Philipps-Universität Marburg

Fachbereich 02

Studiengang: Economics & Institutions

MASTERARBEIT

Incentives for Recycling and Product Lifetime

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1 INTRODUCTION

The environmental issues arising from e-waste have not gotten much attention for quite a while. Although awareness increased somewhat following China's prohibition on importing electronic waste, significant quantities of electronics used in Europe continue to end up as e-waste in developing countries like Vietnam, where only valuable resources are reclaimed, leaving hazardous residuals in landfills (Forti et al., 2020). While this issue currently has limited direct consequences in Western countries, its potential long-term implications are becoming increasingly relevant. Furthermore, there is a substantial political debate surrounding the limited availability of essential resources required for producing consumer electronics, particularly rare earth metals. Recent geopolitical developments, such as attempts by US President Donald Trump to secure agreements regarding rare earth metals in Ukraine, have highlighted the strategic importance of these resources. Given that these resources are inherently limited and extraction is costly, the discussion frequently emphasizes recycling as a viable solution. This aligns with broader ideas surrounding the circular economy, where recycling resources is central (Wang et al., 2024).

From an economic perspective, two main strategies could reduce e-waste generation and resource depletion. The first strategy is increasing recycling rates, which directly reduces e-waste and the demand for new resource extraction. The second strategy is extending product lifetimes[1], thereby decreasing the frequency of replacements and, consequently, both e-waste generation and initial resource consumption. If policymakers aim to implement these strategies, the key question becomes how to effectively achieve these objectives. Current regulatory approaches, such as European Union regulations mandating user-replaceable batteries in phones, aim to extend product lifetimes ("Council adopts new regulation on batteries and waste batteries," 2023). However, these are static regulations and do not fundamentally alter producers' incentives. Firms with market power might still prefer shorter-

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[1] In this thesis, product lifetime refers to the period during which a product is actively used for its intended purpose. Even if a phone remains operational, its lifetime is considered to have ended once it is no longer in active use, regardless of whether it is scrapped or stored. This distinction is relevant because when a product ceases to fulfill its original function, it is likely to be replaced by a new one. While related, durability is not perfectly synonymous with product lifetime, as it refers to the physical longevity of a product rather than the duration of its use. However, in the economic models considered in this thesis, durability directly determines how long a product is used. Since these models assume a simplified world where product usage aligns with its durability, the terms can be used interchangeably within this thesis.

than-optimal product lifetimes, actively practicing planned obsolescence.[2] Thus, this thesis investigates the following research question: "How would a tax on virgin inputs[3] affect the incentives of consumer electronics producers, which are practicing planned obsolescence, and would such a policy be socially desirable?" The focus on consumer electronics is deliberate, as foundational literature on durable goods theory largely predates the substantial technological advancements and widespread proliferation of contemporary electronics. Unlike simple household appliances, modern consumer electronics frequently depend on proprietary software, granting producers greater control over how and how long their products are used. This increased control differentiates consumer electronics significantly from simpler durable goods. To clearly illustrate this distinction, Chapter 2 presents a comprehensive literature review on durable goods theory, specifically addressing durability considerations. The field has seen limited attention over the past two decades, yet substantial potential exists for its expansion to address contemporary market dynamics, particularly in consumer electronics. This literature review sets the stage for Chapter 3, which introduces and analyzes a formal economic model of a monopolist producer deciding on the proportion of recycled inputs, product lifetime, and price in response to a tax on virgin inputs. These findings will subsequently guide a discussion in the final part of the thesis, where implications for designing an optimal tax policy are analyzed, comparing distortions present in monopolistic durable goods markets with those in perfectly competitive markets for non-durable goods.

2 DURABLE GOODS THEORY

2.1 MONOPOLISTS DURABILITY CHOICE

Early research on the durability choices of monopolists examined whether a monopolist would select the same level of durability as a competitive market. Kleiman and Ophir (1966), Levhari and Srinivasan (1969), and Schmalensee (1970) all conclude that monopolists tend to choose lower durability than competitive producers. However, their models differ in how they characterize durable goods. Kleiman and Ophir (1966) propose a framework where a product's

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[2] *"Planned Obsolescence is the production of goods with uneconomically short useful lives so that customers will have to make repeat purchases."* (Bulow, 1986)

[3] Virgin inputs refer to inputs that have not been previously used or recycled.

service stream[4] depends on its age relative to its total lifespan. Levhari and Srinivasan (1969) focus on a so-called "one-hoss shay" good, which provides constant service streams over its lifespan before abruptly failing. Meanwhile, Schmalensee (1970) models durability using a "radioactive decay" approach, where the service streams gradually decline over time but never fully disappear. When considering which model best describes consumer electronics, both the “one-hoss shay” and the radioactive decay models are appealing. It is reasonable to assume that a product’s service stream declines over time—consumers derive progressively less value in each period due to factors such as decreasing battery life, perceived inferiority to newer models, or increased computational demands of current software. Nonetheless, consumer electronics are typically used until they break down or are replaced. For example, in the case of a smartphone, consumers are unlikely to operate two aging devices simultaneously as a substitute for a new one; instead, a single device is used over a period during which it functions as a portable communication, information, and entertainment tool despite its declining performance. While some consumers own multiple devices, they usually serve distinct purposes—such as a private versus work phone or a portable laptop for travel versus a larger one for home and office use—so that each device effectively represents a different category of durable good. This supports the simplifying assumption in theoretical models that each consumer desires only one unit of a particular durable good. Under this framework, an older phone offers less value than a new one because, although it fulfills the same functions, its performance is inferior. Since consumers typically use only one phone at a time, new and old phones are best understood as imperfect substitutes—they serve the same purpose, but one is clearly superior. This notion of imperfect substitutability will be important moving forward.

Swan (1970) challenges earlier findings by arguing that Levhari and Srinivasan (1969) mis-specified the monopolist’s objective function. He contends that, under his assumptions, a monopolist would select the same durability level as a perfectly competitive market, as this minimizes production costs. To support this claim, he examines both "one-hoss shay" goods

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[4] Service stream refers to the level of service a product provides within a given period. This service may deteriorate over time; for example, wireless headphones may initially offer a battery life of five hours but only last four hours after a year of use. This reduction in functionality represents a decline in the service stream. The service stream is identical for all consumers owning the same product of the same age, assuming deterioration is nonrandom. However, individual consumers may assign different values to the service stream based on their preferences and usage patterns.

and those that follow a radioactive decay pattern, showing that in either case, a profit-maximizing monopolist would align durability with the cost-minimizing level observed in competitive markets. In two subsequent papers (Sieper and Swan, 1973; Swan, 1971), further analysis explores why his conclusions diverge from prior research and how various regulatory interventions might influence durability choices and policy outcomes. Waldman (2003) illustrates Swan's argument with the example of a lightbulb manufacturer, where consumers care only about the total amount of light produced before the bulb fails. Under this assumption, a monopolist seeking to maximize profits would not reduce durability but instead lower the quantity of bulbs produced, ensuring that each bulb operates at the lowest possible cost per unit of light output. Swan extends this argument beyond "one-hoss shay" goods, applying it to any durable good where multiple used units can act as perfect substitutes for a single new unit. However, Waldman (2003) critiques this assumption, and it is also an unsuitable framework for consumer electronics, which exhibit imperfect substitutability. Moreover, it is unlikely that consumers are fully informed about a product's durability at the time of purchase. If durability is not transparently conveyed, consumers cannot optimize their decisions based on the total cost of service flows, as Swan's (1970) model suggests. This issue is particularly relevant in the consumer electronics market, where producers may actively influence product durability even after purchase, especially when devices rely on software updates. For example, Apple has been accused of deliberately degrading older iPhones' performance through software updates, effectively shortening their functional lifespan.(Allyn, 2020) Despite these limitations, Swan's (1970) concept of a cost-minimizing durability level remains useful. In this context, the cost-minimizing level refers specifically to the durability that minimizes the private costs of service flows—for instance, the total cost of producing light in the case of a lightbulb. However, while this level may be optimal from a private cost perspective, it can still be socially suboptimal due to the under-internalization of environmental externalities. Nevertheless, it provides a valuable benchmark for assessing firms' durability choices. If a product's durability falls below the private cost of service flow minimizing lifetime (PCSFML), it is also likely to be below the socially optimal level.

Swan's (1970) findings have been further examined by multiple authors. Schmalensee (1974) investigates whether the possibility of maintenance alters Swan's conclusions and finds that when a monopolist sells its output rather than rents it, and consumers are responsible for maintenance decisions, the monopolist no longer produces at the durability level that minimizes the overall cost of service flows. While the idea of maintenance is relevant to consumer electronics—particularly in the context of repairs—its direct applicability to this thesis is less clear. In the paper by Schmalensee (1974), the monopolist "offloads" the responsibility for durability onto consumers by allowing them to decide whether to maintain or repair the product. In contrast, the central question in this thesis concerns a scenario where the producer actively reduces a product's effective lifetime, potentially even obstructing maintenance efforts. This aligns with concerns raised by "right to repair" advocates, who argue that firms intentionally limit repairability to shorten product lifespans (Ozturkcan, 2024). Su (1975) contradicts Swan's results for durable goods that degrade over time, rather than "one-hoss shay" goods, and are sold instead of rented. A particularly noteworthy insight from this literature is that Swan's (1970) result is only contradicted when the monopolist sells its products rather than rents them. This distinction is relevant, as consumer electronics markets overwhelmingly operate on a sales model. More broadly, the durability literature places significant emphasis on the contrast between sales and rental markets, particularly on why rental markets are not more widespread despite their potential to enhance monopolist profitability. One common explanation attributes this to moral hazard on the part of consumers (Bulow, 1982), which reduces the viability of rental agreements. Some studies suggest, however, that leasing has gained traction in specific sectors, such as the automotive market (Johnson and Waldman, 2010). Nonetheless, since consumer electronics are primarily sold rather than rented, the analysis in this thesis will focus on sales markets, with only brief mentions of rental models where relevant.

2.2 TIME INCONSISTENCY

A key concept in understanding planned obsolescence is the role of time inconsistency in durable goods markets. One of the most influential contributions to this topic comes from Coase (1972), who examines how time inconsistency affects a durable goods monopolist's pricing decisions. He uses the example of land, which serves as an instance of a perfectly durable good since it does not degrade over time. Coase (1972) argues that if a monopolist owned all available land and could commit in the first period, they would sell the monopoly-optimal quantity upfront and withhold the rest. However, in subsequent periods, the monopolist would have an incentive to sell additional land, as past sales no longer constrain their pricing behavior. Rational buyers, anticipating future price reductions, would be unwilling to pay the monopoly price in the first period. As a result, Coase (1972) concludes that the price of a perfectly durable good would immediately drop to marginal cost, effectively erasing the monopolist's market power. Coase's (1972) argument remains a conjecture, as he did not formally model this mechanism. However, Bulow (1982) later formalized the time inconsistency problem in a two-period framework. He demonstrates that a monopolist selling a durable good retains less market power than one selling a non-durable good. To counteract this, the monopolist may adopt an inefficient production technology with lower fixed costs and higher marginal costs. By increasing marginal costs, the monopolist credibly commits to limiting future output, which helps sustain higher prices over time. Bond and Samuelson (1984) extend previous findings, which focused on perfectly durable goods, by introducing depreciating durable goods into the analysis. They find that when a good deteriorates over time, the monopolist retains some degree of market power because replacement sales partially mitigate the time inconsistency problem. Since consumers eventually need to replace their worn-out goods, the monopolist considers how current sales will influence future demand, reducing their incentive to flood the market with excess supply in any given period. This insight has important implications for planned obsolescence. If a product naturally depreciates, this creates a built-in mechanism for repeat sales. In turn, a monopolist may have an incentive to intentionally lower durability even further as a way to regain market power. Kahn (1986) demonstrates that Coase's (1972) conjecture no longer holds when production is subject to increasing marginal costs. When marginal costs rise with output, producing a large quantity in a single period becomes prohibitively expensive. As a result, the monopolist has a strong incentive to spread production over multiple periods, since it is more cost-effective to produce smaller quantities gradually rather than flooding the market all at once. This gradual release of output prevents prices from falling to marginal cost, allowing the monopolist to retain some market power and avoid the extreme price erosion predicted by Coase (1972). An interesting extension of this idea is provided by Karp and Perloff (1996), who examine whether a monopolist might deliberately adopt an inferior, higher-cost production technology despite having access to a more efficient alternative. They show that when consumers are unaware of the superior technology's existence—meaning only the monopolist possesses this information—certain equilibria arise in which it is optimal for the firm to initially use the worse technology before later switching to the better one. The key mechanism behind this strategy is that consumers, believing the firm to be constrained by high production costs, do not anticipate a future increase in output. This enables the monopolist to maintain higher prices for a longer period, preserving market power. Once demand has been shaped by these expectations, the monopolist can strategically introduce the superior technology, expanding output beyond what consumers had initially anticipated. If consumers had been fully informed from the outset, their willingness to pay in earlier periods would have been lower. The concept of asymmetric information regarding a producer's technology is particularly relevant in certain consumer electronics markets. A notable example is Nvidia's dominance in the graphics card market. With the release of the RTX 3000 series, Nvidia introduced DLSS 2, an upscaling technology that improves performance by rendering at a lower resolution and then enhancing the image through artificial intelligence. This technology was further refined in the RTX 4000 generation with DLSS 3, which featured significant improvements. However, despite the apparent compatibility, DLSS 3 was exclusively made available for the newer generation, even though there was no clear technological reason preventing its implementation on older GPUs. This raises an important issue of information asymmetry: Nvidia likely had prior knowledge of these technological advancements but chose not to disclose them, allowing for a larger-than-expected performance gap between product generations. By selectively withholding software upgrades, Nvidia might have influenced consumer expectations regarding the relative quality of their new and old products. The firm's strategic decision-making at the point of sale of the newer generation does not only determine the quality of the new product but also affects the perceived and actual performance of the previous generation by controlling the availability of software updates. This is particularly relevant because graphics cards rely on frequent driver updates from the manufacturer to maintain full functionality, and previous iterations of DLSS had received within-generation improvements.

Several studies have examined further conditions under which Coase's conjecture fails to hold, identifying key limitations in its assumptions. Three notable examples illustrate different mechanisms that allow a monopolist to retain market power despite selling durable goods. Gul (1987) extends the analysis to an oligopoly setting, showing that when firms can make offers arbitrarily frequently, they enhance their ability to commit to higher prices. In the limit, this enables them to sustain total market profits equivalent to those of a full-commitment monopoly, effectively negating Coase's (1972) prediction. Bagnoli et al. (1989) demonstrate that Coase's (1972) conjecture breaks down when the number of potential buyers is finite, even if the market is large. Ausubel and Deneckere (1989) argue that Coase's conjecture fails when buyers condition their behavior on the monopolist’s past pricing decisions. In this framework, if a monopolist adheres to a rigid pricing strategy, consumers recognize a stable price trajectory and continue purchasing at elevated prices. However, if the monopolist deviates from this strategy and lowers prices, rational consumers revise their expectations and begin delaying purchases, anticipating further price drops. As a result, the monopolist has a strong incentive to commit to a stable pricing policy to avoid triggering this "punishment" mechanism, thereby preserving some or even all of their market power.

While the general concept of time inconsistency is highly relevant, the markets examined in this thesis are not characterized by perfectly durable goods or pricing at marginal cost. In this regard, Waldman (2003) argued that much of the research on Coase's (1972) conjecture has been misfocused. According to Waldman (2003), firms have numerous ways to commit to a legally binding production plan, reducing the practical relevance of some time inconsistency arguments. He further contends that the implications of time inconsistency extend far beyond the narrow scope within which some of this literature has confined it. This will become clear in the subchapter on planned obsolescence.

2.3 ADVERSE SELECTION AND SECONDARY MARKETS

Since the discussion of planned obsolescence is closely tied to the existence of secondary markets, it is essential to first examine Akerlof's (1970) influential work on the "Market for Lemons", which serves as a foundational framework for understanding how asymmetric information affects markets (Waldman, 2003). Akerlof (1970) demonstrates that when buyers cannot accurately assess the quality of a used car, they will only be willing to pay a price that reflects the expected average quality of cars available in the market. However, this creates an adverse selection problem: sellers of high-quality used cars, knowing their vehicle's true value, will be unwilling to sell at the market price, while sellers of low-quality cars (or "lemons") will readily accept it. As a result, high-quality cars are systematically withdrawn from the market, leaving only lower-quality vehicles in circulation. This dynamic leads to market failure, as the inability to distinguish between good and bad cars reduces trade, even though there are owners of high-quality vehicles and buyers who would be willing to trade if they had full information. In such cases, both parties would benefit from the transaction, but asymmetric information prevents these mutually beneficial exchanges from occurring. Kim (1985) extends Akerlof's (1970) model by incorporating both primary and secondary markets into the analysis.

Unlike Akerlof's (1970) framework, which assumes a one-sided market where sellers differ in product quality, Kim's (1985) model allows agents to act as both buyers and sellers depending on their circumstances. A key innovation of his approach is the endogenization of used goods' quality, which depends on the maintenance decisions of the first owner. While maintenance slows the rate of decay, it cannot entirely prevent deterioration. One particularly interesting result of Kim's (1985) model is that, under certain assumptions, the average quality of used cars sold may actually be higher than that of those retained by their original owners. This stands in stark contrast to Akerlof's (1970) lemons problem, which predicts a negative selection process that drives high-quality goods out of the secondary market. Kim (1985) attributes this reversal to differences in valuation and maintenance incentives: consumers who place a high value on service flows are more likely to invest in maintenance, keeping their cars in better condition. However, these high-valuation consumers are also more likely to replace their vehicles with new ones in subsequent periods. As a result, they are willing to sell their well-maintained used cars at the market price for an average-quality used vehicle, despite knowing their car is of higher-than-average quality. They accept a lower resale price in exchange for the utility gains from upgrading to a new car. Kim's (1985) model thus presents a key contrast to Akerlof's (1970) framework. Akerlof's (1970) model assumes that secondary market transactions occur between sellers with a low valuation and buyers with a high valuation. However, in a durable goods market where consumers own only one unit at a time, the opposite might hold: high-valuation consumers, who maintain their goods well, become sellers, while lower-valuation consumers, who do not prioritize maintenance, become buyers. This idea was further developed by Hendel and Lizzeri (1999a), who describe the motivation to sell as the opportunity cost of keeping a lower-value product, rather than the direct benefit of selling to someone with a higher valuation, as considered by Akerlof (1970). As previously established, consumer electronics markets are likely characterized by imperfect substitutability, with consumers typically using only one device per specific function. Particularly for higher-cost electronics, it seems plausible that buyers take resale value into account when making a purchase, especially if they plan to upgrade frequently. However, secondary markets for electronics might also be subject to significant information asymmetry. Given the technical complexity of many electronic products, potential defects or performance issues may only become apparent after a period of use. Additionally, there may be heterogeneity in product quality even in the primary market, which could further amplify asymmetric information in resale markets. A notable example is the "silicon lottery" in processors, where variations in semiconductor chip performance occur due to random manufacturing differences. This means that two identical-looking processors can exhibit different performance levels, which are only observable over time and by users with sufficient technical knowledge. While these differences may be minor for most consumers, they illustrate how hidden quality variation could introduce additional uncertainty into secondary markets for consumer electronics, possibly reinforcing information asymmetry.

A significant portion of the durable goods literature assumes the existence of perfect secondary markets. This assumption is central to the analyses of Bulow (1982) and Coase (1972), as it allows for a clear valuation of used durable goods. In Coase's (1972) case, where goods are assumed to be perfectly durable, used products are indistinguishable from new ones, meaning that secondary market prices would necessarily align with those in the primary market. However, when considering a realistic depiction of consumer electronics markets, the assumption of perfect secondary markets is unlikely to hold. These markets are more plausibly characterized by some degree of asymmetric information, as buyers often lack full knowledge about the condition of used products.

2.4 PLANNED OBSOLESCENCE

The literature identifies three distinct explanations for the occurrence of planned obsolescence. The first considers it as a strategy to overcome the time inconsistency problem, where firms actively reduce product durability to maintain pricing power. The second perspective suggests that planned obsolescence arises as a consequence of time inconsistency, particularly through the introduction of new products that diminish the value of older ones. Finally, some research examines planned obsolescence as a strategic response to heterogeneous consumer valuations of quality, where firms tailor durability decisions to segment the market. Each of these explanations will be discussed in the following sections.

2.4.1 To overcome Time Inconsistency

The literature on planned obsolescence in the context of durability often contrasts with the view presented by Swan (1972), who argues that the existence of a secondary market does not necessarily constrain a monopolist’s pricing decisions or profitability. According to Swan (1972), a durable goods monopolist receives payment that reflects the net present value of the entire stream of services the product will provide, potentially to multiple future owners. As long as competitive secondhand dealers can resell the product indefinitely, the monopolist's market power remains intact, as resale does not inherently reduce their market power.[5] In contrast, Rust (1986) summarizes what he describes as the the conventional view that secondary markets create competitive pressure on primary market sales, reducing monopolist profits and creating incentives for planned obsolescence. By limiting product durability, a monopolist may ensure that used goods serve as worse substitutes for new products, increasing replacement frequency and sustaining higher sales. Rust (1986) also discusses an extreme form of planned obsolescence, where a firm could theoretically eliminate the secondary market altogether by producing goods with zero durability, meaning they would last for only a single period before requiring replacement. While Rust's (1986) model does not fully align with the conventional wisdom he describes, his findings suggest that both intermediate levels of planned obsolescence and extreme reductions in durability may emerge as equilibrium outcomes. However, he also identifies scenarios in which firms may have an incentive to offer durability levels exceeding the socially optimal level. His analysis relies on a Stackelberg game framework, where the monopolist acts as the Stackelberg leader.[6] Bulow (1986) further explores planned obsolescence and argues that, under what he considers weak assumptions, a monopolist not facing the threat of market entry will have an incentive to shorten product lifespan. In an oligopoly setting, the same incentive exists but is counterbalanced by another incentive: firms may extend product lifetimes to deter future entry and competition. Bulow (1986) attributes this incentive to the commitment problem first described by Coase (1972)—because a monopolist cannot credibly commit to future output restrictions, consumers anticipate future price reductions and adjust their purchasing behavior accordingly. By shortening product lifespan, the firm partially mitigates this issue. However, Bulow (1986) also acknowledges that durability reductions are just one of many potential strategies that firms might use to address the commitment problem. Additionally, he notes that his model considers planned obsolescence only through

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[5] Another study that partially aligns with Swan's assertion is (Miller, 1974), which models the market for textbooks. (Miller, 1974) finds no definitive evidence that frequent textbook revisions are intended to eliminate the secondhand market. However, rather than concluding that this strategy is absent, he characterizes the issue as inconclusive.

[6] An earlier study by (Liebowitz, 1982) found less ambiguous results than Rust's, suggesting a clear incentive for a monopolist to reduce product durability.

the lens of durability in a setting with a perfect secondary market, which may limit its applicability to real-world secondary markets where information asymmetries and transaction costs exist.

2.4.2 As a Result of Time Inconsistency

Levinthal and Purohit (1989) examine how new product introductions contribute to the obsolescence of older products, either through style changes or quality improvements. They argue that durability is an inadequate proxy for obsolescence, as the decline in a product's value often stems not from physical wear but rather from the availability of improved versions. Their model explicitly incorporates the extent to which a new product competes with its predecessor, describing obsolescence as a function of these two factors. The greater the technological improvement and the stronger the competition between new and old products, the more pronounced the decline in the value of older products. Their calculations suggest that as the level of product improvement increases, the cost of older products cannibalizing sales of newer ones rises, incentivizing the monopolist to limit sales of older models and potentially introduce buyback programs. Waldman (1993) also explores planned obsolescence driven by new product introductions. His findings suggest that, under his assumptions, a monopolist has an incentive to introduce new products that accelerate the obsolescence of older ones beyond what would be socially optimal. He attributes this to time inconsistency, as the firm does not internalize the effect of a new product's introduction on the declining value of its existing products. This contrasts with Bulow (1986), where planned obsolescence is used to mitigate time inconsistency rather than arising as a consequence of it. Choi (1994) considers a related scenario in which a monopolist introduces a new product that is deliberately incompatible with the old one, when network externalities are present. His analysis focuses on the incentives of a firm seeking to induce repeat purchases, particularly from consumers who already own the previous product. According to Choi (1994), such a monopolist may find it beneficial to design the new product in a way that lowers the value of the old one, thereby increasing demand for replacements. This behavior is again linked to time inconsistency, as the firm's inability to commit to a long-term production strategy prevents it from fully internalizing the decline in value of its older products. Waldman (1996a) extends this discussion to the context of R&D investment decisions. He finds that, under his assumptions, a monopolist that cannot commit in the first period has an incentive to overspend on R&D, introducing innovations at a faster rate than would be socially optimal. However, unlike previous findings (Choi, 1994; Waldman, 1993), this increased R&D spending may actually enhance overall welfare, as the monopolist's optimal R&D investment in a commitment scenario would be below the social optimum.

2.4.3 As a Strategy when facing Heterogenous Consumers

Kim (1989) examines the role of consumer heterogeneity and inelastic demand in shaping durability choices. He considers a setting in which consumers demand only one unit of a given good, either new or used, and where new and used goods are imperfect substitutes. Specifically, he assumes that new goods provide a higher service stream than used goods and that service streams cannot be combined. Under these conditions, he finds that durability falls below the social optimum in both competitive markets and monopolies, but is even lower in the case of a monopoly. Anderson and Ginsburgh (1994) extend the analysis by incorporating transaction costs in secondary markets. Their results suggest that a monopolist might use the secondhand market as a form of second-degree price discrimination. They draw an analogy to quality differentiation in monopoly pricing, where firms sell products of varying quality levels to segment the market. In this context, used goods serve as the lower-quality alternative, catering to consumers with lower willingness to pay. According to their findings, a monopolist may adjust durability to influence the quality of used goods, optimizing profitability based on the structure of demand. Depending on the market setting, this could create incentives for either higher or lower durability. Waldman (1996b) further explores the relationship between durability choices and consumer heterogeneity, considering a setting with two distinct consumer types: high-valuation consumers who place a strong emphasis on quality and low-valuation consumers who care less about quality. In his model, new goods provide higher quality, while older products deteriorate over time, with the rate of deterioration depending on the monopolist's investment in durability. This framework implies that new and used goods are imperfect substitutes, as they offer different quality levels and are valued differently by consumers. However, consumers can only own and consume one unit at a time, meaning they cannot accumulate multiple used products to replicate the service flow of a new one. This setup generates two countervailing forces that influence the price a monopolist can charge for new products. Since all high-valuation consumers purchase in the first period, they own a used product at the start of the second period. If durability is high, these consumers can sell their used goods at a higher price on the secondary market, increasing their willingness to pay for a new product, as they effectively face an "upgrade" decision. However, higher durability also implies that used products remain relatively high in quality, reducing the surplus utility derived from upgrading. Since the high-valuation group values quality more, this second effect dominates[7], leading to an incentive for the monopolist to decrease durability. In extreme cases, if low-valuation consumers place little value on quality, the monopolist may even find it optimal to eliminate the secondary market entirely by minimizing durability. Hendel and Lizzeri (1999b) build on Waldman's (1996b) model by considering a continuous distribution of consumer valuations, rather than a discrete two-group structure. In their framework, marginal consumers separate those who purchase new goods, used goods, or no goods at all. Additionally, unlike Waldman (1996b), their model allows the monopolist to adjust output levels, which introduces further complexity. As a result, their findings suggest that durability could be higher, lower, or equal to the social optimum, depending on market conditions. However, Waldman (2003) directly comments on the findings of Hendel and Lizzeri (1999b), arguing that when focusing on the actual quantity sold by the monopolist, durability always falls below the social optimum, which he considers the more relevant observation. He points out that while Hendel and Lizzeri (1999b) allow for cases where durability may be above, below, or equal to the social optimum, their results show that whenever output remains constant, the monopolist’s durability choice is always below the socially optimal level. Their reasoning is closely related to Waldman's (1996b): they describe two opposing effects—the "substitution effect", where higher durability increases competition between new and used goods, and the "resale value effect", where higher durability raises the resale price of used goods, increasing consumers’ willingness to pay for new ones. While these effects counteract each other from the monopolist’s perspective, Hendel and Lizzeri (1999b) emphasize that a social planner would disregard the substitution effect, leading to a further distortion in the monopolist’s durability decision.

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[7] To illustrate this, consider a high-valuation customer purchasing an iPhone in period 1 for $1,000. Apple determines the product's durability, which results in the iPhone being valued at either $300 or $100 in the second period. However, these resale values are based on the valuations of low-valuation customers. The original high-valuation owner may personally value the used iPhone at $600 or $200, respectively. These valuations serve as their reservation utility, influencing their upgrade decision. The willingness to pay for a new iPhone is determined by the surplus value gained from upgrading, plus the resale value of the used device. If Apple chooses the higher durability level instead of the lower one, the willingness to pay increases by $200 due to the higher resale value but decreases by $300 because the surplus utility from upgrading is reduced.

2.5 HOW DOES THIS APPLY TO CONSUMER ELECTRONICS?

To illustrate the key insights from the literature on planned obsolescence as they relate to consumer electronics, consider the cases of Apple selling iPhones and Nvidia selling GPUs.

Private Cost of Service Flow Minimizing Lifetime (PCSFML):

The concept of Private Cost of Service Flow Minimizing Lifetime (PCSFML) is introduced in this thesis to describe the durability level that minimizes the private cost of providing overall service flows. Consider Apple producing an iPhone. It seems reasonable to assume that designing and manufacturing an iPhone that functions for only one month before breaking down—while maintaining the same functionality, build quality, and overall user experience as a typical modern iPhone—would not be significantly cheaper than producing one that lasts for two years. Thus, one month should not be the PCSFML of an iPhone, as the overall service flow provided by the product could be increased considerably at minimal or no additional cost. At the same time, producing an iPhone that remains fully functional for 50 years would likely be prohibitively expensive or even technically infeasible if the goal is to maintain the same experience over that time span. A device that needs to be ruggedized to last several decades may require expensive materials or engineering choices that significantly increase its production cost. This would place it above the PCSFML, as the additional durability would not justify the increased costs from a private cost-minimization perspective. A key factor influencing the effective lifetime of iPhones and similar products is their reliance on operating system (OS) updates and security patches. If Apple discontinues software updates for a given iPhone model, the effective lifetime of the product decreases, even if the hardware remains functional. Without OS updates, app compatibility declines, and without security patches, consumers concerned about data protection may deem the device unusable.(Møller et al., 2021) From a PCSFML perspective, these updates remain cost-minimizing as long as enough consumers continue using older iPhones to justify the cost of providing them. For battery-powered devices, another complication arises due to the degradation of lithium-ion batteries. Over time, battery capacity inevitably declines, limiting the realistic effective lifetime of a product. However, this constraint need not determine the PCSFML if devices are designed with replaceable batteries. For instance, imagine Apple selling an iPhone with a guaranteed battery replacement within five years of purchase. In this case, Apple is essentially selling one product, but this product would have a higher effective lifetime than an iPhone without battery replacement.

Used Goods as Imperfect Substitutes for New Goods:

A key aspect of durable goods theory is whether used products are perfect substitutes for new ones. This would be the case if they were perfectly durable or if they decayed in a way that allowed service streams to be combined, such that multiple used goods could replicate the utility of a new product. However, as extensively argued in this thesis, this assumption appears highly unrealistic in the case of consumer electronics. Consumer electronics typically serve a distinct purpose, and in most cases, service streams cannot be combined in a way that makes multiple used devices equivalent to a new one. Consider an individual who owns a four-year-old iPhone and is contemplating whether to purchase the latest model. Their decision is not based solely on the absolute value of the new phone's service streams but rather on the incremental value it offers relative to their current phone. Thus, their willingness to pay for a new phone is influenced by the price of the new device minus the resale value of their current iPhone. Even in cases where consumers use their phone until it breaks and requires replacement, new and used iPhones remain imperfect substitutes. A consumer deciding between purchasing a new or used iPhone will compare the utility derived from the service streams of each option, relative to their respective prices, and select the one that offers the highest net utility. This reasoning assumes a rational actor, but in reality, purchasing decisions might be influenced by a combination of research, perceptions, and subjective preferences regarding product quality, reliability, and personal preferences. However, it appears plausible that many consumers take both new and used products into account when making a purchase decision, suggesting that they perceive them as imperfect substitutes, even if their decision-making process does not fully align with rational choice theory.

Asymmetric Information and Heterogenous Consumers in Secondary Markets:

When a person considers selling their phone or buying a used phone, they will encounter the issue of asymmetric information. As previously discussed, there is a possibility that the same product has different inherent quality levels even at the point of primary market sale, due, for example, to the "silicon lottery". This initial heterogeneity might be further compounded by how the first buyer uses the product. Consider the case of Nvidia selling GPUs. A recent shortage of GPUs in the primary market occurred because GPUs could be used for cryptocurrency mining. When mining became less profitable, many of these GPUs entered the secondary market. However, there is no easy way for buyers to determine whether a used GPU was previously used for mining, where it would have been running continuously, or if it was used intermittently for gaming or engineering applications, in which case it would likely have more remaining lifespan. A seller looking to upgrade their GPU may find that they are unable to sell their device at a price that reflects its true quality, as buyers will likely are only willing to pay for an average used GPU because they cannot verify whether it was previously used for mining. If the seller is a high-valuation consumer, they may still proceed with the sale at a lower price because the perceived benefit of upgrading to a new GPU outweighs the price reduction they face in the secondary market. While it seems plausible that secondary markets for consumer electronics exhibit significant asymmetric information, whether this leads to a "lemons" problem depends on the specific characteristics of the market. The degree of information asymmetry, transaction costs, and buyer's expectations all influence whether high-quality used goods are systematically undervalued or if the market remains functional despite these imperfections.

Planned Obsolescence to overcome Time inconsistency:

Since Bond and Samuelson (1984) found that a monopolist retains some market power when selling a decaying durable good, subsequent research suggests that reducing durability may help mitigate the time inconsistency problem. Consider Apple selling an iPhone. If an iPhone were infinitely durable and no new generations were released, Apple would eventually lose its market power, leading to a situation where iPhones are sold at cost. However, because iPhones are not infinitely durable, Apple retains market power and may even have an incentive to reduce durability below the PCSFML to strengthen its position. One possible strategy could involve making battery replacements difficult, limiting the effective lifetime of an iPhone. Consumers who might have continued using their device with a simple battery replacement could instead be pushed toward purchasing a new device earlier, increasing Apple's ability to sustain higher prices over time.

Planned Obsolescence as a Result of Time Inconsistency:

Planned obsolescence arising from time inconsistency occurs after the product has already been sold. Classical research in this area focuses on how producers can reduce the value of used goods, primarily through the introduction of new products. Consider Apple selling iPhones. One possibility is that Apple releases new models at a frequency that is greater than necessary, primarily to lower the value of older iPhones. This could happen through substantial technological improvements that make older models seem outdated, potentially driven by excessive R&D spending. Alternatively, Apple might introduce stylistic changes that make previous models visibly distinct from newer ones. If consumers perceive their iPhone as a status symbol, they may be incentivized to upgrade not because of technological necessity, but because their older model is now visibly outdated, reducing its perceived value. An additional channel through which consumer electronics producers may influence the value of used goods is software updates. If a firm has an incentive to accelerate the obsolescence of used devices, it might choose to end software support earlier than necessary or introduce updates that degrade user experience. For example, if Apple or Nvidia aimed to lower the value of used iPhones or GPUs, they might cease providing updates or introduce updates that alter performance in ways that reduce the perceived usability of older devices, indirectly influencing consumers to purchase new products. A related concept in literature is found in Choi (1994), who examines the introduction of incompatible new products. In the case of software updates, firms can make strategic decisions regarding whether to maintain compatibility between older products and new software, such as DLSS3, which is otherwise only available to buyers of newer devices. At the same time, there are counterexamples to this practice. Some smartphone manufacturers now offer extended software support guarantees, ensuring updates for several years after purchase. While this does not eliminate the possibility of malicious updates, it suggests that legally enforceable commitments regarding software support duration could serve as a commitment mechanism and therefore as a constraint on planned obsolescence driven by time inconsistency.

3 MODEL

As demonstrated in the previous chapter, there are various ways in which firms can engage in planned obsolescence, and extensive research suggests that firms with market power have incentives to do so. The purpose of this model is to take this incentive as a given, assuming a monopolist producer who earns higher profits by practicing planned obsolescence. In this context, planned obsolescence is defined as reducing the effective product lifetime below its private cost-minimizing level (PCSFML). The PCSFML represents the durability level that minimizes the private cost of providing service flows, while the socially optimal level will be considered in the next chapter. A key assumption in this model is that reducing product lifetime below the PCSFML does not incur significant costs or savings. In reality, two opposing forces would likely influence the cost of decreasing durability. On one hand, using cheaper materials might lower costs, while on the other hand, actively designing measures to reduce product lifetime (e.g., making batteries difficult to replace) could impose additional costs. However, since the model considers a producer offering a specific type of product valued for its characteristics (such as performance, display quality, or material feel) and only differentiated by its effective lifetime, the assumption is that any cost savings from durability reduction are minimal, just as the expenses associated with actively limiting lifetime are also assumed to be insignificant. The PCSFML represents the cost-minimizing durability level that would emerge in a perfectly competitive market with perfectly informed and rational consumers. This serves as the benchmark from which the market is analyzed. However, while the PCSFML minimizes the cost of providing service flows, it does not necessarily minimize the cost per unit of output. The assumption that reducing durability below the PCSFML incurs negligible costs does not imply that increasing durability beyond this level would also have negligible cost implications. Raising durability above the PCSFML increases the costs of overall service flows, which must also increase the cost per unit of output. Maintaining a constant level of service flows would require producing fewer units, and since the PCSFML represents the cost-minimizing durability level, fewer units would be produced at a higher overall cost for the same total service flows. As a result, the cost per unit must be higher. The assumption of negligible costs from lowering durability below the PCSFML applies only on a per-unit basis. However, the total costs of providing service flows would still increase because more products must be produced to achieve the same level of overall service flows, even if per-unit costs remain constant.

This assumption contrasts with prior literature, where lower durability is typically associated with lower per-unit costs. However, this relationship seems more relevant to industries where durability affects production costs more directly, such as automotive manufacturing. Thus, the model assumes that durability reduction does not provide the same cost-reduction incentives observed in other sectors. The first and primary variable of interest is L, which represents product lifetime relative to the PCSFML, where $L = 0$ indicates the PCSFML. Any value $L < 0$ implies that the monopolist deliberately reduces product lifetime despite there being no associated cost advantage.[8] The second variable of interest is r, where $r \in [0,1]$ represents the proportion of recycled materials used in production. A tax on virgin inputs applies to the complementary share (1 - r). The final strategic variable is p, the price of the product. The producer's decision variables are therefore L, r, and p, and the primary focus is on how an increase in t—a tax on virgin inputs—affects these choices. Given the complexity of modeling three strategic variables, a two-period monopoly model is employed, as commonly seen in durable goods literature. The assumption of a monopolist is useful because the results for a monopolist tend to be similar to those of a firm with market power (Waldman, 2003). This allows for insights into the dynamics of some consumer electronics markets, where producers hold significant levels of market power and can influence pricing and durability strategies.

Demand Function:

The demand function the monopolist faces is asymmetric across the two periods, with demand levels dependent on L and p, the product lifetime and price chosen in the first period. This represents a simplification of the considerations discussed in the previous chapter. There, it was argued that producers of some consumer electronics might actively influence product lifetime post-purchase and may have incentives to do so due to time inconsistency. However, this model provides insight into how a monopolist would behave if they were able to commit to durability choices in the first period, which could be feasible through commitments such as guaranteed software updates. Another simplification is that the monopolist sets one price for both periods rather than adjusting prices dynamically over time. This assumption is necessary to keep the complexity manageable while still capturing the core strategic decisions regarding durability. In the first period, product lifetime positively affects demand, as durability is a desirable characteristic of the good. However, in the second period, this positive effect is outweighed by the reduction in replacement demand—a longer-lasting product means fewer consumers need to replace their existing units. The monopolist's decision on L is therefore made before sales in the first period, balancing these opposing effects. Both effects are modeled linearly to maintain tractability. These effects are incorporated into a standard linear demand function with baseline demand A and parameter B, which represents the slope of the

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[8] Since increasing L is associated with higher per-unit costs, any $L>0$ implies positive costs. This means that even if L is positive in the model, it cannot be analyzed in the same way as a negative LLL. The reason is that additional costs, which are not explicitly modeled, would need to be considered when determining the extent to which LLL would actually increase beyond PCSFML.

demand curve. This approach allows for explicit solutions to the monopolist's optimal choices while maintaining interpretability in the model's results. The resulting Demand functions are:

$$\text{Period 1 Demand: } \mathrm{A - Bp + CL} \tag{1}$$

$$\text{Period 2 Demand: } \mathrm{A - Bp - dL} \tag{2}$$

Revenue per Unit:

To derive the revenue component of the profit function, the demand functions must be multiplied by the revenue the monopolist receives per unit sold. This revenue consists of the price paid by consumers minus the tax payment per unit. The price is denoted as p, while the tax payment is determined by the tax rate t, applied to the proportion of virgin inputs (1 - r) used in production. Thus, the revenue per unit is given by:

$$\text{Revenue per Unit: } \mathrm{p - t(1 - r)} \tag{3}$$

This formulation reflects how the producer's input choices influence the effective price received per unit. The inclusion of r highlights the extent to which recycling mitigates tax burdens, creating an incentive for the monopolist to adjust the proportion of recycled materials in response to taxation.

Revenue per Period:

Multiplying the respective period demands by the revenue per unit yields the total revenue per period:

$$\text{Period 1 Revenue: } \mathrm{(A - Bp + CL)(p - t(1 - r))} \tag{4}$$

$$\text{Period 2 Revenue: } \mathrm{(A - Bp - dL)(p - t(1 - r))} \tag{5}$$

Cost Function:

The cost function consists of two components. First, the cost of recycling, which is quadratic in r and proportional to demand in the respective period. Second, a general cost component, which is quadratic in demand for the respective period. These elements together define the cost function as follows:

$$\text{Cost Functioon: } \alpha D_i r^2 + \beta D_i{}^2 \tag{6}$$

where α represents the cost parameter for recycled inputs, β captures the general cost of production, and D_i denotes the demand in period i.

Profit Function:

Combining all elements—revenues and costs—while discounting second-period payoffs and costs by the discount factor δ, the resulting profit function is:

$$\begin{aligned}\Pi(r, p, L) = (A - Bp + CL)(p - t(1 - r)) - \alpha(A - Bp + CL)r^2 - \beta(A - Bp + CL)^2 + \delta((A - Bp - dL)(p - t(1 - r)) \\ - \alpha(A - Bp - dL)r^2 - \beta(A - Bp - dL)^2)\end{aligned} \tag{7}$$

This formulation explicitly captures how the monopolist's choices regarding product lifetime (L), price (p), and the percentage of recycled inputs (r) influence overall profitability. The quadratic cost terms ensure that increasing the percentage of recycled inputs or demand levels leads to increasing marginal costs. The discount factor δ accounts for the time preference of the firm, emphasizing how second-period revenue and costs are weighted relative to the present.

To simplify the following analysis, I will assume that the parameters A, B, C, d, α, β are all greater than 1, and that the discount factor δ is within the range 0.125 ≤ δ ≤ 1. The restriction on the discount factor is fairly weak, as this allows for a period-over-period discount rate of up to 87.5%, which seems like a reasonable assumption for many economic settings. The assumptions regarding the other parameters, in contrast, are more restrictive. If there were no quadratic cost components, it could be argued that achieving this assumption would only require scaling up the smallest parameter to 1 and considering a market for fractions of the durable good. For a sufficiently large market, this would be a reasonable approximation. Since the model includes quadratic elements, this assumption is less trivial. The following analysis is therefore limited to markets where the smallest parameter is greater than 1, or where the model can be scaled appropriately so that representing the market as one for fractional goods remains a solid approximation of the actual market.

Optimal Level of Recycled Inputs

Since the three variables of interest are r, p, and L, the next step is to take the first-order conditions (FOCs) for each variable and solve for their respective optimal levels. However, to simplify this process, it is useful if at least one of these variables can be solved independently of the others. This is the case for r. The first-order condition with respect to r is:

$$\frac{\partial \Pi}{\partial r} = (\mathrm{A} + \mathrm{CL} - \mathrm{Bp})\mathrm{t} - 2(\mathrm{A} + \mathrm{CL} - \mathrm{Bp})\mathrm{r}\alpha + ((\mathrm{A} - \mathrm{dL} - \mathrm{Bp})\mathrm{t} - 2(\mathrm{A} - \mathrm{dL} - \mathrm{Bp})\mathrm{r}\alpha)\delta = 0 \tag{8}$$

Solving for r and simplifying, the optimal level of r is:

$$\mathrm{r}^* = \frac{\mathrm{t}}{2\,\alpha} \tag{9}$$

This equation highlights that r will be exactly 0 in the absence of any tax, as t = 0 leads directly to r = 0. For r to have an interior solution (i.e., be between 0 and 1), t must be strictly smaller than 2α.[9] To develop an intuition about the plausible range of t that leads to an interior solution, we consider the expected magnitude of α. The cost of using recycled inputs is given by $\alpha \mathrm{D_i r^2}$, meaning the per-unit cost of recycled inputs is αr^2. The marginal cost of recycled inputs per unit is therefore 2αr. A reasonable assumption is that the cost of achieving high levels of recycled input usage is prohibitively high, meaning that relying exclusively on recycled materials should be very costly. This implies that α should be quite large, which in turn allows t to be relatively high without leading to a corner solution. If a corner solution were to occur, i.e., r* = 1, the tax burden on producers would be completely eliminated, as the tax only applies to virgin inputs.

Optimal Levels of Price and Lifetime

Since the optimal level of r is independent of both p and L, it is possible to substitute this result into the original profit function to simplify solving for the remaining two variables. The adjusted profit function is:

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[9] For the following analysis on whether an expression is positive or negative, this assumption will be taken as given, and only interior solutions for r will be considered. Consequently, the percentage of virgin inputs, represented by $(1 - \mathrm{r})$, will have an interior solution within the range (0,1), and t will be strictly less than 2α.

$$\Pi(r^{*},p,L)=(A-Bp+CL)(p-t(1-\frac{t}{2\,\alpha}))-\alpha(A-Bp+CL)\frac{t}{2\,\alpha}r^{2}-\beta(A-Bp+CL)^{2}+\delta((A-Bp-dL)(p-t(1-\frac{t}{2\,\alpha}r))-\alpha(A-Bp-dL){\frac{t}{2\,\alpha}}^{2}-\beta(A-Bp-dL)^{2}) \tag{10}$$

Taking the first-order conditions (FOCs) with respect to p and L yields:

$$\frac{\partial\Pi}{\partial p}=A+CL-Bp-B(p-t(1-\frac{t}{2\alpha}))+\frac{Bt^{2}}{4\alpha}+2B(A+CL-Bp)\beta+(A-dL-Bp-B(p-t(1-\frac{t}{2\alpha}))+\frac{Bt^{2}}{4\alpha}+2B(A-dL-Bp)\beta)\delta=0 \tag{11}$$

$$\frac{\partial\Pi}{\partial L}=C(p-t(1-\frac{t}{2\alpha}))-\frac{Ct^{2}}{4\alpha}-2C(A+CL-Bp)\beta+(-d(p-t(1-\frac{t}{2\alpha}))+\frac{dt^{2}}{4\alpha}+2d(A-dL-Bp)\beta)\delta=0 \tag{12}$$

This system of linear equations is solved for p* and L*, yielding:

$$p^{*}=\frac{C^{2}t(t-4\alpha)+2(-Cdt(t-4\alpha)-(C+d)^{2}(Bt(t-4\alpha)-4A\alpha)\beta+8AB(C+d)^{2}\alpha\beta^{2})\delta+d^{2}t(t-4\alpha)\delta^{2}}{-4C^{2}\alpha+8\alpha(Cd+2B(C+d)^{2}\beta+2B^{2}(C+d)^{2}\beta^{2})\delta-4d^{2}\alpha\delta^{2}} \tag{13}$$

$$L^{*}=\frac{(Bt(\frac{t}{4\alpha}-1)+A)(1+\delta)(C-d\delta)}{(2Cd(1+2B\beta)^{2}\delta+d^{2}(4B\beta+4B^{2}\beta^{2}-\delta)\delta+C^{2}(-1+4B\beta\delta+4B^{2}\beta^{2}\delta))}<0 \tag{14}$$

While the interpretation of the optimal price p* is difficult due to its complexity, this is not a primary concern, as the focus of this thesis is not the absolute value of p* but rather how it changes in response to the tax t. For L*, however, the absolute level is more relevant, as the model is intended to represent a market characterized by planned obsolescence. That is, the optimal L* must be negative in the absence of a tax to align with the assumption that firms have an incentive to reduce product lifetime below the PCSFML. A key determinant of whether this is the case is the expression (C - dδ). This term captures the relative importance of the positive effect of L on first-period demand versus the discounted negative effect of L in the second period. Since this model assumes the existence of planned obsolescence, the value of this expression must be negative—planned obsolescence only occurs if the future discounted loss of repurchases outweighs the first-period demand increase from higher L. This implies that dδ must be strictly greater than C. [10] If this condition holds, L* is negative in the absence of any tax. This conclusion follows because the numerator of L* is clearly negative[11], while the

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[10] From this point onward, this assumption will be taken as given when analyzing the sign of any expression and will not be explicitly restated.

[11] If there is no tax t, the numerator simplifies to $A(1+\delta)(C-d\delta)$, which is strictly negative.

denominator is strictly positive.[12] Thus, under this assumption, the model demonstrates that firms have an incentive to reduce product lifetime below the PCSFML, i.e., engage in planned obsolescence, whenever the expected future reduction in replacement sales due to a longer-lasting product outweigh the short-term increase in demand from higher durability.

The Effect of the Tax

The central question in this thesis is how a tax on virgin inputs would influence the firm's incentives. Since t does not appear in the denominator, its impact is concentrated within the expression $(\mathrm{Bt}(\frac{\mathrm{t}}{4\alpha} - 1) + \mathrm{A})$ which can be rearranged to become $(\mathrm{B}(\frac{\mathrm{t}^2}{4\alpha} - \mathrm{t}) + \mathrm{A})$. As t increases from zero, this expression initially decreases because the bracketed term is negative. This reduction leads to a decrease in the absolute value of the numerator, which, as it is inherently negative in the absence of a tax, results in an increase toward a less negative value. However, it remains negative overall as long as A is larger than the absolute value of $\mathrm{B}(\frac{\mathrm{t}^2}{4\alpha} - \mathrm{t})$. For the firm to have an incentive to increase product lifetime above the PCSFML, this inequality must reverse. To determine whether this reversal is possible, r can be substituted for its optimal value, yielding $\mathrm{B}(\mathrm{r}^{*2}\alpha - \mathrm{t})$.Since r^{*2} is less than 1, this expression can take on negative values. If the absolute value of this term exceeds A, then it becomes possible for the firm's optimal strategy to involve increasing L beyond the PCSFML. To formally analyze the relationship, the derivatives of r, p, and L with respect to t can be taken, yielding:

$$\frac{\partial \mathrm{r}^*}{\partial \mathrm{t}} = \frac{1}{2\,\alpha} \tag{15}$$

$$\frac{\partial \mathrm{p}^*}{\partial \mathrm{t}} = -\frac{(\mathrm{t} - 2\alpha)(2\mathrm{Cd}(1 + 2\mathrm{B}\beta)\delta + \mathrm{d}^2(2\mathrm{B}\beta - \delta)\delta + \mathrm{C}^2(-1 + 2\mathrm{B}\beta\delta))}{2\alpha(2\mathrm{Cd}(1 + 2\mathrm{B}\beta)^2\delta + \mathrm{d}^2(4\mathrm{B}\beta + 4\mathrm{B}^2\beta^2 - \delta)\delta + \mathrm{C}^2(-1 + 4\mathrm{B}\beta\delta + 4\mathrm{B}^2\beta^2\delta))} \tag{16}$$

$$\frac{\partial \mathrm{L}^*}{\partial \mathrm{t}} = \frac{\mathrm{B}(\mathrm{t} - 2\alpha)(1 + \delta)(\mathrm{C} - \mathrm{d}\delta)}{2\alpha(2\mathrm{Cd}(1 + 2\mathrm{B}\beta)^2\delta + \mathrm{d}^2(4\mathrm{B}\beta + 4\mathrm{B}^2\beta^2 - \delta)\delta + \mathrm{C}^2(-1 + 4\mathrm{B}\beta\delta + 4\mathrm{B}^2\beta^2\delta))} \tag{17}$$

The interpretation of t's effect on r is straightforward, as it equates the marginal cost of using additional recycled inputs with the tax. However, the effects of t on p and L are more complex. The relationships become clearer when substituting r back in for $\frac{\mathrm{t}}{2\alpha}$, which yields:

$$\frac{\partial \mathrm{p}^*}{\partial \mathrm{t}} = \frac{(1 - \mathrm{r})(2\mathrm{Cd}(1 + 2\mathrm{B}\beta)\delta + \mathrm{d}^2(2\mathrm{B}\beta - \delta)\delta + \mathrm{C}^2(-1 + 2\mathrm{B}\beta\delta))}{(2\mathrm{Cd}(1 + 2\mathrm{B}\beta)^2\delta + \mathrm{d}^2(4\mathrm{B}\beta + 4\mathrm{B}^2\beta^2 - \delta)\delta + \mathrm{C}^2(-1 + 4\mathrm{B}\beta\delta + 4\mathrm{B}^2\beta^2\delta))} > 0 \tag{18}$$

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[12] There are only two bracketed terms that contain negative elements. The term $(4\mathrm{B}\beta + 4\mathrm{B}^2\beta^2 - \delta)$ is positive since the sum of the first two terms exceeds 1. The second negative term, once isolated, is $-\mathrm{C}^2$, which has an absolute value that is strictly smaller than $2\mathrm{Cd}(1 + 2\mathrm{B}\beta)^2\delta$, since $\mathrm{Cd}\delta > \mathrm{C}^2$.

$$\frac{\partial L^*}{\partial t} = \frac{B(1-r)(1+\delta)(d\delta - C)}{(2Cd(1+2B\beta)^2\delta + d^2(4B\beta + 4B^2\beta^2 - \delta)\delta + C^2(-1 + 4B\beta\delta + 4B^2\beta^2\delta))} > 0 \tag{19}$$

Since both derivatives have a denominator that is clearly positive[13], their signs depend on the numerator. A key takeaway from these relationships is that both p and L respond linearly to the percentage of virgin inputs (1 - r). This result is intuitive, as the tax is applied only to virgin inputs, meaning that only this fraction of inputs influences the changes in p and L. A significant implication of this relationship is that if r reaches 1, further increases in the tax rate would have no additional effect on p or L. However, the absolute levels of p and L would still differ from those in the absence of a tax.

The numerator of $\frac{\partial p^*}{\partial t}$ is clearly positive[14] , meaning that the firm raises prices in response to the tax, aligning with economic intuition. This increase occurs only based on actual taxes paid, which explains the linear dependence on the percentage of virgin inputs. Similarly, the numerator of $\frac{\partial L^*}{\partial t}$ is strictly positive.[15] This finding is particularly relevant because it demonstrates that, under the assumptions of this model, a tax on virgin inputs increases product lifetime, thereby reducing planned obsolescence.

The Effect of the Parameters

While the results derived so far provide insights into the direction of the effects of the tax, they do not provide information regarding their magnitude. However, it is possible to analyze how the parameters within the model, as well as t itself through second-order derivatives of the optimal values, influence the size of these effects. To examine this, derivatives of $\frac{\partial r^*}{\partial t}$, $\frac{\partial p^*}{\partial t}$ and $\frac{\partial L^*}{\partial t}$ are taken with respect to the parameters present in their respective expressions as well as t. The term $\frac{\partial r^*}{\partial t}$ contains only α as a parameter, leading to a single derivative:

$$\frac{\partial(\frac{\partial r^*}{\partial t})}{\partial \alpha} = \frac{1}{-2\,\alpha^2} < 0 \tag{20}$$

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[13] Same denominator as before.

[14] There are only two bracketed terms that contain negative elements. The term $(2B\beta - \delta)$ is positive because $2B\beta > \delta$. The second negative term, once isolated, is $-C^2$, which has an absolute value that is strictly smaller than $2Cd(1+2B\beta)^2\delta$, since $Cd\delta > C^2$.

[15] Every element of the product is individually positive.

This effect is clearly negative, which is intuitive. The derivative states that the higher α is, the smaller the increase in r in response to an increase in t. Since a higher α implies that the costs of using recycled inputs are greater and increase more rapidly, the producer—who equalizes the marginal cost of recycled inputs with the tax cost—will adjust r less when facing higher α.

For $\frac{\partial p^*}{\partial t}$ and $\frac{\partial L^*}{\partial t}$, all parameters except A are present. To simplify the interpretation of these effects, the signs of the respective derivatives are summarized in a table, while the full derivations are provided in the appendix.

Table 1: Second Order Derivatives

Derivative with respect to …	$\frac{\partial L^*}{\partial t}$		$\frac{\partial p^*}{\partial t}$	
	Unconditional Sign	**Conditional Sign**	**Unconditional Sign**	**Conditional Sign**
t	$-$		$-$	
B	$-$		?	$-$ if $\delta < 0.8$ or $B\beta > \approx 1.2071$
C	$-$		$+$	
d	?	$+$ if $C > \frac{d\delta}{2}$	$-$	
α	$+$		$+$	
β	$-$		?	$-$ if $\delta < 0.8$ or $B\beta > \approx 1.2071$
δ	$+$		$-$	

Note. The table is structured as follows. The leftmost column lists the parameter with respect to which the second-order derivative is taken. The top row is divided into $\frac{\partial L^*}{\partial t}$ and $\frac{\partial p^*}{\partial t}$. For each of these, there are two sub columns: Unconditional Sign and Conditional Sign. The unconditional sign column describes the sign of the derivative when only the established assumptions about the parameter ranges are considered. The derivative may be unambiguously positive, which is denoted by a plus sign, unambiguously negative, which is denoted by a minus sign, or ambiguous, which is denoted by a question mark. The conditional sign column specifies the conditions under which the sign of an ambiguous derivative in the unconditional column becomes unambiguous.

It is important to note that these conditions do not represent the full range of cases under which the signs hold. For $\frac{\partial(\frac{\partial L^*}{\partial t})}{\partial d}$ the actual condition is less restrictive but also more complex. This explains why the condition only implies a positive sign, while the reverse condition does not necessarily imply a negative sign. The same applies to $\frac{\partial(\frac{\partial p^*}{\partial t})}{\partial B}$ and $\frac{\partial(\frac{\partial p^*}{\partial t})}{\partial \beta}$, as there are two conditions under which they are negative. Meeting either of these conditions implies a negative sign, but if neither condition is met, that is not a sufficient condition for the sign being positive. A more detailed explanation of these relationships is provided in the appendix.

The first row in the table represents the second-order derivatives of L* and p* with respect to t. Both of these second-order derivatives are negative, indicating that as the tax increases, the magnitude of the firm's response in terms of increasing both lifetime and price diminishes. This result is intuitive, as t appears in the first-order derivatives in a way that allows it to be substituted with the optimal level of r. A higher tax increases r, which reduces the effective tax burden on virgin inputs. As the tax rate increases, a smaller share of the tax applies to each product sold, explaining why the firm’s response in terms of price and lifetime adjustments becomes less pronounced.

Since A does not appear in either first-order derivative, the next row concerns the second-order derivative with respect to B. There is an unambiguous negative effect for $\frac{\partial L^*}{\partial t}$ and a likely negative effect for $\frac{\partial p^*}{\partial t}$. The latter is considered likely because the condition for a negative sign requires $B\beta > 1.2071$, which is reasonable given that B and β both have a lower bound of 1. The negative effect on $\frac{\partial p^*}{\partial t}$ aligns with economic intuition, as B represents the slope of the demand curve in this model. When demand is more elastic, consumers bear a smaller share of the tax burden, leading to a smaller increase in the price they actually pay. However, the effect on $\frac{\partial L^*}{\partial t}$ is less clear. It may be that under this model’s assumptions, the firm needs to adjust lifetime less because demand already decreases significantly due to the price increase, reducing the necessity for further changes in L*. This requires further investigation.

An increase in C introduces the first case in which the second-order derivatives of L and p have opposite signs. A higher C leads to a smaller increase in L and a larger increase in p following a tax increase. This result is noteworthy because it suggests a reaction that is undesirable from a consumer welfare perspective. From a consumer welfare standpoint, a tax is more beneficial when it leads to a greater increase in L, as this enhances the value consumers derive from the product, even if they are initially unaware of its lifetime. Additionally, a tax is more desirable if it results in a smaller increase in p. This implies that the lower C is—meaning consumers care less about lifetime at the time of purchase—the more desirable the tax is from a consumer welfare perspective. However, this does not mean that a tax is necessarily beneficial in a market with low C. It is possible that the tax negatively impacts consumer welfare overall, but its negative impact would be smaller in a market with a low C compared to one with a high C.

For d, the opposite effect of C is observed under certain conditions. If these conditions hold, a higher d implies a market in which a tax is more beneficial for consumer welfare. However, this effect is only valid up to a certain threshold of d. While failing to meet the given condition does not necessarily result in a negative sign, increasing d beyond some level eventually reverses the effect. This means that while the negative effect on $\frac{\partial p^*}{\partial t}$ is beneficial from a consumer welfare perspective, the effect on $\frac{\partial L^*}{\partial t}$ is only beneficial within a specific range.

For α, there is an unambiguous positive effect on both first-order derivatives. This result is intuitive, as a higher α implies a smaller increase in r, leading to a higher effective tax per unit. This results in a stronger response from the firm in terms of increasing both p and L.

For β, there is likely a negative effect across both first-order derivatives. This is because the condition for a negative sign is the same as for B, making it likely that higher production costs lead to a smaller increase in both p and L in response to the tax. While no clear economic explanation is immediately apparent, it may be related to the fact that a higher β results in a lower optimal quantity, which could influence the firm's decision-making process. Further investigation is needed to confirm this.

For δ, the effect mirrors that of d, but here it holds unconditionally for both first-order derivatives. This suggests that a tax is relatively more beneficial in a market where there is little discounting between periods. The interesting aspect is that the condition for planned obsolescence to occur is $d\delta > C$. Furthermore, the incentive to engage in planned obsolescence strengthens as the gap between dδ and C increases. Since a lower C and a higher δ make a tax more desirable from a consumer welfare perspective, while a higher d does so only within a certain range, this suggests that the stronger the incentive for planned obsolescence, the more effective the tax is in improving consumer welfare—at least under certain conditions.

4 TAX UNDER PLANNED OBSOLESCENCE AND E-WASTE EXTERNALITY

Beyond the emissions from production, consumer electronics generate a negative environmental externality through e-waste, which accumulates at the end of a product's life cycle. This implies that the product lifetime that minimizes the private costs of service flows is likely shorter than the socially optimal lifetime, as producers do not internalize the externality. Under the assumptions of the previous chapter's model, a tax on virgin inputs increases product lifetime whenever a producer has an incentive to engage in planned obsolescence. However, even if the tax fully eliminates the incentive for planned obsolescence, product lifetime remains below the socially optimal level because the producer does not internalize the environmental externality. This raises the question of how a tax on virgin inputs should be structured to align durability incentives with the socially optimal level. A seemingly logical starting point for designing an optimal tax is the Pigouvian tax, which directly corrects for externalities by imposing a cost equal to the marginal social damage, which is the optimal tax under perfect competition for non-durable goods. However, multiple factors limit the applicability of this approach in this scenario. To explore these limitations, the following analysis begins with a simplified setting and progressively introduces additional complexities step by step.

4.1 RENTING MONOPOLIST

The simplest scenario to consider is that of a renting monopolist, whose durable good generates a negative environmental externality. In this context, the good in question is a type of consumer electronics composed of materials that eventually become e-waste. Goering and Boyce (1999) (henceforth GB) analyze this scenario while examining the socially optimal level of an output-based tax. Although their analysis is built upon a complex vintage model to determine the effect of an output tax on product durability and quantity in an oligopoly setting, the core intuition behind their findings remains straightforward.

The oligopolist considered by GB either rents out their durable goods or can perfectly commit to a production plan that consumers believe to be credible. Under these conditions, the firm internalizes the impact of its current production decisions on the value of past output. Overproduction in any period leads to a decline in the value of the entire stock of the durable good, thereby lowering the rental price consumers are willing to pay. This mechanism eliminates the time inconsistency issue, ensuring that in the absence of an output tax, both monopolists and symmetric oligopolists choose the private cost-minimizing level of durability for the desired level of overall service flow. This occurs because they are directly selling service flows rather than the durable good itself.

GB find that introducing a per-unit output tax leads the oligopolist to increase durability beyond the level that originally minimized the private cost of providing service flows. This result follows naturally, as an output tax effectively raises the cost of production. To minimize the cost of service flows, the firm equalizes the marginal cost of additional service flows across both dimensions—enhancing durability and producing additional durable goods. Since the tax increases the cost of producing additional goods but does not directly affect the cost of increasing durability, the firm optimally raises durability while reducing output to restore cost efficiency.[16] However, as long as the tax is levied on a per-unit basis and remains independent of output levels, the durability choice remains the same whether the market structure is a monopoly or a symmetric oligopoly

This scenario highlights two key distortions that are central to this thesis. The first is the environmental externality associated with e-waste. Producers of durable goods do not internalize the social costs of e-waste, leading them to produce more than the socially optimal quantity. The second distortion arises from market power. Firms with market power restrict output below the socially optimal level to charge higher prices compared to a competitive market. This results in a welfare loss relative to perfect competition. In a perfectly competitive setting, the optimal tax would correspond precisely to the marginal social cost of the externality, known as a Pigouvian tax. However, Barnett (1980) and Buchanan (1969) show that when both market power and externalities are present, the optimal tax is actually lower

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[16] GB employ an emissions tax that directly targets emissions. In their analysis, they assume that emissions depend on output but do not necessarily impose a linear relationship between emissions and quantity. They find that if emissions are nonlinear in output, the number of competitors may influence product durability. For simplicity, I present only their results for the case in which each unit of output generates the same amount of emissions. This assumption aligns more closely with the focus of this thesis, as it examines e-waste, which is presumably independent of the number of units produced. Under this condition, an emissions tax that depends solely on output effectively functions as a per-unit tax. This assumption will also be used moving forward.

than the Pigouvian tax. This result is intuitive because market power counteracts some of the overproduction induced by the unpriced externality. Furthermore, if the influence of market power is sufficiently strong and the externality is relatively minor, it is even possible that production falls below the social optimum in the absence of a tax.

GB differentiate their research by considering the optimal tax when product durability is endogenous. They argue that prior studies on optimal taxation overlooked the possibility that firms can adjust both output and durability. Their findings suggest that, when durability is endogenous, the optimal tax may exceed the Pigouvian tax. They attribute this to firms underinvesting in durability, resulting in an additional inefficiency beyond the overproduction of durable goods. However, this conclusion appears counterintuitive. Since both durability and output are means of generating service flows for the renting firm, both should be distorted by the externality. In the renting scenario, durability is not subject to additional distortions, as there is no planned obsolescence, and firms select the private cost-minimizing level of durability. Thus, the claim that the optimal tax should exceed the Pigouvian level seems questionable. This issue was addressed by Runkel (1999), who reexamined GB's findings and demonstrated that, for renting firms, the optimal tax is in fact lower than the Pigouvian tax. [17] While this correction simplifies GB's conclusions, it still implies that even in a renting model without planned obsolescence, there are already two opposing forces shaping the optimal tax level for consumer electronics firms with market power that generate e-waste. Beyond what GB considered, this thesis incorporates two additional differences: first, an endogenous mechanism in the form of the decision on recycled inputs, and second, a sales market structure that includes planned obsolescence.

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[17] Except under highly specific and implausible assumptions, which also contradict the assumption that e-waste per unit of output is independent of both durability and the number of units produced.

4.2 SALES MARKET

Runkel (2004) extends this discussion by analyzing a sales market for durable goods in which product durability is endogenously determined. He demonstrates that a monopolist selling a durable good may optimally be taxed above the marginal environmental damage caused by their product. [18] This phenomenon, referred to as over-internalization, arises due to the monopolist's incentive to engage in planned obsolescence. By deliberately reducing durability below the level that minimizes the private cost of providing service flows, the firm introduces an additional distortion that influences the optimal tax rate. This additional distortion serves as a third force driving the optimal tax level upward. Since an output tax increases durability, it indirectly mitigates planned obsolescence by discouraging firms from deliberately reducing product lifetime. If this effect is sufficiently strong, it may justify setting the tax rate above the level of marginal environmental damage, even in cases where market power alone would suggest a lower-than-Pigouvian tax. The reasoning behind this result is intuitive. The tax incentivizes firms to reduce production volumes, which, under normal circumstances, would lead to a welfare loss when output is already below the socially optimal level due to market power. However, in a market characterized by planned obsolescence, an additional increase in durability can partially counteract the negative effects of market power. By making the good last longer, the firm increases the overall service flow provided to consumers while simultaneously reducing the environmental harm associated with frequent product replacement.

4.3 ENDOGENOUS RECYCLING

The closest paper to the question of this thesis was written by Eichner and Runkel (2003)(henceforth ER). They examine the relationship between durability and recyclability in durable goods markets, explicitly modeling the input mix as a combination of virgin and recycled materials to derive optimal government policies. Their framework is based on a vintage model, which describes the production of durable goods using labor and materials (a mix of virgin and recycled inputs). To manage model complexity, they assume that product

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[18] (Runkel, 2004) explores several additional considerations in this paper, including the idea that, in an oligopolistic setting, producers may have an incentive to oversell in the current period to compete with future market entrants. If this effect outweighs the reduction in quantity caused by market power, it would provide an even stronger argument for over-internalization. While this is a compelling perspective, it does not apply to the model analyzed in Chapter 3 and is therefore relegated to this footnote.

weight is positively correlated with both recyclability and durability. Specifically, they equate product weight with recyclability and model durability as a function of weight. While this simplification may be valid for some durable goods, it is questionable for consumer electronics, particularly for products where durability is more dependent on software updates than on physical attributes. Furthermore, even for the physical characteristics of consumer electronics, one could argue that less repairable products might be lighter than modular, repairable designs. Since repairable products can be used longer due to ease of maintenance, this could imply an inverse relationship between weight and durability—contrary to the assumption in ER’s model. However, there may still be an argument for a positive correlation between recyclability and durability through a different channel—namely, repairability. A product designed to be modular and easily repairable may also be more recyclable, as its components can be easily disassembled and repurposed. Unlike the model in this thesis, ER directly model the extraction of recyclable materials, making recyclability an inherent property of the production process. This contrasts with this thesis, which assumes a market for recycled inputs, where firms purchase recycled materials rather than extracting them from existing products. If a tax on virgin inputs were implemented in reality, firms would likely face incentives to design products with higher recyclability to reduce reliance on taxed virgin materials. This could even lead to vertical integration of recycling processes, where firms recycle their own products to reduce input costs. Over time, this dynamic could lead to an increasing share of recycled materials in production as they become cheaper relative to virgin inputs. While this perspective is not directly incorporated into the model in this thesis, it is an interesting consideration for the long-run evolution of recycling markets. Unlike the approach taken in this thesis, ER develop a general equilibrium framework, explicitly modeling the representative household’s utility function. Household utility is positively related to consumption of durable goods and negatively related to labor supply, which is endogenously determined by the labor required to produce durable goods. While this broad approach might provide useful insights, it also introduces limitations. Specifically, in their model, the production of recycled materials is labor-intensive, meaning that an increase in recycling reduces household utility by increasing labor requirements. This assumption makes sense in their model, where the entire economy revolves around durable goods production, but it is less applicable to real-world consumer electronics markets, where the production of recycled inputs is unlikely to affect consumer labor supply in any meaningful way, as most consumers do not engage in the production process of the goods they purchase. ER also assume that the price households are willing to pay depends on recyclability, since a higher recyclability increases the residual value of the good when it reaches the end of its life cycle. While this might be relevant for large machinery or automobiles, it seems less applicable to consumer electronics markets. For example, while a modern high-end smartphone can cost well over a thousand euros, its residual material value is negligible in comparison. As a result, recyclability is unlikely to significantly affect consumer willingness to pay, at least through the channel of residual value. Despite these simplifications, ER do extend their analysis to cases where markets for recyclability and durability are absent, either individually or simultaneously. They demonstrate that when at least one of these markets is missing, both recyclability and durability become inefficient, even in the absence of market power and externalities. This is one of their most relevant findings as it concerns this thesis, as it suggests that the set of distortions affecting the socially optimal level of durability may be even broader than previously considered. While ER's work provides valuable insights, their model is both highly complex—attempting to describe a closed economic system—and overly simplistic in some of its assumptions. Perhaps the most problematic assumption for applicability to consumer electronics is their equation of weight with recyclability and durability. This simplification limits the extent to which their findings can inform the role of input mix choices in distorting the optimal tax level beyond the distortions already discussed in this thesis. Additionally, the authors themselves acknowledge limitations in their model, noting that many of their results are indeterminate—meaning that the optimal policy recommendations lack clear directional results. They suggest that further research should focus on more applied general equilibrium models to generate clearer policy implications. Given these limitations, it can only be inferred that incorporating endogenous recycled inputs would introduce additional complexities to the determination of the optimal tax level, further complicating the analysis beyond the three distortions from the Pigouvian tax previously discussed. However, the precise manner in which these additional complexities would manifest remains unclear, highlighting the need for further research in this area.

4.4 INTERTEMPORAL PREFERENCES

While no clear implications could be derived from the paper by (Eichner and Runkel, 2003), it does provide valuable perspectives, particularly regarding the type of intertemporal preferences a social planner should adopt when deriving optimal policy. The authors consider two distinct preference structures. The first follows utilitarian preferences, characterized by a constant discount rate applied uniformly across all future periods. However, this approach leads to a rapid decline in the valuation of future periods. For example, with a 10% annual discount rate, costs incurred 30 years in the future would be valued at less than 5% of their present value. This presents a significant issue when evaluating long-term environmental concerns, such as e-waste, which is both highly toxic and persistent in its ecological effects (Forti et al., 2020). While the utilitarian approach was used in the model in Chapter 3 to reflect the perspective of a profit-maximizing firm, the same assumption may not be appropriate for a social planner, whose objectives depend on normative considerations. The appropriate choice of discounting ultimately depends on the norms guiding policy decisions. There is an argument that democratic decision-makers tend to act with short time horizons(Ogami, 2024), but there is also substantial concern for climate change and long-term environmental risks. If policymakers seek to incorporate such long-term concerns, a constant discount rate may not be the best representation of their preferences. Although Eichner and Runkel (2003) do not explicitly make this argument, they consider an alternative preference structure based on Chichilnisky preferences. Chichilnisky (1996) introduces two axioms that a welfare criterion must satisfy to be consistent with sustainable development. The first axiom states that "a welfare criterion used to evaluate sustainable development must be a complete order over utility paths which does not assign a dictatorial role either to the present or to the future, and which increases with increases in the welfare of any generation." This directly contradicts utilitarian discounting, which assigns nearly zero weight to the welfare of future generations, making current generations disproportionately influential in decision-making. The second axiom requires that "the welfare criterion should not be dictated by the long-run future, and thus requires sensitivity to the present." This rules out approaches that completely ignore discounting, as they would make present concerns irrelevant when optimizing over an infinite horizon. Even a substantial increase in the welfare of a finite number of generations would be inconsequential compared to a small improvement spread across an infinite number of future generations. In such a scenario, no finite number of present or near-future generations could meaningfully impact the overall welfare calculation. Using these axioms, Chichilnisky (1996) derives a welfare criterion that balances present and future welfare considerations, leading to policy paths that differ significantly from those implied by standard utilitarian discounting. In particular, natural resources with high long-term value—such as clean drinking water—are significantly undervalued under conventional discounting approaches. This has important implications for e-waste, as its long-lasting toxicological effects may not be adequately considered when applying short-term discounting. Imagine, for example, the value of clean drinking water. Such an essential part of human life would have a very large value in the long run, which might not be considered properly when generating and disposing of e-waste, which might have long-lasting toxicological effects on the ecosystem, including freshwater sources (Forti et al., 2020).

A related concept, extinction discounting, was explored in a later paper co-authored by Chichilnisky (Chichilnisky et al., 2020). This approach aligns with the previously discussed axioms but introduces a stochastic extinction process, which is strictly exogenous and does not consider self-inflicted extinction risks. The key idea is that any individual is equally likely to be born into any generation, similar to the "veil of ignorance" concept in welfare economics. Without modification, this assumption would lead to the same issues as pure non-discounting. However, by introducing exogenous extinction risk, the model incorporates a finite expected number of future generations. Since human civilization is unlikely to survive indefinitely—due to risks from cosmic events and eventually the death of the universe—only the welfare of actual future individuals is considered in social welfare maximization. The authors argue that this process is based on reality, as humanity will be unable to survive the eventual death of all stars in the universe and will inevitably face extinction due to external, non-human-driven events. Based on this, they assume that the utility for everyone out of the infinitely many humans who might have been born but were not before extinction can be normalized to zero. Therefore, only the utility of those humans actually born will be relevant to the maximization problem. This approach maintains sensitivity to intergenerational equity while ensuring that present welfare is not entirely overshadowed by the needs of an infinite future. Given the long-term consequences of e-waste accumulation, these alternative discounting models may be more appropriate when considering environmental policy design than conventional utilitarian frameworks. While this is just an exemplary view into the field of welfare criteria within the vast field of welfare economics, it illustrates that the choice of how to model intertemporal costs and benefits might complicate the derivation of optimal policy even further

5 CONCLUSION

This thesis provided an extensive overview of the development of durable goods theory, particularly from the late 1960s to the early 2000s, with a focus on its relevance to consumer electronics markets. The analysis began with a review of early literature on whether a monopolist would choose a lower durability than what would emerge in a competitive market. This was followed by an examination of Coase's (1972) time inconsistency problem, which proposed that a monopolist’s market power would diminish due to consumer expectations about future price reductions. However, subsequent research demonstrated that Coase's (1972) extreme case does not occur when durable goods depreciate rather than being perfectly durable. The thesis then explored the role of asymmetric information in secondary markets, examining how market dynamics change when the primary and secondary markets are considered simultaneously. Three distinct motivations for planned obsolescence were derived from the literature: (1) planned obsolescence to overcome time inconsistency, (2) planned obsolescence as a result of time inconsistency, and (3) planned obsolescence as a second-degree price discrimination strategy when facing heterogeneous consumers. These theoretical insights were then applied to markets for consumer electronics, where firms may actively reduce durability to influence repurchase behavior. Following this literature review, the thesis introduced and analyzed a formal monopoly model, in which a producer simultaneously decides on product lifetime, pricing, and the share of recycled inputs while facing both an incentive for planned obsolescence and a tax on virgin inputs. The model demonstrated a positive relationship between the tax rate and product lifetime when planned obsolescence was initially present. However, the analysis was limited to the firm’s optimization problem, and did not explicitly incorporate the social planner’s perspective. While no formal extension to the model was made in the final chapter, it included an extensive discussion on the factors influencing the socially optimal tax in a durable goods sales market. The analysis considered both the environmental externality of e-waste and the presence of producers with market power, who have an incentive to engage in planned obsolescence. Additionally, the chapter explored how the endogenous decision on recycled inputs might affect the optimal tax. Finally, the discussion examined how different intertemporal welfare criteria influence optimal policy design.

What I find very interesting is that when looking at the classical literature, there is a considerable discussion of asymmetric information on secondary markets. The same, however, is not true for primary markets. While there is one paper discussing asymmetric information about production technology, there is no research on the role of how well consumers are informed about the actual durability of a product. I believe that this represents a significant gap in the study of durable goods. This is especially interesting because, as argued in this thesis, it is likely that producers of some consumer electronics could have much more granular control over the effective durability of their products through software updates. While the classical literature considered how new product introductions might make used products obsolete by lowering their value on secondary markets, the use of malicious updates or the ending of software support altogether can affect the durability of a product directly, rather than indirectly making it worth less compared to newer products.[19] While this could relate to the idea of non-compatible new product introductions when network externalities are present, the role of software updates in directly influencing the durability of durable goods appears to be a particularly promising area for future research. If producers therefore can control the durability of their products even after the point of sale, consumers inherently cannot be perfectly informed about the lifetime of the products they purchase. This observation is highly relevant for the model presented in this thesis. Specifically, I found that the lower the parameter C, the relatively better the introduction of a tax on virgin inputs is from a consumer welfare perspective. Poorly informed consumers, who have limited ability to differentiate product durability at the time of purchase, would characterize a market with a low parameter C, as they would be unwilling to pay a premium for increased product lifetimes. While the model developed in Chapter 3 did not explicitly consider the producer's ability to alter the lifetime of period 1 products before sales in period 2, this represents an interesting perspective worthy of further exploration. It is also noteworthy that any planned obsolescence strategy arising from time inconsistency or aimed at avoiding time inconsistency inherently results in lower profits for the monopolist than if commitment were possible. Given this, it is intriguing to observe that rental markets for consumer electronics are quite rare,

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[19] While new product introductions can shorten a product’s effective lifetime—such as when consumers replace older models due to stylistic preferences—this does not necessarily render the product nonfunctional. In contrast, discontinuing software updates can make a product entirely unusable for its intended purpose. This not only reduces the product’s effective lifetime but also limits the duration for which it remains viable for its original function.

despite renting outputs entirely circumventing the issue of time inconsistency. If software updates exacerbate time inconsistency, it would logically become even more beneficial to rent rather than sell consumer electronics. Investigating the motivations behind companies' reluctance to adopt rental models could be a fruitful direction for empirical or qualitative research. From a theoretical perspective, it might be interesting to further explore planned obsolescence strategies when considering heterogeneous consumers. While challenging, I intend to extend the basic framework presented in Chapter 3 to include discrete or continuous consumer heterogeneity. Currently, the literature typically focuses solely on heterogeneity in consumer valuation. However, extending this to consider heterogeneity in consumer switching costs appears to be a promising avenue for future research, though it may necessitate different methodological approaches. Although I aim to extend my analysis to optimal taxation by incorporating a social planner's perspective into the model, based on the complexities discussed in Chapter 3, this might prove to be quite complicated and thus represents another important avenue for future investigation.

Overall, while the classical literature on durable goods continues to offer meaningful foundational insights, it is clear that significant potential remains for further theoretical advancement. Adapting and expanding these frameworks to address the nuanced dynamics of contemporary consumer electronics markets could be a particularly promising direction for future research.

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ACKNOWLEDGEMENT OF THE USE OF SOFTWARE AND AI

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Mathematica was used for all calculations for the model in Chapter 3 as well as the Appendix.

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APPENDIX

Derivatives of $\frac{\partial L^*}{\partial t}$:

- $\frac{\partial(\frac{\partial L^*}{\partial t})}{\partial t} = \frac{B(1+\delta)(C-d\delta)}{-2C^2\alpha+4\alpha(Cd+2B(C+d)^2\beta+2B^2(C+d)^2\beta^2)\delta-2d^2\alpha\delta^2} < 0$
    - The numerator is strictly negative because B and $(1+\delta)$ are positive while the factor $(C-d\delta)$ is negative. For the denominator, note that it comprises negative terms $-2C^2\alpha$ and $-2d^2\alpha\delta^2$ and a positive term $4\alpha(Cd+2B(C+d)^2\beta+2B^2(C+d)^2\beta^2)\delta$. In particular, the negative element $-2C^2\alpha$ is outweighed by the positive term $4\alpha Cd\delta$, and the negative element $-2d^2\alpha\delta^2$ is exceeded by the positive term $4\alpha 2B(C+d)^2\beta\delta$. Thus, each negative component is counterbalanced by a larger positive component, ensuring that the overall denominator is strictly positive, and therefore the entire fraction is negative.
- $\frac{\partial(\frac{\partial L^*}{\partial t})}{\partial A} = 0$
    - A is not present in the first order derivative
- $\frac{\partial(\frac{\partial L^*}{\partial t})}{\partial B} = \frac{(t-2\alpha)(1+\delta)(-C+d\delta)(2Cd(-1+4B^2\beta^2)\delta+d^2\delta(4B^2\beta^2+\delta)+C^2(1+4B^2\beta^2\delta))}{2\alpha(C^2-2(Cd+2B(C+d)^2\beta+2B^2(C+d)^2\beta^2)\delta+d^2\delta^2)^2} < 0$
    - The denominator is positive because the bracketed expression is squared and then multiplied by 2α. In the numerator, $(1+\delta)$, $(-C+d\delta)$, and the large bracketed term are positive, while $(t-2\alpha)$ is negative. Therefore, the only negative factor is $(t-2\alpha)$, which makes the overall numerator negative. Dividing a negative numerator by a positive denominator gives a strictly negative fraction.
- $\frac{\partial(\frac{\partial L^*}{\partial t})}{\partial C} = \frac{B(t-2\alpha)(1+\delta)\left(C^2(1-4B\beta(1+B\beta)\delta)+2Cd\delta(-1+4B\beta(1+B\beta)\delta)+d^2\delta\left(\delta+4B\beta(1+B\beta)(1+2\delta)\right)\right)}{2\alpha(C^2-2(Cd+2B(C+d)^2\beta+2B^2(C+d)^2\beta^2)\delta+d^2\delta^2)^2} <$
0
    - The denominator is positive because the entire bracketed expression is squared and then multiplied by 2α. In the numerator, the factors B and $(1+\delta)$

are positive, and the large bracketed term is also positive, while $(\mathrm{t}-2\alpha)$ is negative. Thus, the only negative factor is $(\mathrm{t}-2\alpha)$, which makes the overall numerator negative. To explain why the large bracketed term is positive: it contains two negative components. The first negative component, $-4\mathrm{B}\beta(1+\mathrm{B}\beta)\delta\mathrm{C}^2$, is outweighed by the corresponding positive component, $\mathrm{d}^2\delta4\mathrm{B}\beta(1+\mathrm{B}\beta)(1+2\delta)$, since $\mathrm{d}^2\delta>\mathrm{C}^2$ and $(1+2\delta)>1$. The second negative component, $(-1+4\mathrm{B}\beta(1+\mathrm{B}\beta)\delta)$, is overall positive because $4\mathrm{B}\beta(1+\mathrm{B}\beta)\delta)>1$.. Dividing a negative numerator by a positive denominator yields a strictly negative fraction.

- $\frac{\partial(\frac{\partial\mathrm{L}^*}{\partial\mathrm{t}})}{\partial\mathrm{d}}=\frac{-\mathrm{B}(\mathrm{t}-2\alpha)\delta(1+\delta)((2\mathrm{Cd}-\mathrm{d}^2\delta)(4\mathrm{B}\beta+4\mathrm{B}^2\beta^2-\delta)+\mathrm{C}^2(1+4\mathrm{B}\beta(2+\delta)+4\mathrm{B}^2\beta^2(2+\delta)))}{2\alpha(2\mathrm{Cd}(1+2\mathrm{B}\beta)^2\delta+\mathrm{d}^2(4\mathrm{B}\beta+4\mathrm{B}^2\beta^2-\delta)\delta+\mathrm{C}^2(-1+4\mathrm{B}\beta\delta+4\mathrm{B}^2\beta^2\delta))^2}$
  - The denominator here is strictly positive because the bracketed term is squared. The numerator has one strictly negative multiplicative element, $(\mathrm{t}-2\alpha)$. Due to the minus sign in the beginning the overall sign of the numerator and therefore the entire fraction depends on the big right hand bracketed term, in that the sign of that bracketed term is the sign of the numerator and therefore the entire fraction. Since $4\mathrm{B}\beta+4\mathrm{B}^2\beta^2-\delta$ is strictly positive there will be only positive additive elements if $2\mathrm{Cd}-\mathrm{d}^2\delta$ is also positive. This is the case as long as $2\mathrm{Cd}>\mathrm{d}^2\delta$ or equivalently $\mathrm{C}>\frac{\mathrm{d}\delta}{2}$. Which is always true when d is at most twice as large as C. However, this does not mean that this condition can be reversed to show that the bracketed term is negative, because there is another strictly positive additive term $\mathrm{C}^2(1+4\mathrm{B}\beta(2+\delta)+4\mathrm{B}^2\beta^2(2+\delta))$. This means $\mathrm{C}>\frac{\mathrm{d}\delta}{2}$ is a sufficient but not a necessary condition for $\frac{\frac{\partial\mathrm{L}}{\partial\mathrm{t}}}{\partial\mathrm{d}}$ to be positive. It is therefore still be possible for the entire fraction to be positive even if this condition is not met. However the full condition is not really interpretable due to its complexity.
- $\frac{\partial(\frac{\partial\mathrm{L}^*}{\partial\mathrm{t}})}{\partial\alpha}=\frac{-\mathrm{Bt}(1+\delta)(-\mathrm{C}+\mathrm{d}\delta)}{2\alpha^2(\mathrm{C}^2-2(\mathrm{Cd}+2\mathrm{B}(\mathrm{C}+\mathrm{d})^2\beta+2\mathrm{B}^2(\mathrm{C}+\mathrm{d})^2\beta^2)\delta+\mathrm{d}^2\delta^2)}>0$
  - The numerator here is clearly negative since all elements of the product are individually positive which turns the entire numerator negative due to the

minus sign. The denominator is negative due to the big right hand bracketed term. The positive additive elements are C^2 and $d^2\delta^2$. The first term has an absolute value which is strictly smaller than $-2Cd\delta$ because $Cd\delta > C^2$. The second term has an absolute value smaller than $\delta(C+d)^2 4(B\beta + B^2\beta^2)$ because $4(B\beta + B^2\beta^2) > 8 > \delta$ and because $\delta(C+d)^2 > d^2\delta$. This means that the negative elements outweigh the positive elements which makes the entire bracket and therefore the denominator negative.

- $\frac{\partial(\frac{\partial L^*}{\partial t})}{\partial \beta} = \frac{-2B^2(C+d)^2(t-2\alpha)(1+2B\beta)\delta(1+\delta)(C-d\delta)}{\alpha(C^2-2(Cd+2B(C+d)^2\beta+2B^2(C+d)^2\beta^2)\delta+d^2\delta^2)^2} < 0$
    - The denominator here is strictly positive because the bracketed term is squared. The numerator is strictly negative because 2 of the multiplicative elements are negative, $(t-2\alpha)$ and $(C-d\delta)$. This combined with the minus means that the overall numerator and therefore the entire fraction is negative.
- $\frac{\partial(\frac{\partial L^*}{\partial t})}{\partial \delta} = \frac{-B(C+d)(t-2\alpha)((C+2BC\beta)^2+d^2(1+2B\beta)^2\delta^2+2Cd(-\delta+2B\beta(1+\delta^2)+2B^2\beta^2(1+\delta^2)))}{2\alpha(2Cd(1+2B\beta)^2\delta+d^2(4B\beta+4B^2\beta^2-\delta)\delta+C^2(-1+4B\beta\delta+4B^2\beta^2\delta))^2} > 0$
    - The denominator here is strictly positive because the bracketed term is squared. The numerator is strictly positive, because only one of the multiplicative elements is negative, $(t-2\alpha)$. The big right hand bracketed term is positive because it has only one negative additive element within the bracketed term $(-\delta + 2B\beta(1+\delta^2) + 2B^2\beta^2(1+\delta^2))$ which is positive overall because $2B\beta(1+\delta^2) + 2B^2\beta^2(1+\delta^2) > 4 > \delta$. This combined with the minus sign in the beginning means that the numerator and therefore the overall fraction is positive.

Derivatives of $\frac{\partial p^*}{\partial t}$:

- $\frac{\partial(\frac{\partial p^*}{\partial t})}{\partial t} = \frac{C^2-2\left(Cd+B(C+d)^2\beta\right)\delta+d^2\delta^2}{-2C^2\alpha+4\alpha(Cd+2B(C+d)^2\beta+2B^2(C+d)^2\beta^2)\delta-2d^2\alpha\delta^2} < 0$
    - The numerator is strictly negative because it has two positive additive elements, C^2 and $d^2\delta^2$. Both have a lower absolute value than the negative additive elements $-2Cd\delta$ and $-2B(C+d)^2\beta\delta$ respectively. The denominator is strictly positive because it has two negative additive elements,

$-2C^2\alpha$ and $-2d^2\alpha\delta^2$, which have smaller absolute values than the positive additive elements $4\alpha Cd$ and $4\alpha 2B(C+d)^2\beta\delta$ respectively. This makes the entire denominator positive and the entire fraction negative

- $\frac{\partial(\frac{\partial p^*}{\partial t})}{\partial A} = 0$
    - A is not present in the first order derivative
- $\frac{\partial(\frac{\partial p^*}{\partial t})}{\partial B} = \frac{(C+d)^2(t-2\alpha)\beta\delta\left(2Cd(1+2B\beta)^2\delta+d^2\delta(4B^2\beta^2-\delta-4B\beta\delta)+C^2(-(1+4B\beta)+4B^2\beta^2\delta)\right)}{\alpha\left(2Cd(1+2B\beta)^2\delta+d^2(4B\beta+4B^2\beta^2-\delta)\delta+C^2(-1+4B\beta\delta+4B^2\beta^2\delta)\right)^2}$
    - The denominator here is strictly positive because the bracketed term is squared. The numerator has one strictly negative multiplicative element, $(t-2\alpha)$. The overall sign therefore depends on the big right hand bracketed term, in that the overall expression has the opposite sign to it. The bracketed term has 3 negative additive elements. One of them is $-C^2(1+4B\beta)$, which has a strictly lower absolute value than the positive additive element $2Cd(1+2B\beta)^2\delta$. The other two are within the bracketed term $(4B^2\beta^2-\delta-4B\beta\delta)$ This will be positive overall as long as $\frac{4B^2\beta^2}{1+4B\beta} > \delta$ which is always true as long as $\delta < 0.8$ or if $B\beta > \frac{1}{2}+\frac{1}{\sqrt{2}} \approx 1.2071$. Both of which seem likely to be true.
- $\frac{\partial(\frac{\partial p^*}{\partial t})}{\partial C} = \frac{2Bd(C+d)(t-2\alpha)\beta(1+2B\beta)\delta(1+\delta)(C-d\delta)}{\alpha(C^2-2(Cd+2B(C+d)^2\beta+2B^2(C+d)^2\beta^2)\delta+d^2\delta^2)^2} > 0$
    - The denominator here is strictly positive because the bracketed term is squared. The numerator is strictly positive, because it has two negative multiplicative elements, $(t-2\alpha)$ and $(C-d\delta)$. This makes the entire fraction positive.
- $\frac{\partial(\frac{\partial p^*}{\partial t})}{\partial d} = \frac{-2BC(C+d)(t-2\alpha)\beta(1+2B\beta)\delta(1+\delta)(C-d\delta)}{\alpha(C^2-2(Cd+2B(C+d)^2\beta+2B^2(C+d)^2\beta^2)\delta+d^2\delta^2)^2} < 0$
    - The denominator here is strictly positive because the bracketed term is squared. The numerator is negative because it has two negative multiplicative elements, $(t-2\alpha)$ and $(C-d\delta)$. This combined with the minus sign in the beginning makes the numerator and therefore the entire fraction negative.

- $\frac{\partial(\frac{\partial p^*}{\partial t})}{\partial \alpha} = \frac{t(C^2-2(Cd+B(C+d)^2\beta)\delta+d^2\delta^2)}{2\alpha^2(C^2-2(Cd+2B(C+d)^2\beta+2B^2(C+d)^2\beta^2)\delta+d^2\delta^2)} > 0$
  - The numerator here is clearly negative because one of the multiplicative elements is negative. This is the big right hand bracketed term which is negative because it has two positive additive elements, C^2 and $d^2\delta^2$. These have strictly lower absolute values than the negative additive terms, $-2Cd\delta$ and $\delta(C+d)^2 2B\beta$ respectively. The denominator is negative due to the big right hand bracketed term. The positive additive elements are C^2 and $d^2\delta^2$. The first term has an absolute value which is strictly smaller than $-2Cd\delta$ because $Cd\delta > C^2$. The second term has an absolute value smaller than $\delta(C+d)^2 4(B\beta + B^2\beta^2)$ because $4(B\beta + B^2\beta^2) > 8 > \delta$ and because $\delta(C+d)^2 > d^2\delta$. This means that the negative elements outweigh the positive elements which makes the entire bracket and therefore the denominator negative. This makes the entire fraction positive.
- $\frac{\partial(\frac{\partial p^*}{\partial t})}{\partial \beta} = \frac{B(C+d)^2(t-2\alpha)\delta\left(2Cd(1+2B\beta)^2\delta+d^2\delta(4B^2\beta^2-\delta-4B\beta\delta)+C^2(-(1+4B\beta)+4B^2\beta^2\delta)\right)}{\alpha\left(2Cd(1+2B\beta)^2\delta+d^2(4B\beta+4B^2\beta^2-\delta)\delta+C^2(-1+4B\beta\delta+4B^2\beta^2\delta)\right)^2}$
  - The denominator here is strictly positive because the bracketed term is squared. The numerator has one strictly negative multiplicative element, $(t-2\alpha)$. The overall sign therefore depends on the big right hand bracketed term, in that the overall expression has the opposite sign to it. The bracketed term has 3 negative additive elements. One of them is $-C^2(1+4B\beta)$, which has a strictly lower absolute value than the positive additive element $2Cd(1+2B\beta)^2\delta$. The other two are within the bracketed term $(4B^2\beta^2 - \delta - 4B\beta\delta)$ This will be positive overall as long as $\frac{4B^2\beta^2}{1+4B\beta} > \delta$ which is always true as long as $\delta < 0.8$ or if $B\beta > \frac{1}{2} + \frac{1}{\sqrt{2}} \approx 1.2071$. Both of which seem likely to be true.
- $\frac{\partial(\frac{\partial p^*}{\partial t})}{\partial \delta} = \frac{-B(C+d)^2(t-2\alpha)\beta(1+2B\beta)(C-d\delta)(C+d\delta)}{\alpha(C^2-2(Cd+2B(C+d)^2\beta+2B^2(C+d)^2\beta^2)\delta+d^2\delta^2)^2} < 0$
  - The denominator here is strictly positive because the bracketed term is squared. The numerator is negative because it has two negative multiplicative

elements, $(\mathrm{t}-2\alpha)$ and $(\mathrm{C}-\mathrm{d}\delta)$. This combined with the minus sign in the beginning makes the numerator and therefore the entire fraction negative.