Self-evaluation test - solutions

Mathematics

Task 1: Write down a general polynomial equation.

$$\text{A general form: } P(x) = \sum_{i=0}^{n} a_i x^i = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$$

Example: $y = 3x^2 + 2x - 5$

Task 1: Write down a differential equation.

$$\text{A general form: } F\left(x, y(x), \frac{dy(x)}{dx}, ..., \frac{d^n(y(x))}{dx^n}\right)$$

Examples: $x + 2y = \frac{dy}{dx}$, Schrödinger equation

Task 3: Differentiate the following functions:

$$\frac{df(x)}{dx} = \frac{d(x^2 \cos(x))}{dx} = 2x\cos(x) - x^2 \sin(x)$$

using the product rule $(uv)' = u'v + uv'$

Task 4: Calculate the following integrals:

$$\int_0^1 xe^x dx = xe^x \Big|_0^1 - \int_0^1 e^x dx = xe^x \Big|_0^1 - e^x \Big|_0^1 = e^x(x-1) \Big|_0^1 = 1$$

using integration by parts: $u = x, du = dx, dv = e^x dx, v = e^x : \int uv'dx = uv - \int vu'dx$

Task 5: There are two ways to express a complex number z. Please write them down. How are they connected?

$$z = a + ib$$

with $a, b \in R$

$$z = re^{i\phi}$$

with $r = \sqrt{a^2 + b^2}$, $\phi = \arctan\left(\frac{b}{a}\right)$, $e^{i\phi} = \cos\phi + i\sin\phi$

Task 6: Simplify the following expression:

$$\frac{4+3i}{1+2i} = \frac{(4+3i)(1-2i)}{(1+2i)(1-2i)} = \frac{4-8i+3i+6}{1-2i+2i+4} = 2-i$$

Task 7: Calculate the result of the following expression:

$$\begin{pmatrix} 3 & 2 & 4 \\ 5 & 8 & 3 \\ 4 & 5 & 7 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 6+2+20 \\ 10+8+15 \\ 8+5+35 \end{pmatrix} = \begin{pmatrix} 28 \\ 33 \\ 48 \end{pmatrix}$$

Physics

Task 1: If a metal is cooled down, how does this affect the conductivity (stays unchanged, increases or decreases)? Explain your answer - what is the physics behind it?

If a metal is cooled down, its conductivity increases. A lower temperature leads to less electron-phonon scattering which is exhibited as an increased conductivity.

Task 2: What is the name of the fundamental equations of electromagnetism? Can you write down any?

Maxwell equations:

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$$
$$\nabla \cdot B = 0$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

Task 3: Please explain in simple words what a semiconductor is?

A semiconductor is a material, which has a higher resistivity than a conductor and a lower resistivity than an insulator. In contrast to the above-discussed metals, the conductivity of semiconductors increase with the temperature. Further, the resistivity can be modified by the controlled introduction of impurity atoms ("doping"). By doping semiconductors with different impurity atoms, devices like diodes and transistors can be realized. The most wide-spread semiconductor is silicon, which is the basis of most modern electronics. Further, semiconductors can be used to build lasers which are also a key element for consumer electronics (CD, DVD, Blu-Ray) and telecommunication.