\right)$ and $c_\Phi^{rw} \equiv \Gamma\left(\frac{Q + a\delta\Phi}{a - \gamma}\right)$.

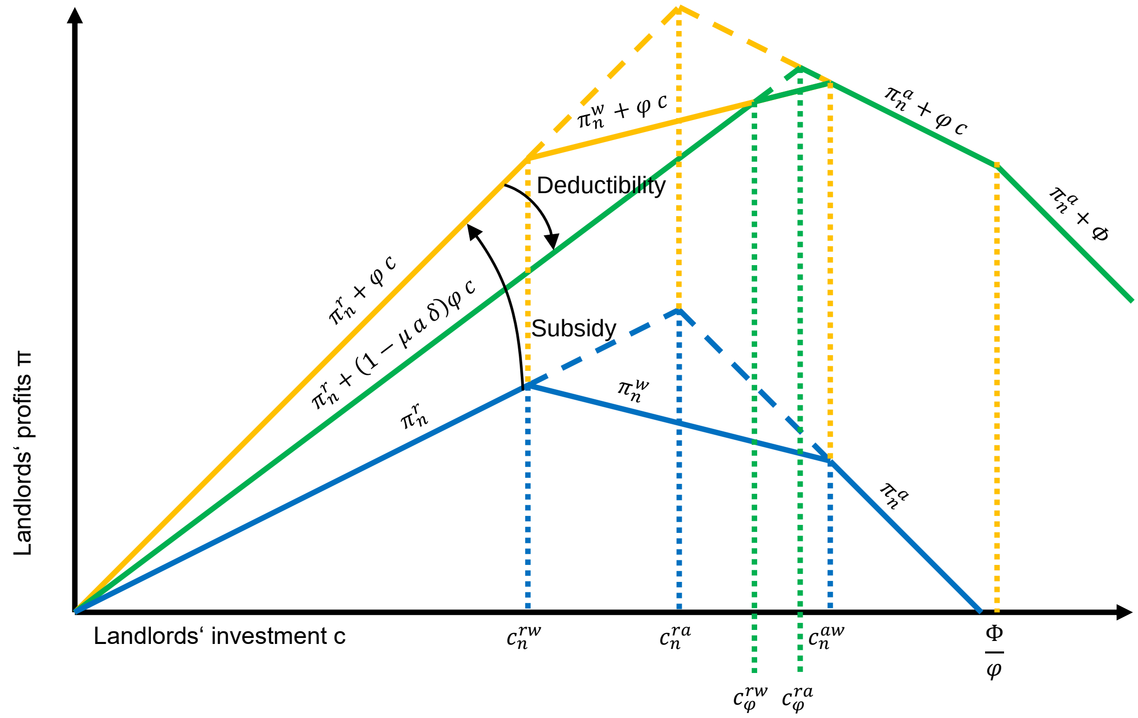

Figure 4: Landlords' profits $\pi_n(c)$ (without subsidies, in blue), $\pi_0(c)$ (with non-deductible subsidies, in orange) and $\pi_\delta(c)$ (with at least some deductibility of subsidies, in green) as a function of investment costs c. Hand-drawn figure qualitatively equivalent to the following parametrization: $a = 1.7$, $\gamma = 0.7$, $L = 380$, $Q = 130$, $\mu = 0.35$, $\varphi = 0.25$, $\Phi = 100$, $\delta = 1$. [Should be printed in color]

These values define the unique intersection of the function describing the relative legal limit of the modernization surcharge with the functions describing the absolute legal limit and the willingness to pay, respectively, depending on whether subsidies are limited by the subsidy quota φ or by the absolute maximum subsidy Φ. Before stating the Lemma, we need to extend the definition of the critical level of φ above which the slope of $\pi_n^r + (1 - \mu\, a\, \delta)\varphi\, c$ becomes positive:

Definition 4: $\underline{\varphi}_\delta \equiv \frac{1-\mu\, a-(1-\mu)\gamma}{1-\delta\, \mu\, a}$.

Note that for $\delta = 0$ this definition of $\underline{\varphi}_\delta$ is consistent with the earlier definition of $\underline{\varphi}_0$ and $c_\delta^{ra} = c_\varphi^{ra} = c_\Phi^{ra} = c_n^{ra}$ as well as $c_\delta^{rw} = c_\varphi^{rw} = c_\Phi^{rw} = c_n^{rw}$.

Lemma 4: With deductible subsidies, the optimal modernization costs is given by:

$$c_\delta^* = \begin{cases} 0 & \text{if } \varphi < \underline{\varphi}_\delta \\ \Phi/\varphi & \text{if } \begin{cases} \varphi \geq \max\left(\underline{\varphi}_\delta, \Phi/\min(c_\delta^{ra}, c_\delta^{rw})\right) + H\left(-\underline{\varphi}_\delta\right) \\ \text{or } \max\left(\underline{\varphi}_\delta, \bar{\varphi}, \Phi/c_n^{aw}\right) \leq \varphi < \Phi/c_\delta^{rw} \\ \text{or } \max\left(\underline{\varphi}_\delta, \bar{\bar{\varphi}}\right) \leq \varphi < \Phi/\min(c_\delta^{ra}, c_\delta^{rw}) \end{cases} \\ c_n^{aw} & \text{if } c_n^{ra} > c_n^{rw} > 0 \text{ and } \varphi \in \left[\max\left(\underline{\varphi}_\delta, \bar{\varphi}\right), \min(\bar{\bar{\varphi}}, \Phi/c_n^{aw})\right) \\ \min(c_\delta^{ra}, c_\delta^{rw}) & \text{otherwise} \end{cases} \qquad (5)$$

Lemma 4 is very similar to Lemma 2. We now first discuss how the introduction and increase of the deductibility factor δ affects the conditions for each line and then examine the effect of δ on the actual optimal modernization extent.

For the first line, deductibility increases $\underline{\varphi}_\delta$, which increases the range for φ in which no modernization is profitable. For the first case of the second line, deductibility increases $\underline{\varphi}_\delta$. However, c_δ^{ra} and c_δ^{rw}

increase in δ which means that $\Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ grows more slowly in Φ for greater values of δ. This implies that increased deductibility has an ambiguous effect on the likelihood of this case to occur. For the second case of line two, the lower bound of the range remains constant with respect to δ whereas the upper bound shrinks, therefore decreasing the case's likelihood. In the last case of the second line, the deductibility again decreases the upper bound, making it less likely that this case occurs. For the third line, the deductibility of subsidies has no effect. Deductibility therefore reduces the likelihood of the first line, has an ambiguous effect on the second and leaves the third unaffected.

Although being "hidden" inside the catch-all fourth line of Lemma 4, introducing and increasing the deductibility δ actually has its greatest effect on landlords' investment decision when it is determined by the intersection of π_δ^r with either π_δ^a or π_δ^w. Where non-deductible subsidies cause the optimal investment to remain at the amount that is optimal without subsidies, subsidies that have to be deducted actually increase the optimal investment from $\min(c_n^{ra}, c_n^{rw})$ to $\min(c_\delta^{ra}, c_\delta^{rw})$. If the rent increase for incumbent tenants is already capped by the absolute legal limit, deductible subsidies mean that for no additional costs for landlords or incumbent tenants, landlords can extend the investment which is profitable due to the increase in future tenants' rent payments. If limited by incumbent tenants' WTP, landlords can use deductible subsidies to extend the investment, only charging incumbent tenants with an additional rent increase according to their WTP and gaining additional profits via increased future rent levels. A positive δ curves both Φ/c_δ^{ra} and Φ/c_δ^{rw} downward so that besides the origin a new intersection emerges at $\tilde{\Phi} \equiv \frac{1}{\delta}\frac{a\,[L-Q]-\gamma\,L}{a\,\gamma}$ and $\tilde{\varphi} \equiv \frac{1}{\delta}\frac{a\,[L-Q]-\gamma\,L}{a\,[L-Q]}$. At $\tilde{\Phi}$ we find $c_\Phi^{ra} = c_\Phi^{rw}$ and at $\tilde{\varphi}$ we find $c_\varphi^{ra} = c_\varphi^{rw}$. Note that both are positive if and only if $c_n^{rw} = Q/(a-\gamma) < L/a = c_n^{ra}$ and that their absolute values decrease in δ. We thus find that if without subsidies only the legal limits on rent increases guide landlords' decisions, increasing δ only affects the optimal investment via the effect on c_δ^{ra} and on the conditions of the first and second line of Lemma 4. The case where $L < Q$ (equivalent to $c_n^{aw} < 0$, which results in $\tilde{\Phi} < 0 < \tilde{\varphi}$) implies that $c_\delta^{ra} < c_\delta^{rw}$ and the preceding argument applies as well. However, if tenants' WTP is binding without subsidies and deductibility is sufficiently large, i.e. δ exceeding $\tilde{\delta} \equiv 1 - \frac{\gamma\,L}{a\,[L-Q]}$ to ensure $\tilde{\varphi} \in (0,1)$, an increased deductibility enables c_δ^{ra} as an optimal investment. Intuitively, if subsidies are sufficiently large it can prove optimal for landlords to extend the investment to a point where incumbent tenants' WTP exceeds the absolute legal maximum and further increases in the investment are profitable.

Therefore, we derive our

> Corollary 2: An increase in the deductibility δ reduces optimal modernization costs to zero if and only if it re-establishes non-profitability. However, even with $\delta = 1$ only a strict subset of cases which are non-profitable without subsidies are non-profitable with deductible subsidies but would be profitable if subsidies were not to be deducted. Otherwise, increasing δ either increases optimal modernization costs or leaves them unaffected.

Figure 5 provides a graphical representation of Lemma 4. It highlights regions depending on φ and Φ exist in which a certain investment is optimal for landlords. Figure 5 is divided into two panels to account for all of the cases laid out in Lemma 4. Panel A shows the profitable case and panel B the unprofitable case. The arrows show the effect of introducing and increasing deductibility. Figure 5 is a transformation of the first row of Figure 3 with $\delta > \tilde{\delta}$ so that $\tilde{\varphi}$ and thus the transition from c_δ^{rw} to c_δ^{ra} can be seen. Lower levels of deductibility leave the first row of Figure 3 qualitatively unchanged. Likewise, the upper right hand corners ($\Phi > \tilde{\Phi}$ and $\varphi > \tilde{\varphi}$) of both panels in Figure 5 appear very similar to the second row of Figure 3.

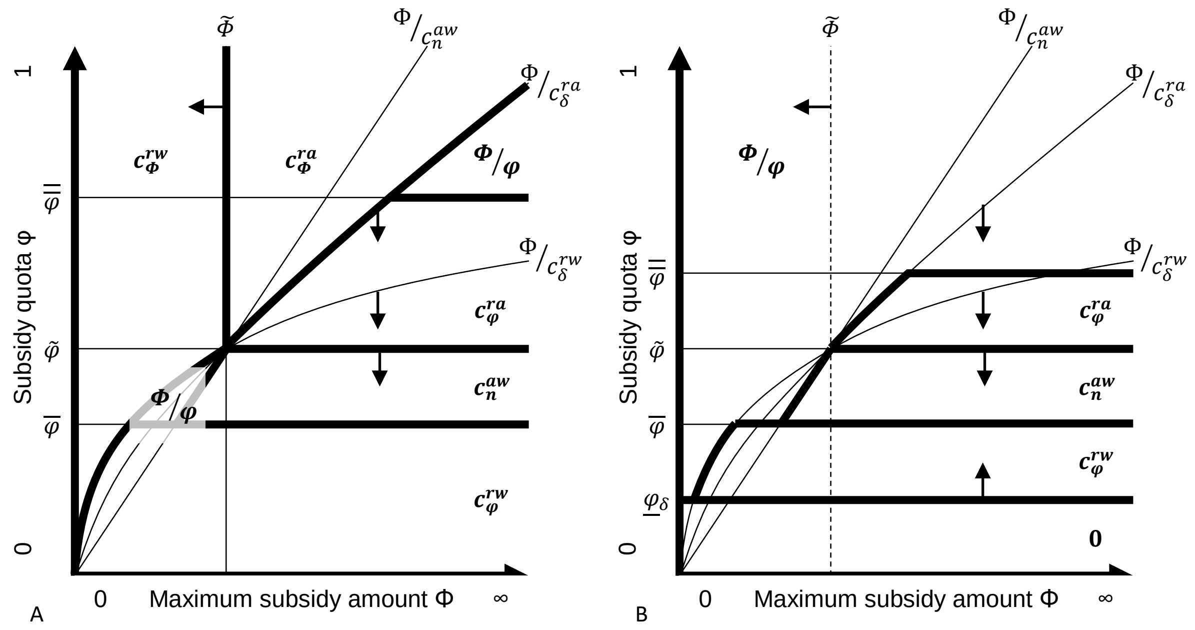

Figure 5: Landlords' optimal investment c_{δ}^{*} depending on the available subsidies with the obligation to deduct a strictly positive share δ of subsidies from the modernization surcharge. Arrows indicate the effect of increasing δ. Panel A shows the profitable case with $\mu\, a + [1-\mu]\,\gamma - 1 \geq 0$. Panel B shows the unprofitable case. Bold lines indicate binding borders, dashed lines are for orientation only. Hand-drawn figure qualitatively equivalent to the following parametrization: $a = 1.7$, $\gamma = 0.7$, $L = 380$, $Q = 130$, Panel A: $\mu = 0.35$, Panel B: $\mu = 0.2$.

From our discussion on landlords' optimal choice of modernization investment without and with subsidies, which may be non-deductible or have to be deducted from the modernization surcharge, we derive our

> Proposition 1: Introducing sufficiently large non-deductible subsidies increases modernization width, i.e. the share profitable modernizations. However, non-deductible subsidies only increase modernization depth, i.e. the optimal modernization extent of already profitable modernizations, if they are sufficiently large to make landlords choose their exact optimal investment solely based on the available subsidies instead of the modernization surcharge. Increasing the deductibility δ of subsidies increases the depth of already profitable modernization for some parameter constellations and never reduces it, but lowers the width of modernization, i.e. deductible subsidies decrease the range of exogenous variables where subsidies cause landlords to undertake otherwise non-profitable investments.

4 Welfare analysis

Now that we have shown that it is indeed profitable for landlords to apply for subsidies when investing in energy efficiency, even when obliged to deduct them from the modernization surcharge, we turn to the welfare effects of the subsidies. We first define our welfare measure and apply it to the case without subsidies. Relying on the previous result that subsidies always benefit modernizing landlords, we then analyze the welfare effects of subsidies on incumbent tenants. We investigate the level of subsidies that ensures the greatest increase in investment before we deduce optimal subsidization strategies for various political goals.

4.1 Measuring welfare

For simplicity we assume homogenous landlords and tenants for each apartment in the rental stock, enabling us to analyze the welfare effects for one representative agent each. Landlords' share in welfare following the investment is simply given by the attainable profits $\pi_x(c_x^*)$. As we assume that landlords only invest when profitable, we know that $\pi_x(c_x^*) \geq 0$. Tenants' welfare from an investment in energy efficiency is given by the difference between their WTP, reflecting their valuation of the investment, and their actual rent payments. By assumption, future tenants always pay their WTP, therefore they are indifferent to any investment. Incumbent tenants, however, might pay less than their WTP when tenancy law protects them from high rent increases. Incumbent tenants' welfare thus differs between the cases of no subsidy $\left(w_n^{it}(c_n^*) \equiv Q + \gamma\, c_n^* - \min(a\, c_n^*, L, Q + \gamma\, c_n^*)\right)$ and the case with subsidies $\left(w_\delta^{it}(c_\delta^*) \equiv Q + \gamma\, c_\delta^* - \min(a\, [c_\delta^* - \delta \min(\varphi\, c_\delta^*, \Phi)], L, Q + \gamma\, c_\delta^*)\ \right)$. As a final party to consider in the welfare analysis we find the government (which we take as representative for the rest of society for simplicity) paying for the subsidies. In return, the government accrues a positive externality from increased energy efficiency in terms of savings in climate change damages. We therefore propose $w_n^{gov}(c_n^*) \equiv EE(c_n^*)$ and $w_\delta^{gov}(c_\delta^*) \equiv EE(c_\delta^*) - \min(\varphi\, c_\delta^*, \Phi)$, where $EE(c)$ is a positive externality gained from increased energy efficiency representing decreasing climate change damages with $\partial EE(c)/\partial c > 0$ and $\partial^2 EE(c)/\partial c^2 < 0$.

Summing up the respective agents' individual welfare measures we obtain the following social welfare functions: $W_x(c_x^*) \equiv EE(c_x^*) + [\gamma - 1]\, c_x^*$. Note that subsidies and rent payments are only transfers between the parties and thus do not appear in the social welfare function, although they obviously have significant effects on the distribution of welfare between these parties. Further note that without any consideration of the externality, due to $\gamma < 1$, i.e. the assumptions that tenants value an investment in energy efficiency below its cost, no investment in energy efficiency creates any welfare between landlords and tenants and landlords are only incentivized to invest by extracting incumbent tenants' quasi-rent. Without consideration of the externality, investing in energy efficiency could thus be considered inefficient in terms of landlords' and tenants' welfare. Only with sufficient internalization of the positive externality, i.e. with $s_\delta^*(c) > [1 - \gamma]\, c$, landlords and tenants find a welfare surplus from investing in energy efficiency that is allocated via the rent payments.

For the time being, we assume in accordance with much of the literature (e.g. dena, 2019, 2016; März, 2018; Stede et al., 2020) that modernization of residential apartment buildings currently faces severe underinvestment with regard to the externality or, equivalently, that the positive externality by far outweighs the undervaluation of modernization costs, i.e. $EE(c) \gg [1 - \gamma]\, c$ and increasing the modernization activity thus improves social welfare.

4.2 The effect of subsidies on landlords' welfare

To analyze how the subsidies affect landlords' welfare, we need to compare their profits without subsidies $\pi_n(c_n^*)$ to their profits with non-deductible subsidies $\pi_s(c_s^*)$. We thus define $\Delta\pi \equiv \pi_s(c_s^*) - \pi_n(c_n^*)$. We know from our previous discussion that $\Delta\pi \geq 0$. More specifically, landlords accrue additional profit whenever they choose to invest something in the presence of subsidies and their difference in profits is only zero when they choose not to invest at all.

4.3 The effect of subsidies on incumbent tenants' welfare

Incumbent tenants' welfare difference is given by $\Delta w^{it} \equiv w_s^{it}(c_s^*) - w_n^{it}(c_n^*)$ which in detail can be expressed by the following equation:

$$\Delta w^{it} = \underbrace{\underbrace{Q + \gamma\, c_\delta^*}_{WTP} - \underbrace{\min(a\, [c_\delta^* - \delta\ \min(\varphi\, c_\delta^*, \Phi)], L, Q + \gamma\, c_\delta^*)}_{\text{rent increase}}}_{w_\delta^{it}(c_\delta^*)} - \underbrace{\left[\underbrace{Q + \gamma\, c_n^*}_{WTP} - \underbrace{\min(a\, c_n^*, L, Q + \gamma\, c_n^*)}_{rent\ increase}\right]}_{w_n^{it}(c_n^*)} \qquad (6)$$

As can be easily seen, both $w_\delta^{it}(c_\delta^*)$ and $w_n^{it}(c_n^*)$ are unambiguously non-negative, i.e. whether with or without a subsidy, any modernization decision individually leaves tenants at worst with rent payments exactly matching their WTP. However, the difference between the case without and with subsidies is not as immediately obvious.

In the profitable case, incumbent tenants gain welfare whenever (a) subsidies cause landlords to expand their investment and (b) then tenants' WTP exceeds the rent increase. Condition (b) is satisfied in two cases: (1) if landlord's maximum permissible profits, $\pi_\delta(c_\delta^{ra})$, are less than what would be achievable from extracting the tenant's WTP with the same amount of investment, $\pi_\delta^w(c_\delta^{ra})$, and (2) if the subsidy quota is, on the one hand, large enough to reduce marginal investment costs enough to induce the landlord to expand his investment to the amount at which the absolute maximum subsidy becomes binding ($\partial\pi_\delta(c)/\partial c > 0$ for all $c < \Phi/\varphi$) but, on the other hand, not so large that the absolute maximum subsidy becomes binding when landlord's profits are still bound by the tenant's WTP ($\Phi/\varphi > \max(c_n^{aw}, c_\delta^{rw})$). Condition (a) translates into the subsidy quota being small enough to allow the profit function being tilted and not just shifted at the original profit-maximizing investment level ($\min(c_n^{ra}, c_n^{rw}) < \Phi/\varphi$) but — if subsidies are non-deductible ($\delta = 0$)[2] — large enough to turn the slope of the profit function positive on both sides of the original profit-maximizing investment level ($\varphi > \bar{\varphi}$ for $c_n^* = c_n^{rw}$ and $\varphi > \bar{\bar{\varphi}}$ for $c_n^* = c_n^{ra}$).

In the unprofitable case without subsidies we find $c_n^* = 0$ and thus $w_n^{it}(0) = Q$: incumbent tenants retain their entire quasi-rent. Then tenants obviously experience no change in welfare if no modernization is performed even with subsidies, i.e. if $\varphi < \underline{\varphi}_\delta$. But if the subsidy incentivizes landlords to invest, incumbent tenants will gain welfare only in two cases that resemble those of the profitable case but are more restrictive: (1) if landlord's maximum permissible profits, $\pi_\delta(c_\delta^{ra})$, exceed the profits achievable from extracting the tenant's WTP with the same amount of investment, $\pi_\delta^w(c_\delta^{ra})$, *by more than the quasi-rent* and (2) if the subsidy quota is large enough to induce the investment at which the absolute maximum subsidy becomes binding ($\partial\pi_\delta(c)/\partial c \geq 0$ for all $c < \Phi/\varphi$). In the latter case, the subsidy quota also has to be small enough to leave a difference *of more than the quasi-rent*, Q, between the WTP and the rent increase at the investment level that makes the absolute maximum subsidy binding, $(Q + \gamma\,\Phi/\varphi) - L$.

We summarize and further specify these results in our:

> Lemma 5: Incumbent tenants' change in welfare due to subsidies depends on whether an investment is already profitable without subsidies and on the size of the government funding according to Table 2.

[2] Due to $c_\delta^{ra} > c_0^{ra}$ and $c_\delta^{rw} > c_0^{rw}$ for all $\delta > 0$, no corresponding condition is needed for deductible subsidies.

Table 2: Incumbent tenants' change in welfare due to subsidization.

	Profitable case	Unprofitable case
	$\mu\, a + [1-\mu]\,\gamma - 1 \geq 0$	$\mu\, a + [1-\mu]\,\gamma - 1 < 0$
$\Delta w^{it} > 0$	$\varphi \in [\bar{\bar{\varphi}}, \Phi / \min(c_n^{ra}, c_n^{rw})) \vee (\varphi > \tilde{\varphi} \wedge \Phi > \tilde{\Phi} \wedge \delta > 0)$	$\varphi > \max\left((a-\gamma)/(a\,\delta), \underline{\varphi_\delta}\right) \vee \varphi \in \left[\max\left(\underline{\varphi_\delta}, \bar{\bar{\varphi}}\right), \Phi\,\gamma/L\right)$
$\Delta w^{it} = 0$	Otherwise, i.e. $(\varphi < \bar{\bar{\varphi}} \vee \varphi \geq \Phi / \min(c_n^{ra}, c_n^{rw})) \wedge (\varphi \leq \tilde{\varphi} \vee \Phi \leq \tilde{\Phi} \vee \delta = 0)$	$\varphi < \underline{\varphi_\delta} \vee \varphi = (a-\gamma)/(a\,\delta) \in \left(\underline{\varphi_\delta}, \bar{\bar{\varphi}}\right) \cup \left[\max\left(\bar{\bar{\varphi}}, \Phi\,\gamma/L, \underline{\varphi_\delta}\right), 1\right] \vee \varphi = \Phi\,\gamma/L \in \left[\max\left(\underline{\varphi_\delta}, \bar{\bar{\varphi}}\right), (a-\gamma)/(a\,\delta)\right]$
$\Delta w^{it} < 0$	Impossible	Otherwise, i.e. $\varphi \in \left[\underline{\varphi_\delta}, \min((a-\gamma)/(a\,\delta), \bar{\bar{\varphi}})\right) \vee \varphi \in \left(\max\left(\bar{\bar{\varphi}}, \Phi\,\gamma/L, \underline{\varphi_\delta}\right), (a-\gamma)/(a\,\delta)\right) \vee \varphi = \bar{\bar{\varphi}} \in \left(\max\left(\Phi\,\gamma/L, \underline{\varphi_\delta}\right), (a-\gamma)/(a\,\delta)\right)$

Incumbent tenants always gain welfare when the subsidy quota is sufficiently but not too large, that is when $\varphi \in [\bar{\bar{\varphi}}, \min(\Phi/\min(c_n^{ra}, c_n^{rw}), \Phi\,\gamma/L)]$. We can furthermore conclude that, unsurprisingly, increasing the share of the subsidy δ that has to be deducted from the modernization surcharge makes it more likely that tenants' welfare grows. Increasing δ decreases $\tilde{\varphi}$ and $(a-\gamma)/(a\,\delta)$, which are both lower bounds for a subsidy that improves tenants' welfare. It furthermore increases $\underline{\varphi_\delta}$ which is a lower bound for a subsidy that harms tenants.

4.4 The effect of subsidies on the optimal investment

As argued above, society as a whole gains welfare from increased investments due to the positive externality in addition to tenants' valuation of energy efficiency. Therefore, any increase in the modernization extent increases social welfare. Thus, we now identify which subsidy offers the greatest increases in the investment. We will first discuss how the subsidies affect the investment within each line of Lemma 2 and Lemma 4, respectively, and then whether and how changing the subsidy can cause qualitative changes in the optimal investment.

When the optimal investment is 0, increasing the subsidy obviously has no marginal effect. When the optimal investment is Φ/φ, increasing the maximum subsidy amount Φ increases the investment. Because $\varphi \leq 1$, the marginal increase in the investment from increasing Φ is greater than unity and thus the subsidy is leveraged. However, increasing the subsidy quota φ actually decreases the optimal investment if it is given by Φ/φ. If the optimal investment is given by c_n^{aw}, neither Φ nor φ affect its size. Lastly, the effect of the subsidies on the investment in the last line of the lemmata depends on whether or not they have to be deducted. Non-deductible subsidies have no effect on c_n^{ra} or c_n^{rw}. However, increasing δ causes φ or Φ, whichever is limiting, to increase c_δ^{ra} and c_δ^{rw}.

Looking once more at Figure 4 we can quickly identify that discontinuous increases in the optimal investment may occur at three critical subsidy quotas: at $\underline{\varphi_\delta}$ where the slope of π_δ^r, if negative, turns positive; at $\bar{\varphi}$ where, if applicable, the slope of π_δ^w turns positive and lastly at $\bar{\bar{\varphi}}$ where even the slope of π_δ^a turns positive. Assuming that if indifferent between two investments landlords choose the larger, the government can leverage the subsidy quota best if setting it to one of these critical values. Combined with the insight that increasing φ above $\bar{\bar{\varphi}}$ marginally decreases the optimal investment, we can infer our

> Lemma 6: Setting the available maximum subsidy Φ above $\bar{\bar{\varphi}} * \min(c_s^{ra}, c_s^{rw}, c_n^{aw})$ and the subsidy quota φ at $\bar{\bar{\varphi}}$ ensures that landlords always invest $\Phi/\bar{\bar{\varphi}} = \frac{\Phi}{1-[1-\mu]\,\gamma}$ – which is the greatest investment for a given value of Φ that can be achieved by choosing φ.

4.5 Optimal subsidization levels

In this section we analyze which subsidy levels could be considered optimal under different measures of optimality. Determining optimal subsidy levels depends on the goal of the policy intervention.

Suppose the goal is to increase modernization width, i.e. to ensure that the policy intervention triggers investments that would not be made without subsidies. Then the subsidy needs to exceed $\underline{\varphi_\delta}$ according to Lemma 2 and Lemma 4. Note that this goal is easier to achieve if subsidies are not deducted from the modernization surcharge.

Now suppose that in line with the general intentions of rent control the goal is to ensure that subsidies never cause loss of welfare to incumbent tenants. Then subsidies need to be sufficiently large, i.e. $\varphi \in [(a-\gamma)/(a\,\delta), 1] \cup [\bar{\bar{\varphi}}, \min(\Phi/\min(c_n^{ra}, c_n^{rw}), \Phi\,\gamma/L)]$ according to Lemma 5. Note that this goal is easier to achieve, if subsidies are fully deducted from the modernization surcharge.

Alternatively, suppose the goal is to achieve a greater modernization depth, i.e. to incentivize substantial extensions of already undertaken modernizations. According to Lemma 6, this can be achieved by setting $\varphi = \bar{\bar{\varphi}} \le \Phi/\min(c_s^{ra}, c_s^{rw}, c_n^{aw})$. Note that this goal is slightly easier to achieve if subsidies are not to be deducted: increasing δ increases c_s^{ra} and c_s^{rw}, weakly decreasing $\Phi/\min(c_s^{ra}, c_s^{rw}, c_n^{aw})$ which is the upper bound for the goal fulfilling subsidy quota.

We can conclude that the policy goals are not mutually exclusive.

> Proposition 2: Setting the available maximum subsidy Φ above $\frac{\bar{\bar{\varphi}}}{\min(1/\min(c_s^{ra}, c_s^{rw}, c_n^{aw}), \gamma/L)}$ and the subsidy quota at $\bar{\bar{\varphi}}$ ensures that landlords always invest $\frac{\Phi}{\bar{\bar{\varphi}}} = \frac{\Phi}{1-[1-\mu]\,\gamma}$ – which is the largest investment for a given value of Φ that can be achieved by choosing φ – and that both landlords and incumbent tenants experience an increase in welfare from the policy intervention.

Note that the question of deductibility finds no direct mention in Proposition 2. That is because in the case described therein, landlords' optimal modernization is based on maximizing the subsidy income, regardless of the rental income from incumbent tenants, which is capped by the absolute legal limit. In a sense, sufficiently large subsidies transcend the boundaries set by the modernization surcharge and the specific question of deductibility.

Our fundamental assumption that landlords currently severely underinvest in energy efficiency compared to the social optimum may imply that it is indeed socially optimal to choose an appropriately large Φ to remain within the bounds of Proposition 2. If this is the case, the welfare-maximizing subsidy has interesting properties:

> Corollary 3: If $EE'(\Phi/\bar{\bar{\varphi}}) > 1-\gamma$ for $\Phi/\bar{\bar{\varphi}} \ge \frac{1}{\min\left(1/\min(c_s^{ra}, c_s^{rw}, c_n^{aw}), \frac{\gamma}{L}\right)}$ then the optimal policy on subsidies for energetic modernization restricts subsidies to the finite maximum amount solving $EE'(\Phi/\bar{\bar{\varphi}}) = 1-\gamma$ and to a share $\bar{\bar{\varphi}}$ of the investment costs. This optimal policy implies that neither landlords nor tenants lose compared to any lower levels of the subsidy parameters φ and Φ. Whether the rest of society (which we called "the government") gains or loses from such subsidies depends on the size of the positive externality, but Social Welfare amongst landlords, tenants, and the rest of society is maximized.

5 Localization to the German case

In this section we show that the core results from the general model hold when adding a layer of complexity to more realistically model German tenancy law, where additional rent revenue after a modernization not only depends on landlords' investment but also on the development of the local rental market.

5.1 Landlords' profits

In Germany, only two situations allow landlords to increase the rent without the consent of incumbent tenants who have to either pay up or terminate the tenure: increasing the monthly rent up to the (generally speaking increasing over time) reference rent customary in the locality (Sec. 558 of the German Civil Code – *Bürgerliches Gesetzbuch, BGB*) and landlords' investment in modernization of the dwelling (Sec. 559 BGB). If the rent is already equal to the reference rent, which we assume with landlords' rationality, landlords may further increase the rent only if the reference rent increases. While strictly speaking a geometric growth of the reference rent must prevail, we simplify the argument substantially by assuming a linear increase by an amount of β every year.[3]

According to Sec. 559 BGB, after a modernization of the apartment landlords may increase the annual rent by 8 per cent of the modernization costs. If the initial monthly rent per square meter is less than 7 EUR, the monthly rent may not be increased by more than 2 EUR, hence the annual rent may not be increased by more than 24 EUR. For a higher initial rent the legal limit is an increase of 3 EUR per month and square meter (36 EUR annually). Once the rent has been increased due to a modernization, it must remain constant until the reference rent catches up and landlords may again increase the rent following the usual development of the reference rent or by means of another modernization.

Formally, the law allows the yearly rent to increase to a level of

$$\max\left(\underbrace{\min(r+\alpha\, c, r+\Lambda)}_{\substack{modernization\\ surcharge}},\ \underbrace{r+\beta t}_{\substack{reference\\ rent}}\right) \tag{7}$$

where r is the annual rent before modernization, α is the share of modernization costs by which the yearly rent may increase (i.e. 8 per cent), Λ is the upper legal limit of such rent increases (24 EUR or 36 EUR, depending on the initial rent), β is the linear increase of the reference rent and t is the time elapsed since modernization. Note that for this localized model, we find $\alpha \in (0,1)$. Figure 6 shows the case of rent after a modernization compared to the rent following the local reference rent.

[3] In fact, looking at the first two decades of the century, linear growth of the reference rent seems to be even closer to empirical observations than geometric growth (Destatis, 2018, pp. 172–177).

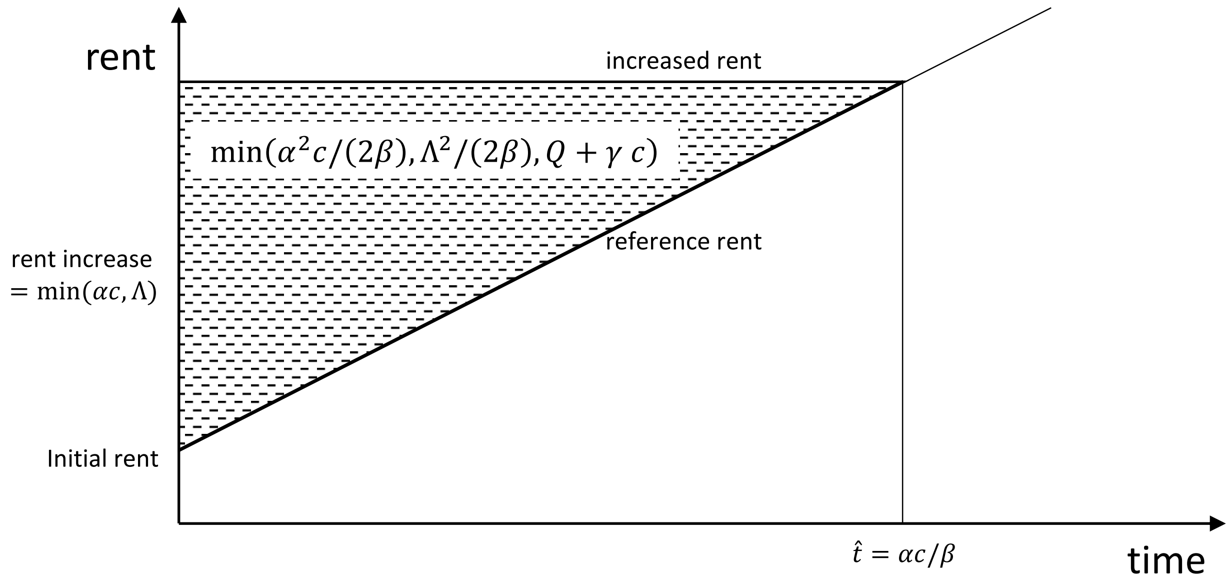

Figure 6: Revenue gained from rent increase after a modernization without subsidies considering the intricate German tenancy law.[4]

As Figure 6 illustrates, the legal modernization surcharge affects the legally permissible additional revenue twice. It defines both the height of the revenue triangle and is proportional to the length of the time until the reference rent catches up with the initial rent augmented by the modernization surcharge ($\hat{t} = \frac{\alpha\, c}{\beta}$, width of the revenue triangle).[5] Thus landlords' profits change to $\pi_n^{GER}(c)$:

$$\pi_n^{GER}(c) \equiv \underbrace{\mu \min(\alpha^2 c/(2\beta), \Lambda^2/(2\beta), Q + \gamma\, c)}_{\text{incumbent tenants}} + \underbrace{[1-\mu]\,\gamma\, c}_{\text{future tenants}} - \underbrace{c}_{\text{costs}} \tag{8}$$

Note that in the German localization we have $a = a^{GER}(c) \equiv \alpha^2 c/(2\beta)$ and $L = L^{GER} \equiv \Lambda^2/(2\beta)$ but otherwise landlords' profits are unchanged compared to the base model.

In this section we focus on the localization of the model. Therefore, we first explore deductible subsidies that more closely fit with the current German regulation, before exploring a potential change in regulation towards non-deductible subsidies. Lemma 3 still holds, which posits that when investing optimally, landlords will apply for the full subsidy, providing the following profit function:

$$\pi_\delta^{GER}(c) \equiv \underbrace{\mu \min(\alpha^2 [c - \delta \min(\varphi\, c, \Phi)]^2/(2\beta), \Lambda^2/(2\beta), Q + \gamma\, c)}_{\text{incumbent tenants}} + \underbrace{[1-\mu]\,\gamma\, c}_{\text{future tenants}} - \underbrace{c}_{\text{costs}} + \underbrace{\min(\varphi\, c, \Phi)}_{\text{subsidies}} \tag{9}$$

5.2 Optimal investments

Looking at landlords' marginal profits to determine their optimal investment reveals a significant difference to the base model: for low investments, where the rent increase is legally capped to be proportional to landlords' costs, marginal profits are negative at $c = 0$ but increase with increasing investment due to the quadratic nature of the additional rent revenue. This means that $c_n^{raGER} \equiv \Lambda/\alpha$

[4] Modernizations may promote the apartment to a higher tier of the reference rent. In that case landlords gain additional revenue that is the difference between the new and the old tier of the reference rent. This additional revenue accrues indefinitely but gains a finite discounted present value given a positive time preference. It thus increases the value of the modernization. For our forthcoming calculations we disregard this case without loss of generality.

[5] The quadratic nature of revenue gained from rent increases following a modernization has been shown similarly by Rehkugler et al. (2014, p. 38).

and $c_n^{rwGER} \equiv \frac{\beta\gamma+\sqrt{2\alpha^2\beta Q+\beta^2\gamma^2}}{\alpha^2}$ are the critical values at which the min-function changes its argument[6], and the smaller of the two maximizes landlords' profits not only locally but also globally if the legal limits and tenants' quasi-rent are sufficiently large (i.e. $\Lambda > \frac{\beta[1-[1-\mu]\gamma]}{\mu\alpha}$ or $Q > \frac{\beta[[1-\gamma]^2-\mu^2\gamma^2]}{2\mu^2\alpha^2}$, respectively). Similarly to the general case, we can define c_φ^{raGER}, c_Φ^{raGER}, c_φ^{rwGER}, and c_Φ^{rwGER} as the respective solutions for c where the min-function in landlords' profits changes its argument in the presence of subsidies. While their exact definitions are drastically different from, and more complicated than, in the general model, the general behavior as presented in Section 3 remains the same, that is to say that there exist two concave functions Φ/c_δ^{raGER} where $c_\varphi^{raGER} = c_\Phi^{raGER}$ and Φ/c_δ^{rwGER} where $c_\varphi^{rwGER} = c_\Phi^{rwGER}$ that delineate the respective regions in the Φ-φ-plane and intersect at $\tilde{\Phi}^{GER}$ with $\tilde{\Phi}^{GER}/c_\delta^{raGER} = \tilde{\Phi}^{GER}/c_\delta^{rwGER} = \tilde{\varphi}^{GER}$. Similar is also how optimal investment is determined for sufficiently large values of Φ and subsidy quotas above $\bar{\varphi}$ and, more importantly, $\bar{\bar{\varphi}}$, where landlords are incentivized to invest as much as to maximize the subsidization. This makes sense intuitively: when the investment is large enough that tenants always pay only the maximum rent increase, it does not matter substantially how exactly that rent increase is being calculated.

The similarities to the general model are slightly weaker for the *unprofitable case* where no investment is profitable without subsidies. Since the marginal profits are not constant over the respective intervals determined by the min-function, no constant profitability threshold can be determined that the subsidy quota needs to overcome. Instead, the subsidies must be sufficiently large to assure that the local maxima yield positive profits. This implies different minimum subsidy quotas as well as minimum values of Φ, depending on whether incumbent tenants' WTP or the absolute legal limit on rent increases is binding. Due to the nonlinear marginal profits, landlords invest at c_Φ^{raGER} and c_Φ^{rwGER} if the respective minimum subsidy quota is exceeded and if the maximum subsidy is sufficiently large, instead of investing at Φ/φ as in the general model. However, for the case where no investment is profitable without subsidies, the general results hold for sufficiently large subsidies by the same argument as outlined above.

5.3 Welfare effects

Similar to the optimal investments, the main results about the welfare effects of subsidies hold in the localized model as well, although the exact details differ. Landlords always profit from subsidies if they invest in their presence. Future tenants, by assumption, remain welfare neutral. Incumbent tenants' welfare difference needs again to be differentiated by whether there is some investment without subsidies or not. In the *profitable case*, they are at least welfare neutral and gain welfare if their rent increase is bound by the absolute legal limit. In the *unprofitable case* they only gain welfare if the subsidies are sufficiently large to induce landlords to invest enough that the rent increase equals the absolute limit and that the difference between that rent increase and tenants' WTP exceeds their quasi-rent. Social Welfare is increased if the optimal modernization extent increases thanks to the subsidies. In the localized model we find a very similar sufficient condition for subsidies benefitting both landlords and tenants as before, that is that the subsidy quota must exceed $\bar{\bar{\varphi}}$ and the maximum subsidy amount must be sufficiently large, with the only difference that this also depends on the development of the reference rent customary in the locality. The reason for this similarity is again that if the rent increase for incumbent tenants is constant at large investments thanks to the legal limit, it is irrelevant how this constant value is reached at lower investments.

[6] c_n^{awGER} where the min function changes between the second and the third argument is irrelevant since marginal profits are negative in both cases.

5.4 Effects of introducing non-deductibility of subsidies to Germany

If, in this localized version of the model, subsidies remained fully with landlords, the effects explored in the general model would remain. If subsidies do not affect the rent increase for incumbent tenants for any given investment, landlords again lose the incentive to marginally deepen their already profitable investment in the presence of subsidies. However, if no investment is profitable without subsidies, they need to be lower to trigger some investment if they directly benefitted landlords. And the same holds as for the previous sections: if the subsidies are sufficiently large that incumbent tenants' rent increase is constant anyway, it is irrelevant whether they would have to be deducted from the modernization surcharge or not.

6 Discussion of limitations

Our results, as any results derived from a theoretical model, must be taken with a few grains of salt and require further research. To attain clear results, several simplifying assumptions had to be made.

Most notable is the assumption of a constant but insufficient willingness to pay of tenants for landlords' investment costs, when it is generally only the investment's outcome, i.e. energy performance, that tenants can observe. Empirical studies ranging from controlled experiments (e.g. Banfi et al., 2008; Carroll et al., 2016; Collins and Curtis, 2018) to hedonic pricing analyses (e.g. Hahn et al., 2018; Hyland et al., 2013; Kholodilin et al., 2017; Taruttis and Weber, 2022) have reliably shown that tenants are indeed willing to pay a rent premium for energy efficient dwellings. Collins and Curtis (2018) have shown that the marginal WTP per unit energy saved is decreasing. Coupled with the standard assumption that the marginal energy savings of an investment are also decreasing we can safely conclude that tenants' marginal WTP for landlords' investment would be more realistically modelled by some positive but decreasing function. This would mainly change our results for the case that the subsidies are sufficiently large to overcome the modernization surcharge where another optimal investment arises from where the marginal WTP plus the subsidy quota no longer exceed the marginal profits even if the absolute maximum subsidy is not yet binding. However, we posit that this would not change the results (1) that deductible subsidies offer an incentive to extend already profitable modernizations, (2) that very large subsidies can trigger investments going beyond the confines of the modernization surcharge, and (3) that sufficiently large subsidies can improve both landlords' and tenants' welfare. A more detailed model incorporating the decreasing marginal WTP for energy efficiency coupled with decreasing marginal energy savings per investment was beyond the scope of this paper but could be tackled in future theoretical analyses building on our research.

Another clearly unrealistic assumption is that the social benefits outweigh the costs of any investment, rendering any increase in the investment socially desirable. This obviously cannot be true considering the decreasing marginal energy effects for greater modernizations. However, there is a broad consensus that at least in Germany, the energetic modernization rate is insufficient to meet the climate protection target of an emission-free building sector by 2045 (e.g. dena, 2019, 2016; German Environment Agency, 2022; März, 2018; Stede et al., 2020). This refers both to the depth and to the width of performed renovations. Generally speaking, German building owners invest far too little. This leads us to the simplifying assumption that the socially optimal investment lies sufficiently far beyond the investment values incentivized by the current modernization surcharge. To determine the socially optimal investment and accordingly the optimal subsidization theoretic modelling is not enough. Rather, exact technological knowledge on the effectiveness of modernizations and reliable estimates of the positive externality created by energy savings are required.

As the goal of this research was to qualitatively analyze the direction in which subsidies interacting with the mechanism of the modernization surcharge affect landlords' behavior, we focused on modelling that mechanism and disregarded the heterogeneity of agents. This mainly applies to the available professional knowledge and the intent to maximize profits that differs between professional real estate companies and small private landlords (Renz and Hacke, 2016). However, besides some communicative effect ("the government promotes one action so it is likely desirable."), subsidization is an inherently financial policy intervention that directly aims at the recipients' bottom line. It is therefore reasonable to closely scrutinize subsidies for that financial effect.

It is for a similar reason that we glossed over the topic of time preferences and discounting. While times preferences certainly differ among building owners (small private landlords, often in the latter half of their lives, tend to shy away from costly investments; Cischinsky et al., 2015), discounting constant returns as in the general model over a finite time period of about 20 years (a good estimate for an average investment cycle in the building sector; Collins and Curtis, 2018) comes down to multiplying the annual rent increase by a constant factor without infringing on the mechanistic interaction of subsidies and the modernization surcharge. We therefore decided to implicitly assume that the modeled landlords consider revenues discounted with their proper individual discount rate. For the localized model that considers the intricacies of German tenancy law, landlords' returns are more time sensitive due to the interaction with the development of the reference rent customary in the locality. Taking this into account surely changes the definitions of the critical values calculated in the model, but the general results remain the same, that is: if subsidies must be deducted from the modernization surcharge they incentivize increasing already profitable investments and if they stay with landlords they reward investing at all.

The last issue potentially limiting the explanatory power of our model and that we are aware of, are the transaction costs that accompany subsidies. In Germany, to qualify for subsidies the intended modernization must abide by certain standards. While this assures the quality of the publicly funded projects, it inflicts an opportunity cost on the building owner who is less free in the exact design of the renovation. Furthermore, subsidies require extensive knowledge on the application process regarding the timing and the content of the necessary forms. These transaction costs are often cited as a major barrier to applying for the subsidies (Renz and Hacke, 2016). Adding these transaction costs into the model, even if only as a constant cost term that applies if subsidies are used, would render Lemma 3 less absolute, implying that it is now only profitable to apply for subsidies if the additional revenue exceeds the subsidy transaction costs. This in turn weakens the core result that if subsidies must be deducted they always lead to an extension of already profitable modernizations. This would tip the scales more towards a modernization surcharge without the obligation to deduct subsidies as being more socially desirable due to its higher capacity to trigger otherwise unprofitable investments. The transaction costs extension would not change the result that the modernization surcharge as a whole is inefficient and subsidies best influence landlords' behavior if sufficiently large to disregard the details of the modernization surcharge.

7 Conclusions

Causing ca. 11 % of Germany's greenhouse gas emissions with only minor reductions for the last decade (German Environment Agency, 2022), heat production for residential buildings is an obvious target for climate protection policies. Reducing those emissions can in principle be achieved via changes in energy-consumption behavior by the occupant, or via modernizations by the owner. In the rental sector, where the two diverge, the landlord-tenant-dilemma is a severe barrier to such investment (Jaffe and Stavins, 1994). It is mostly the existing building stock with its long investment cycles that severely that lacks advancements in energy efficiency (dena, 2019). In Germany, tenancy law allows for extraordinary rent increases beyond general rent control after energy efficiency modernizations. This is intended to incentivize socially desirable investments and to overcome the landlord-tenant-dilemma (Klinski, 2010). Another policy measure aiming to increase building owners' investment are public subsidies (Bird and Hernández, 2012; MacAskill et al., 2019). In Germany, to avoid double financing of the investment, subsidies need to be deducted from the investment costs for the modernization surcharge, apparently decreasing the appeal of subsidies to landlords. We developed a theoretical microeconomic model to analyze the interaction of the modernization surcharge with public subsidies.

Based on numerous empirical studies eliciting tenants' WTP for energy efficiency (e.g. Banfi et al., 2008; Carroll et al., 2016; Collins and Curtis, 2018; Hahn et al., 2018; Hyland et al., 2013; Kholodilin et al., 2017; Taruttis and Weber, 2022), we assumed that tenants are indeed willing to pay for landlords' investments resulting in decreasing energy costs, but below the actual investment costs. Allowing landlords to extraordinarily increase the rent following a modernization enables them to extract some of a quasi-rent that tenants accrue thanks to rent control. In essence, landlords can overcharge for energy efficiency because they generally have to undercharge for the apartment itself due to tenancy law, leaving tenants worse off after a modernization. We have shown that small non-deductible subsidies increase modernization width but not as much modernization depth, exacerbating the undesirable effects of the modernization surcharge. Deductibility improves the positive effect on modernization depth at the cost of less impact on modernization width, remedying some of the welfare problems. Only sufficiently large subsidies help overcome the inefficient incentives of the modernization surcharge enabling widespread deep modernizations benefitting both landlords and tenants as well as the climate.

Acknowledging the limitations of a theoretical model, our analysis can be summarized into three political implications. First, our model reconfirms the critique that many have voiced against the modernization surcharge: that it incentivizes inefficient investments to circumvent rent control and accrue additional profits from tenants where improved energy performance is at most a welcome by-product (Gaßner et al., 2019; Hallof, 2013; Klinski, 2010; Kossmann et al., 2016; Mellwig and Pehnt, 2019). Therefore, efforts should be made to drastically change the modernization surcharge to only incentivize modernizations that effectively reduce energy consumption, or maybe even to explore entirely different measures to overcome the landlord-tenant-dilemma such as a partial inclusive rent system. Second, our model shows that to ensure improvements for both landlords and tenants from subsidies, they need to be sufficiently large to overcome the misaligned incentives of the modernization surcharge thanks to the absolute limit on rent increases. Thus, if substantial reforms of the modernization surcharge are not attainable, political capital should be focused on severely increasing the subsidization, assuming that much more investment than can be currently found is socially optimal considering the externality. Third, our model shows that if subsidies are sufficiently large to overcome the modernization surcharge, it does not make too much of a difference regarding modernization extent and the qualitative distribution of welfare whether they have to be deducted or not. Hence, focusing the political discussion on the exact design of the modernization surcharge misses the bigger picture.

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Appendix

Appendix A: Proof to Lemma 1

In the first line, $\partial\pi_n/\partial c < 0$ for any positive c and no investment is profitable. In the second line, $\partial\pi_n/\partial c = \mu\, a + [1-\mu]\,\gamma - 1 \geq 0$ for $c < \min(c_n^{ra}, c_n^{rw})$ and thus extending the investment is at least profit-neutral or profitable. According to our assumption, landlords – if indifferent between several modernization options – choose the largest. As soon as that extension causes landlords' profits to be limited by either the absolute legal limit or the incumbent tenants' WTP, extending the investment is no longer profitable; $\partial\pi_n/\partial c = \begin{cases} \gamma - 1\ if\ c \in (\,c_n^{rw}, c_n^{aw}) \\ -1\ if\ c > \max(c_n^{ra}, c_n^{aw}) \end{cases} < 0$. Therefore, the kink in the profit function causes marginal profits to change the sign from non-negative to negative, indicating a profit maximum. Whether the optimal investment is given by c_n^{ra} or c_n^{rw} depends on whichever is smaller. □

Appendix B: Proof to Lemma 2

The first line corresponds to the first line of Lemma 1. $\partial\pi_0/\partial c = \partial\pi_n/\partial c + \varphi$ for $c < \min(c_n^{ra}, c_n^{rw}, \Phi/\varphi)$. If inequality (3) does not hold, that derivative is negative for $\varphi < \underline{\varphi}_0$ and thus these sufficiently small subsidies incentivize no modernization.

The second line comprises the cases where marginal profits shift from positive to negative at Φ/φ, that is where larger investments no longer cause larger subsidies. Landlords optimally choose to invest at $c = \Phi/\varphi$ if marginal profits are positive for smaller investments and negative for larger ones. The first condition refers to the case that $\partial\pi_n/\partial c < 0$ for any positive c. Complementary to the first line, φ must at least exceed $\underline{\varphi}_0$ to trigger an investment. As long as φ also exceeds $\Phi/\min(c_n^{ra}, c_n^{rw})$, it is ensured that $\Phi/\varphi\ < \min(c_n^{ra}, c_n^{rw})$ and that therefore marginal profits turn negative caused by subsidies shifting from relative to absolute. The second condition refers to cases where the relative subsidy is large enough to turn marginal profits positive if landlords' optimal investment is determined by the incumbent tenants' WTP; $\partial\pi_0/\partial c = \gamma - 1 + \varphi$ for $c \in (\,c_n^{rw}, c_n^{aw})$. $\bar{\varphi} \leq \varphi$ ensures $\partial\pi_0/\partial c > 0$ for $c < \Phi/\varphi$, $\Phi/c_n^{aw} \leq \varphi$ ensures $\Phi/\varphi \leq c_n^{aw}$ and $\varphi < \Phi/c_n^{rw}$ ensures $\Phi/\varphi > c_n^{rw}$ clearly placing the changing sign of marginal profits at Φ/φ. The third condition refers to cases where the relative subsidy is large enough to turn marginal profits positive if landlords' optimal investment is determined by the absolute legal limit on the modernization surcharge; $\partial\pi_0/\partial c = [1-\mu]\,\gamma - 1 + \varphi$ for $c > \max(c_n^{ra}, c_n^{aw})$. $\bar{\bar{\varphi}} \leq \varphi$ ensures $\partial\pi_0/\partial c > 0$ for $c < \Phi/\varphi$ while $\varphi < \Phi/\min(c_n^{ra}, c_n^{rw})$ ensures $\Phi/\varphi > \min(c_n^{ra}, c_n^{rw})$ clearly placing the changing sign of marginal profits at Φ/φ.

The third line comprises the cases where the relative subsidy is sufficiently large to turn marginal profits positive when bound by the relative legal limit and the incumbent tenants' WTP but not large enough to extend the modernization beyond the amount where the absolute legal limit on the modernization surcharge becomes binding; $\partial\pi_0/\partial c = \gamma - 1 + \varphi > 0$ for $c \in (c_n^{rw}, c_n^{aw})$ but $\partial\pi_0/\partial c = [1-\mu]\gamma - 1 + \varphi < 0$ for $c >\ c_n^{aw}$. $c_n^{ra} > c_n^{rw} > 0$ ensures that the optimal investment without subsidies is determined by the relative legal limit and the incumbent tenants' WTP. $\varphi > \bar{\varphi}$ ensures $\partial\pi_0/\partial c = \gamma - 1 + \varphi > 0$ for $c > c_n^{rw}$. $\varphi < \bar{\bar{\varphi}}$ ensures $\partial\pi_0/\partial c = [1-\mu]\gamma - 1 + \varphi < 0$ for $c >\ c_n^{aw}$. $\varphi < \Phi/c_n^{aw}$ ensures $c_n^{aw} < \Phi/\varphi$.

The fourth line comprises the cases where subsidies do not cause marginal profits to change signs and therefore the positive optimal investments remain as in the third line of Lemma 1. □

Appendix C: Proof to Lemma 3

The contrapositive proof is simple: suppose for some $c_\delta^+ > 0$ the subsidy $s^*(c_\delta^+) < \min(\varphi\, c_\delta^*, \Phi)$ maximized landlords' profits $\pi_\delta\left(c_\delta^+, s^*(c_\delta^+)\right)$. Then landlords could increase their investment by Δc and the subsidies by Δs in a ratio that leaves the costs that form the basis of the maximal modernization surcharge constant, i.e. by the amounts $\Delta s = \Delta c/\delta \leq \min(\varphi\, c_\delta^*, \Phi) - s^*(c_\delta^+)$. As a consequence, the min-function in landlords' profit function (2) would grow by $\gamma\, \Delta c > 0$ or remain constant and the remaining sum of three terms would strictly increase by $([1-\mu]\,\gamma + [1/\delta - 1])\Delta c > 0$, obviously increasing the profits. Hence c_s^+ cannot be a profit maximum and therefore choosing the profit maximizing investment c_s^* implies to take advantage of the entire offered subsidies, i.e. $s^*(c_\delta^*) = \min(\varphi\, c_\delta^*, \Phi)$. □

Appendix D: Proof to Lemma 4

The first line corresponds to the first line of Lemma 1. $\partial\pi_\delta/\partial c = \partial\pi_n/\partial c + (1 - \mu\, a\, \delta)\varphi$ for $c < \min(c_\delta^{ra}, c_\delta^{rw}, \Phi/\varphi)$. If inequality (3) does not hold, this derivative is negative for $\varphi < \underline{\varphi}_\delta$ and thus these sufficiently small subsidies incentivize no modernization.

The second line comprises the cases in which marginal profits turn from positive to negative at Φ/φ, that is when larger investments no longer cause larger subsidies. Landlords optimally choose to invest at $c = \Phi/\varphi$ if marginal profits are positive for smaller investments and negative for larger ones. The first condition refers to the case that $\partial\pi_n/\partial c < 0$ for any positive c. Complementary to the first line, φ must at least exceed $\underline{\varphi}_\delta$ to trigger an investment. As long as φ also exceeds $\Phi/\min(c_\delta^{ra}, c_\delta^{rw})$, it is ensured that $\Phi/\varphi < \min(c_\delta^{ra}, c_\delta^{rw})$ and that therefore marginal profits turn negative caused by subsidies shifting from relative to absolute. The second condition refers to cases where the relative subsidy is large enough to turn marginal profits positive if landlords' optimal investment is determined by the incumbent tenants' WTP; $\partial\pi_\delta/\partial c = \gamma - 1 + \varphi$ for $c \in (\, c_\delta^{rw}, c_\delta^{aw})$. $\bar{\varphi} \leq \varphi$ ensures $\partial\pi_\delta/\partial c > 0$ for $c < \Phi/\varphi$, $\Phi/c_n^{aw} \leq \varphi$ ensures $\Phi/\varphi < c_\delta^{aw}$ and $\varphi < \Phi/c_\delta^{rw}$ ensures $\Phi/\varphi > c_\delta^{rw}$ clearly placing the changing sign of marginal profits at Φ/φ. Note that if $\Phi/c_n^{aw} = \Phi/c_\delta^{rw}$ where the range collapses we find $c_n^{aw} = c_\delta^{rw}$ which implies $c_\delta^{rw} = c_\delta^{ra}$.The third condition refers to cases where the relative subsidy is large enough to turn marginal profits positive if landlords' optimal investment is determined by the absolute legal limit on the modernization surcharge; $\partial\pi_\delta/\partial c = [1-\mu]\,\gamma - 1 + \varphi$ for $c > \max(c_\delta^{ra}, c_n^{aw})$. $\bar{\bar{\varphi}} \leq \varphi$ ensures $\partial\pi_\delta/\partial c > 0$ for $c < \Phi/\varphi$ while $\varphi < \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ ensures $\Phi/\varphi > \min(c_\delta^{ra}, c_\delta^{rw})$ clearly placing the changing sign of marginal profits at Φ/φ.

The third line comprises the cases where the relative subsidy is sufficiently large to turn marginal profits positive when bound by the relative legal limit and the incumbent tenants' WTP but not large enough to extend the modernization beyond the amount where the absolute legal limit on the modernization surcharge becomes binding; $\partial\pi_\delta/\partial c = \gamma - 1 + \varphi > 0$ for $c \in (c_\delta^{rw}, c_n^{aw})$ but $\partial\pi_\delta/\partial c = [1-\mu]\gamma - 1 + \varphi < 0$ for $c > c_n^{aw}$. $c_\delta^{ra} > c_\delta^{rw} > 0$ ensures that the optimal investment without subsidies is determined by the relative legal limit and the incumbent tenants' WTP. $\varphi > \bar{\varphi}$ ensures $\partial\pi_\delta/\partial c = \gamma - 1 + \varphi > 0$ for $c > c_\delta^{rw}$. $\varphi < \bar{\bar{\varphi}}$ ensures $\partial\pi_\delta/\partial c = [1-\mu]\gamma - 1 + \varphi < 0$ for $c > c_n^{aw}$. $\varphi < \Phi/c_n^{aw}$ ensures $c_n^{aw} < \Phi/\varphi$.

The fourth line comprises the cases where subsidies do not cause marginal profits to change signs. However, since the obligation to deduct subsidies from the modernization surcharge includes the subsidy into the min-function that provides the additional revenue from incumbent tenants in equation (2), the kinks in the piecewise linear profit function change and likewise the optimal investment when determined by that first kink. Therefore, the optimal investment in that case is given by $\min(c_\delta^{ra}, c_\delta^{rw})$ by the same argument as in Lemma 1. Which of the set $\{c_\delta^{ra}, c_\delta^{rw}\} =$

$\{\min(c_\varphi^{ra}, c_\Phi^{ra}), \min(c_\varphi^{rw}, c_\Phi^{rw})\}$ is smallest depends on the subsidy quota φ and the maximum subsidy Φ. We therefore define $\tilde{\varphi}_\delta \equiv \frac{1}{\delta}\frac{a\,[L-Q]-\gamma\,L}{a\,[L-Q]}$ as that subsidy quota below which we find $c_\varphi^{rw} < c_\varphi^{ra}$. Similarly, we define $\tilde{\Phi}_\delta \equiv \frac{1}{\delta}\frac{a\,[L-Q]-\gamma\,L}{a\,\gamma}$ below which $c_\Phi^{rw} < c_\Phi^{ra}$ holds. Note that Φ/c_δ^{rw} and Φ/c_δ^{ra} are concave increasing functions that intersect at the origin of the Φ-φ-plane and at $(\tilde{\Phi}_\delta, \tilde{\varphi}_\delta)$. Furthermore, note that $\Phi/c_\delta^{rw} > \Phi/c_\delta^{ra}$ if and only if $\tilde{\Phi} > 0$ and the maximum subsidy amount Φ is in the interval $(0, \tilde{\Phi})$. We note that $\Phi/\min(c_\delta^{ra}, c_\delta^{rw}) \geq \Phi/c_\delta^{rw} > 0$ due to $\Phi > 0$ by assumption. The minimum of the set $\{c_\varphi^{ra}, c_\Phi^{ra}, c_\varphi^{rw}, c_\Phi^{rw}\}$ provides maximum profits according to the fourth line of Lemma 4. It depends on φ and Φ as indicated by Table D.1:

Table D.1: Minimum of the set $\{c_\varphi^{ra}, c_\Phi^{ra}, c_\varphi^{rw}, c_\Phi^{rw}\}$ depending on the subsidy as characterized by φ and Φ with the obligation to deduct the share δ of the subsidies from the modernization surcharge.

$\varphi > \Phi / \min(c_\delta^{ra}, c_\delta^{rw})$	$\Phi < \tilde{\Phi}_\delta \Rightarrow c_\Phi^{rw} < \min(c_\varphi^{ra}, c_\Phi^{ra}, c_\varphi^{rw})$	$\Phi = \tilde{\Phi} \Rightarrow c_\Phi^{rw} = c_\Phi^{ra} < \min(c_\varphi^{ra}, c_\varphi^{rw})$	$\Phi > \tilde{\Phi}_\delta \Rightarrow c_\Phi^{ra} < \min(c_\varphi^{ra}, c_\Phi^{rw}, c_\varphi^{rw})$
$\varphi = \Phi / \min(c_\delta^{ra}, c_\delta^{rw})$	$(\Phi < \tilde{\Phi}_\delta \Leftrightarrow \varphi < \tilde{\varphi}_\delta) \Rightarrow c_\Phi^{rw} = c_\varphi^{rw} < \min(c_\varphi^{ra}, c_\Phi^{ra})$	$(\Phi = \tilde{\Phi}_\delta \Leftrightarrow \varphi < \tilde{\varphi}_\delta) \Rightarrow c_\Phi^{rw} = c_\varphi^{rw} = c_\varphi^{ra} = c_\Phi^{ra}$	$(\Phi > \tilde{\Phi}_\delta \Leftrightarrow \varphi > \tilde{\varphi}_\delta) \Rightarrow c_\Phi^{ra} = c_\varphi^{ra} < \min(c_\varphi^{rw}, c_\Phi^{rw})$
$\varphi < \Phi / \min(c_\delta^{ra}, c_\delta^{rw})$	$\varphi < \tilde{\varphi}_\delta \Rightarrow c_\varphi^{rw} < \min(c_\Phi^{rw}, c_\varphi^{ra}, c_\Phi^{ra})$	$\varphi = \tilde{\varphi}_\delta \Rightarrow c_\varphi^{rw} = c_\varphi^{ra} < \min(c_\Phi^{rw}, c_\Phi^{ra})$	$\varphi > \tilde{\varphi}_\delta \Rightarrow c_\varphi^{ra} < \min(c_\Phi^{rw}, c_\varphi^{rw}, c_\Phi^{ra})$

We prove the validity of Table D.1 as a whole by first proving each of the nine cells individually using sublemmata and then connecting them.

Sublemma 1: For $\varphi > \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ and $\Phi < \tilde{\Phi}_\delta$ we find $c_\varphi^{ra} > c_\Phi^{ra} > c_\Phi^{rw} < c_\varphi^{rw}$, implying $c_\Phi^{rw} = min(c_\varphi^{ra}, c_\Phi^{ra}, c_\varphi^{rw}, c_\Phi^{rw})$.

Proof: From $\Phi < \tilde{\Phi}_\delta$ we know that $c_\Phi^{ra} > c_\Phi^{rw}$ and $\Phi/c_\delta^{rw} > \Phi/c_\delta^{ra}$. From $\varphi > \max(\Phi/c_\delta^{rw}, \Phi/c_\delta^{ra}) = \Phi/c_\delta^{rw}$ we know that $c_\Phi^{rw} < c_\varphi^{rw}$. Since $\varphi > \Phi/\min(c_\delta^{ra}, c_\delta^{rw}) > \Phi/c_\delta^{ra}$ we also know that $c_\varphi^{ra} > c_\Phi^{ra}$. Combining these yields $c_\varphi^{ra} > c_\Phi^{ra} > c_\Phi^{rw} < c_\varphi^{rw}$. □

Sublemma 2: For $\varphi > \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ and $\Phi = \tilde{\Phi}_\delta$ we find $c_\varphi^{ra} > c_\Phi^{ra} = c_\Phi^{rw} < c_\varphi^{rw}$, implying $c_\Phi^{ra} = c_\Phi^{rw} = min(c_\varphi^{ra}, c_\Phi^{ra}, c_\varphi^{rw}, c_\Phi^{rw})$.

Proof: From $\Phi = \tilde{\Phi}_\delta$ we know that $c_\Phi^{ra} = c_\Phi^{rw}$ and $\Phi/c_\delta^{rw} = \Phi/c_\delta^{ra}$. From $\varphi > \Phi/\min(c_\delta^{ra}, c_\delta^{rw}) = \Phi/c_\delta^{ra}$ we know that $c_\varphi^{ra} > c_\Phi^{ra}$. Since $\varphi > \Phi/\min(c_\delta^{ra}, c_\delta^{rw}) = \Phi/c_\delta^{rw}$ we also know that $c_\Phi^{rw} < c_\varphi^{rw}$. Combining these yields $c_\varphi^{ra} > c_\Phi^{ra} = c_\Phi^{rw} < c_\varphi^{rw}$. □

Sublemma 3: For $\varphi > \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ and $\Phi > \tilde{\Phi}_\delta$ we find $c_\varphi^{ra} > c_\Phi^{ra} < c_\Phi^{rw} < c_\varphi^{rw}$, implying $c_\Phi^{ra} = min(c_\varphi^{ra}, c_\Phi^{ra}, c_\varphi^{rw}, c_\Phi^{rw})$.

Proof: From $\Phi > \tilde{\Phi}_\delta$ we know that $c_\Phi^{ra} > c_\Phi^{rw}$ and $\Phi/c_\delta^{rw} < \Phi/c_\delta^{ra}$. From $\varphi > \Phi/\min(c_\delta^{ra}, c_\delta^{rw}) = \Phi/c_\delta^{ra}$ we know that $c_\varphi^{ra} > c_\Phi^{ra}$. Since $\varphi > \Phi/\min(c_\delta^{ra}, c_\delta^{rw}) > \Phi/c_\delta^{rw}$ we also know that $c_\Phi^{rw} < c_\varphi^{rw}$. Combining these yields $c_\varphi^{ra} > c_\Phi^{ra} < c_\Phi^{rw} < c_\varphi^{rw}$. □

Sublemma 4: For $\varphi = \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ and $\Phi < \tilde{\Phi}_\delta$ we find $c_\varphi^{ra} > c_\Phi^{ra} > c_\Phi^{rw} = c_\varphi^{rw}$, implying $c_\Phi^{rw} = c_\varphi^{rw} = min(c_\varphi^{ra}, c_\Phi^{ra}, c_\varphi^{rw}, c_\Phi^{rw})$.

Proof: From $\Phi < \tilde{\Phi}_\delta$ we know that $c_\Phi^{ra} > c_\Phi^{rw}$ and $\Phi/c_\delta^{rw} > \Phi/c_\delta^{ra}$. From $\varphi = \Phi/\min(c_\delta^{ra}, c_\delta^{rw}) = \Phi/c_\delta^{rw}$ we know that $c_\Phi^{rw} = c_\varphi^{rw}$. Since $\varphi = \Phi/\min(c_\delta^{ra}, c_\delta^{rw}) > \Phi/c_\delta^{ra}$ we also know that $c_\varphi^{ra} > c_\Phi^{ra}$. Combining these yields $c_\varphi^{ra} > c_\Phi^{ra} > c_\Phi^{rw} = c_\varphi^{rw}$. □

Sublemma 5: For $\varphi = \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ and $\Phi = \widetilde{\Phi}_\delta$ we find $c_\varphi^{ra} = c_\Phi^{ra} = c_\Phi^{rw} = c_\varphi^{rw}$.

Proof: From $\Phi = \widetilde{\Phi}_\delta$ we know that $c_\Phi^{ra} = c_\Phi^{rw}$ and $\Phi/c_\delta^{rw} = \Phi/c_\delta^{ra}$. From $\varphi = \Phi/\min(c_\delta^{ra}, c_\delta^{rw}) = \Phi/c_\delta^{ra}$ we know that $c_\varphi^{ra} = c_\Phi^{ra}$. Since $\varphi = \Phi/\min(c_\delta^{ra}, c_\delta^{rw}) = \Phi/c_\delta^{rw}$ we also know that $c_\Phi^{rw} = c_\varphi^{rw}$. Combining these yields $c_\varphi^{ra} = c_\Phi^{ra} = c_\Phi^{rw} = c_\varphi^{rw}$. □

Sublemma 6: For $\varphi = \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ and $\Phi > \widetilde{\Phi}_\delta$ we find $c_\varphi^{ra} = c_\Phi^{ra} < c_\Phi^{rw} < c_\varphi^{rw}$, implying $c_\varphi^{ra} = c_\Phi^{ra} = min(c_\varphi^{ra}, c_\Phi^{ra}, c_\varphi^{rw}, c_\Phi^{rw})$.

Proof: From $\Phi > \widetilde{\Phi}_\delta$ we know that $c_\Phi^{ra} > c_\Phi^{rw}$ and $\Phi/c_\delta^{rw} < \Phi/c_\delta^{ra}$. From $\varphi = \Phi/\min(c_\delta^{ra}, c_\delta^{rw}) = \Phi/c_\delta^{ra}$ we know that $c_\varphi^{ra} = c_\Phi^{ra}$. Since $\varphi = \Phi/\min(c_\delta^{ra}, c_\delta^{rw}) > \Phi/c_\delta^{rw}$ we also know that $c_\Phi^{rw} < c_\varphi^{rw}$. Combining these yields $c_\varphi^{ra} = c_\Phi^{ra} < c_\Phi^{rw} < c_\varphi^{rw}$. □

Sublemma 7: For $\varphi < \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ and $\varphi < \tilde{\varphi}_\delta$ we find $c_\varphi^{rw} < c_\Phi^{rw}$, $c_\varphi^{rw} < c_\varphi^{ra}$, and $c_\varphi^{rw} < c_\Phi^{ra}$implying $c_\varphi^{rw} = min(c_\varphi^{ra}, c_\Phi^{ra}, c_\varphi^{rw}, c_\Phi^{rw})$.

Proof: From $\varphi < \tilde{\varphi}_\delta$ we know that $c_\varphi^{ra} > c_\varphi^{rw}$. From $\varphi < \tilde{\varphi}_\delta$ and $\varphi < \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ we know that $\varphi < \Phi/c_\delta^{rw}$ as $\Phi/c_\delta^{rw} = \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ for $\Phi \le \widetilde{\Phi}_\delta$ and $\tilde{\varphi}_\delta < \Phi/c_\delta^{rw} < \Phi/\min(c_\delta^{ra}, c_\delta^{rw}) = \Phi/c_\delta^{ra}$ for $\Phi > \widetilde{\Phi}_\delta$. $\varphi < \Phi/c_\delta^{rw}$ implies $c_\Phi^{rw} > c_\varphi^{rw}$. To show that $c_\varphi^{rw} < c_\Phi^{ra}$, we differentiate into two cases. Case 1: suppose $\Phi/c_\delta^{ra} < \varphi < \Phi/c_\delta^{rw}$ which can only occur if $\Phi < \widetilde{\Phi}_\delta$. From $\Phi/c_\delta^{ra} < \varphi$ we know that $c_\varphi^{ra} > c_\Phi^{ra}$. From $\Phi < \widetilde{\Phi}_\delta$ we know that $c_\Phi^{ra} > c_\Phi^{rw}$ which combines with $c_\varphi^{ra} > c_\Phi^{ra}$ to $c_\varphi^{rw} < c_\Phi^{ra}$. Case 2: suppose $\tilde{\varphi}_\delta > \varphi < \Phi/c_\delta^{ra} < \Phi/c_\delta^{rw}$. from $\varphi < \Phi/c_\delta^{ra}$ we know that $c_\varphi^{ra} < c_\Phi^{ra}$. Combined with $c_\varphi^{ra} > c_\varphi^{rw}$ from $\varphi < \tilde{\varphi}_\delta$, we get $c_\varphi^{rw} < c_\Phi^{ra}$. □

Sublemma 8: For For $\varphi < \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ and $\varphi = \tilde{\varphi}_\delta$ we find $c_\varphi^{rw} < c_\Phi^{rw}$, $c_\varphi^{rw} = c_\varphi^{ra}$, and $c_\varphi^{rw} < c_\Phi^{ra}$implying $c_\varphi^{rw} = c_\varphi^{ra} = min(c_\varphi^{ra}, c_\Phi^{ra}, c_\varphi^{rw}, c_\Phi^{rw})$.

Proof: From $\varphi = \tilde{\varphi}_\delta$ we know that $c_\varphi^{ra} = c_\varphi^{rw}$. From $\varphi = \tilde{\varphi}_\delta$ and $\varphi < \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ we know that $\Phi > \widetilde{\Phi}_\delta$ as $\tilde{\varphi}_\delta$ is a constant, $\Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ strictly increases over Φ, and both intersect at $\widetilde{\Phi}_\delta$. This also implies $\tilde{\varphi}_\delta < \Phi/c_\delta^{rw} < \Phi/\min(c_\delta^{ra}, c_\delta^{rw}) = \Phi/c_\delta^{ra}$. $\varphi < \Phi/c_\delta^{rw}$ implies $c_\Phi^{rw} > c_\varphi^{rw}$. $\tilde{\varphi}_\delta < \Phi/c_\delta^{rw} < \Phi/\min(c_\delta^{ra}, c_\delta^{rw}) = \Phi/c_\delta^{ra}$ implies $c_\varphi^{ra} < c_\Phi^{ra}$. Combined with $c_\varphi^{ra} = c_\varphi^{rw}$ from $\varphi = \tilde{\varphi}_\delta$, we get $c_\varphi^{rw} = c_\Phi^{ra}$. □

Sublemma 9: For $\varphi < \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ and $\varphi > \tilde{\varphi}_\delta$ we find $c_\varphi^{ra} < c_\Phi^{ra}$, $c_\varphi^{ra} < c_\varphi^{rw}$, and $c_\varphi^{ra} < c_\Phi^{rw}$, implying $c_\varphi^{ra} = min(c_\varphi^{ra}, c_\Phi^{ra}, c_\varphi^{rw}, c_\Phi^{rw})$.

Proof: From $\varphi > \tilde{\varphi}_\delta$ we know that $c_\varphi^{ra} < c_\varphi^{rw}$. From $\varphi > \tilde{\varphi}_\delta$ and $\varphi < \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ we know that $\Phi > \widetilde{\Phi}_\delta$ as $\tilde{\varphi}_\delta$ is a constant, $\Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ strictly increases over Φ, and both intersect at $\widetilde{\Phi}_\delta$. This also implies $\tilde{\varphi}_\delta < \Phi/c_\delta^{rw} < \Phi/\min(c_\delta^{ra}, c_\delta^{rw}) = \Phi/c_\delta^{ra}$. From $\varphi < \Phi/\min(c_\delta^{ra}, c_\delta^{rw}) = \Phi/c_\delta^{ra}$ we know that $c_\varphi^{ra} < c_\Phi^{ra}$. To show that $c_\varphi^{ra} < c_\Phi^{rw}$, we differentiate into two cases. Case 1: suppose $\Phi/c_\delta^{rw} < \varphi < \Phi/c_\delta^{ra}$. From $\Phi/c_\delta^{rw} < \varphi$ we know that $c_\varphi^{rw} > c_\Phi^{rw}$. From $\Phi > \widetilde{\Phi}_\delta$ we know that $c_\Phi^{rw} > c_\Phi^{ra}$ which combines with $c_\varphi^{rw} > c_\Phi^{rw}$ to $c_\varphi^{ra} < c_\Phi^{rw}$. Case 2: suppose $\tilde{\varphi}_\delta < \varphi < \Phi/c_\delta^{rw} < \frac{\Phi}{c_\delta^{ra}}$. from $\varphi < \Phi/c_\delta^{rw}$ we know that $c_\varphi^{rw} < c_\Phi^{rw}$. Combined with $c_\varphi^{ra} < c_\varphi^{rw}$ from $\varphi > \tilde{\varphi}_\delta$, we get $c_\varphi^{ra} < c_\Phi^{rw}$. □

The three rows of Table partition the relevant first quadrant of the Φ-φ-plane horizontally into three regions where we have $\varphi > \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$, $\varphi = \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$, and $\varphi < \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$, respectively. Since $\Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ is continuous and increasing for $\Phi > 0$ and ranges between 0 and 1, we know that those three rows cover the entire quadrant. Within the first two rows, i.e. for $\varphi > \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ and $\varphi = \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$, we partition the respective regions vertically with $\Phi < \widetilde{\Phi}_\delta$, $\Phi = \widetilde{\Phi}_\delta$, and $\Phi > \widetilde{\Phi}_\delta$, respectively. Since $\widetilde{\Phi}_\delta$ is a straight vertical line, we know that this partitioning covers the entire regions with $\varphi > \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$ and $\varphi = \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$, respectively. We then partition the region of the third row, defined by $\varphi < \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$, horizontally, with $\varphi < \tilde{\varphi}_\delta$, $\varphi = \tilde{\varphi}_\delta$, and $\varphi > \tilde{\varphi}_\delta$, respectively. Since $\tilde{\varphi}_\delta$ is a straight horizontal line, we know that this partitioning covers the entire region with $\varphi < \Phi/\min(c_\delta^{ra}, c_\delta^{rw})$. Therefore, we know that Table D.1 covers the entire relevant first quadrant of the Φ-φ-plane. Thus, the fourth line of Lemma 4 is complete and therefore the entire Lemma 4 holds. □

Appendix E: Proof to Lemma 5

For the profitable case we first show that tenants never lose welfare from the subsidy. It is easy to see that for $c_n^* > 0$, $w_n^{it}(c_n^*) \geq 0$. Likewise, $w_\delta^{it}(c_\delta^*) \geq 0$ for $c_\delta^* > 0$. Note that $w_\delta^{it}(c) \geq w_n^{it}(c)$ for any investment. From Corollary 1 and Corollary 2 we know that $c_\delta^* \geq c_n^*$. Therefore, $\Delta w^{it} \geq 0$ in the profitable case. We now identify the conditions for an actual increase in incumbent tenants' welfare. For $\varphi \in [\bar{\bar{\varphi}}, \Phi/\min(c_n^{ra}, c_n^{rw}))$ we have $c_\delta^* = \Phi/\varphi > \min(c_n^{ra}, c_n^{rw}) = c_n^*$ as follows from Lemma 4. This yields $\Delta w^{it} = \underbrace{\underbrace{Q + \gamma\, \Phi/\varphi}_{WTP} - \underbrace{L}_{\text{rent increase}}}_{w_\delta^{it}(\Phi/\varphi)} - \underbrace{\left[\underbrace{Q + \gamma\, c_n^*}_{WTP} - \underbrace{\min(a\, c_n^*, L, Q + \gamma\, c_n^*)}_{rent\ increase}\right]}_{w_n^{it}(c_n^*)}$. The latter reduces to $w_n^{it}(c_n^*) = \max(Q + \gamma\, c_n^* - L, 0)$. As $c_\delta^* > c_n^*$ we know that $\Delta w^{it} \geq \gamma[\Phi/\varphi - c_n^*] > 0$.

For $\varphi > \tilde{\varphi} \wedge \Phi > \tilde{\Phi}$ we have $\Delta w^{it} = \underbrace{\underbrace{Q + \gamma\, c_\delta^*}_{WTP} - \underbrace{L}_{\text{rent increase}}}_{w_\delta^{it}(c_\delta^*)} - \underbrace{\left[\underbrace{Q + \gamma\, c_n^*}_{WTP} - \underbrace{\min(a\, c_n^*, L, Q + \gamma\, c_n^*)}_{rent\ increase}\right]}_{w_n^{it}(c_n^*)}$ according to our discussion of Lemma 4, where the latter again reduces to $w_n^{it}(c_n^*) = \max(Q + \gamma\, c_n^* - L, 0)$. Due to $\delta > 0$ and thus $c_\delta^* > c_n^*$ we thus know that $\Delta w^{it} \geq \gamma[c_\delta^* - c_n^*] > 0$.

To achieve welfare-neutrality for incumbent tenants in the profitable case, to sets of conditions must be fulfilled, of which each consists of several sufficient conditions. Obviously, if $c_\delta^* = c_n^*$ then nothing changes for incumbent tenants and they remain welfare neutral regardless of subsidies. If $\delta = 0$ we know that $c_0^* > c_n^*$ only for sufficiently large subsidies, where, if applicable, $\varphi \geq \bar{\varphi}$ incentivizes $c_0^* = c_n^{aw}$ and $\varphi \geq \bar{\bar{\varphi}}$ incentivizes $c_0^* = \Phi/\varphi$. In the former case, tenants pay according to their WTP and thus $\Delta w^{it} = 0$. In the latter case, if $\varphi \geq \Phi/\min(c_n^{ra}, c_n^{rw})$ we still find that tenants pay according to their WTP and thus $\Delta w^{it} = 0$. If $\Phi \leq \tilde{\Phi}$ as well as if $\varphi \leq \tilde{\varphi}$, we find that $c_\delta^{rw} < c_\delta^{ra}$. If then also $\varphi < \bar{\bar{\varphi}}$, optimal investment is given by c_δ^{rw} or c_n^{aw}, which makes tenants pay according to their WTP leaving them welfare neutral. If alternatively $\varphi \geq \Phi/\min(c_n^{ra}, c_n^{rw})$ we again still find that tenants pay according to their WTP and thus $\Delta w^{it} = 0$.

We now turn to the unprofitable case. We know that $c_n^* = 0$ and thus $w_n^{it}(0) = Q$. For $\varphi > \max\left((a-\gamma)/(a\,\delta), \underline{\varphi}_\delta\right)$ we have $\Delta w^{it} = \underbrace{\underbrace{Q + \gamma\, c_\delta^*}_{WTP} - \underbrace{\min(a\,[c_\delta^* - \delta\, \min(\varphi\, c_\delta^*, \Phi)], L, Q + \gamma\, c_\delta^*)}_{\text{rent increase}}}_{w_\delta^{it}(c_\delta^*)} - \underbrace{Q}_{w_n^{it}(c_n^*)}$ where due to $\varphi > \underline{\varphi}_\delta$ we have $c_\delta^* > 0$. Because of $\mu\, a + [1-\mu]\, \gamma - 1 < 0$ we have $c_\delta^* \leq \Phi/\varphi$, thus $\varphi\, c_\delta^* \leq \Phi$. Combined with $\varphi > (a-\gamma)/(a\,\delta)$ we have $\min(a\,[c_\delta^* - \delta\, \min(\varphi\, c_\delta^*, \Phi)], L, Q + \gamma\, c_\delta^*) = \min(a\,[c_\delta^* - \delta\, \varphi\, c_\delta^*], L, Q + \gamma\, c_\delta^*) < \min(a\,[c_\delta^* - \delta\, (a-\gamma)/(a\,\delta)\, c_\delta^*], L, Q + \gamma\, c_\delta^*) = \min(a\, c_\delta^* - [a\, c_\delta^* - \gamma\, c_\delta^*], L, Q + \gamma\, c_\delta^*) = \min(\gamma\, c_\delta^*, L, Q + \gamma\, c_\delta^*) = \min(\gamma\, c_\delta^*, L)$. We therefore have $\underbrace{Q + \gamma\, c_\delta^*}_{WTP} - \underbrace{\min(\gamma\, c_\delta^*, L)}_{\text{rent increase}} > Q$ and thus $\Delta w^{it} > 0$.

For $\varphi \in \left[\max\left(\underline{\varphi}_\delta, \bar{\bar{\varphi}}\right), \Phi\,\gamma/L\right)$ we know that $c_\delta^* = \Phi/\varphi$ due to Lemma 4. Therefore, we have $\Delta w^{it} = \underbrace{\underbrace{Q+\gamma\,\Phi/\varphi}_{WTP} - \underbrace{\min(a\,[\Phi/\varphi - \delta\,\Phi], L, Q+\gamma\,\Phi/\varphi)}_{\text{rent increase}}}_{w_\delta^{it}(c_\delta^*)} - \underbrace{Q}_{w_n^{it}(c_n^*)}$. For $\varphi < \Phi\,\gamma/\mathrm{L}$ we have $Q+\gamma\,\Phi/\varphi > Q+\gamma\,\frac{\Phi}{\Phi\,\gamma/\mathrm{L}} = Q+L$. Therefore, we have $\Delta w^{it} > \underbrace{\underbrace{Q+L}_{WTP} - \underbrace{\min(a\,[\Phi/\varphi - \delta\,\Phi], L, Q+L)}_{\text{rent increase}}}_{w_\delta^{it}(c_\delta^*)} - \underbrace{Q}_{w_n^{it}(c_n^*)} = \underbrace{\underbrace{Q+L}_{WTP} - \underbrace{\min(a\,[\Phi/\varphi - \delta\,\Phi], L)}_{\text{rent increase}}}_{w_\delta^{it}(c_\delta^*)} - \underbrace{Q}_{w_n^{it}(c_n^*)} \geq \underbrace{\underbrace{Q+L}_{WTP} - \underbrace{L}_{\text{rent increase}}}_{w_\delta^{it}(c_\delta^*)} - \underbrace{Q}_{w_n^{it}(c_n^*)} = 0.$

We now identify the conditions for $\Delta w^{it} = 0$.

For $\varphi < \underline{\varphi}_\delta$ we have $c_\delta^* = 0$ and thus obviously $\Delta w^{it} = 0$.

$\varphi = (a-\gamma)/(a\,\delta) \in \left(\underline{\varphi}_\delta, \bar{\bar{\varphi}}\right) \cup \left[\max\left(\bar{\bar{\varphi}}, \Phi\,\gamma/L, \underline{\varphi}_\delta\right), 1\right]$ covers the edge-cases of $\varphi > \max\left((a-\gamma)/(a\,\delta), \underline{\varphi}_\delta\right)$ where $\varphi \in \left[\max\left(\underline{\varphi}_\delta, \bar{\bar{\varphi}}\right), \Phi\,\gamma/L\right)$ is excluded.

Likewise, $\varphi = \frac{\gamma}{L}\Phi \in \left[\max\left(\underline{\varphi}_\delta, \bar{\bar{\varphi}}\right), (a-\gamma)/(a\,\delta)\right]$ covers the edge-case of $\varphi \in \left[\max\left(\underline{\varphi}_\delta, \bar{\bar{\varphi}}\right), \Phi\,\gamma/L\right)$ where $\varphi > \max\left((a-\gamma)/(a\,\delta), \underline{\varphi}_\delta\right)$ is excluded.

For incumbent tenants to lose welfare due to subsidies in the unprofitable case, we have three sufficient conditions, which each necessarily include $\varphi > \underline{\varphi}_\delta$ to find $c_\delta^* > 0$. First, for $\varphi \in \left[\underline{\varphi}_\delta, \min\left((a-\gamma)/(a\,\delta), \bar{\bar{\varphi}}\right)\right)$ we know that $\min(a\,[c_\delta^* - \delta\,\min(\varphi\,c_\delta^*, \Phi)], L, Q+\gamma\,c_\delta^*) = a[c_\delta^* - \delta\,\min(\varphi\,c_\delta^*, \Phi)]$ from $\varphi < \bar{\bar{\varphi}}$. Lastly, from $\varphi < (a-\gamma)/(a\,\delta)$ we know that $a[c_\delta^* - \delta\,\min(\varphi\,c_\delta^*, \Phi)] > a\,c_\delta^* - \min(a\,c_\delta^* - \gamma\,c_\delta^*, (a-\gamma)\Phi) \geq \gamma\,c_\delta^*$ and therefore $\Delta w^{it} < 0$.

Second, for $\varphi \in \left(\max\left(\bar{\bar{\varphi}}, \Phi\,\gamma/L, \underline{\varphi}_\delta\right), (a-\gamma)/(a\,\delta)\right)$ we know that $c_\delta^* = \Phi/\varphi$ from $\varphi > \max\left(\bar{\bar{\varphi}}, \underline{\varphi}_\delta\right)$. From $\varphi > \Phi\,\gamma/L$ we know that $Q+\gamma\,\Phi/\varphi < Q+L$. Indicating that whenever $\min(a\,[c_\delta^* - \delta\,\min(\varphi\,c_\delta^*, \Phi)], L, Q+\gamma\,c_\delta^*) = L$ we have $\Delta w^{it} < 0$. Furthermore, from $\varphi < (a-\gamma)/(a\,\delta)$ we have that $a[c_\delta^* - \delta\,\min(\varphi\,c_\delta^*, \Phi)] > a\,c_\delta^* - \min(a\,c_\delta^* - \gamma\,c_\delta^*, (a-\gamma)\Phi) \geq \gamma\,c_\delta^*$ indicating that whenever $\min(a\,[c_\delta^* - \delta\,\min(\varphi\,c_\delta^*, \Phi)], L, Q+\gamma\,c_\delta^*) = a\,[c_\delta^* - \delta\,\min(\varphi\,c_\delta^*, \Phi)]$ we have $\Delta w^{it} < 0$. Since always at least one of the relative legal limit and the absolute legal limit are binding, we have $\Delta w^{it} < 0$.

Third, for $\varphi = \bar{\bar{\varphi}} \in \left(\max\left(\Phi\,\gamma/L, \underline{\varphi}_\delta\right), (a-\gamma)/(a\,\delta)\right)$ the same arguments as for the previous case are applicable. □

Appendix 1: Proof to Lemma 6

Lemma 6 consists of two statements. The first is that $\varphi = \bar{\bar{\varphi}}$ with $\Phi > \bar{\bar{\varphi}} * \min(c_s^{ra}, c_s^{rw}, c_n^{aw})$ ensures an investment of $\Phi/\bar{\bar{\varphi}} = \frac{\Phi}{1-[1-\mu]\,\gamma}$. This directly follows from Lemma 2 and Lemma 4. Second, Lemma 6 states that for any given maximum subsidy Φ no investment greater than $\Phi/\bar{\bar{\varphi}} = \frac{\Phi}{1-[1-\mu]\,\gamma}$ can be achieved by setting φ. On the one hand, larger values of φ cause the optimal investment to remain at Φ/φ according to Lemma 2 and Lemma 4 which decreases with increasing φ as φ is in the denominator. On the other hand, no lower levels of φ induce larger investments. Above $\bar{\bar{\varphi}}$, π_s^a turns positive up to Φ/φ. π_s^a always has the lowest slope of the three linear segments of landlords' profit function and always relevant for large investments. Therefore, if π_s^a turns positive it is always profitable to extend the investment as much as possible up to the point where π_s^a again assumes a negative slope, because the other segments relevant for lower investments already are positive. Therefore, $\bar{\bar{\varphi}}$ with $\Phi > \bar{\bar{\varphi}} * \min(c_s^{ra}, c_s^{rw}, c_n^{aw})$ causes the largest investment for any sufficiently large maximum subsidiy. □