

Models of Income Distributions for Knowledge Discovery

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Descriptions of income distributions using a single distribution, like Lognormal or Gamma are often quite poor in describing the tails of the distribution [1]. This led to separate models for the upper vs. lower parts of income distributions [2]. For example [3-5] describe the high-income region with the Pareto power laws. Other authors model the low to medium income region using Exponential [6], Lognormal [7] or Gamma distributions [8, 9]. The high income range is often modeled using the cumulative distribution function (cdf) [10], whereas the low to medium income regions are modeled using the probability density function (pdf) [11]. Usually no systematic limits between low, medium and high income are defined [3]. A goal for a valid and suitable model for income distributions is to derive a theory of the mechanisms which operate in a society (Computational Social Science) and explains the observed distribution [12]. Here a model for income distributions as a mixture of components is proposed. The model is derived using the Pareto Density Estimation (PDE) [14] for an estimation of the pdf. PDE has been designed in particular to identify groups/classes in a dataset [13]. Precise limits for the classes can be calculated using the theorem of Bayes. Our model suggests that there are different groups/classes in a society, which contribute to the total distribution of income in their own way. The approach is demonstrated on several real world data sets including actual income data from Germany.

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