Traces of shearlet coorbit spaces on domains

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Abstract

We study traces of certain subspaces of shearlet coorbit spaces on smooth domains in $\mathbb{R}^d$ with $d = 2, 3$. Our results are based on embedding theorems into Besov spaces which enable us to establish embedding relations of traces on the boundary of these domains.

1 Introduction

In recent years shearlets have shown the potential to retrieve directional information so that they became interesting for various applications. Moreover, quite surprisingly, the shearlet transform has the outstanding property to stem from a square integrable group representation [1]. This remarkable fact provides the opportunity to design associated canonical smoothness spaces, so-called shearlet coorbit spaces [4,5] by applying the general coorbit theory derived by Feichtinger and Gröchenig [6–9]. To understand the structure of shearlet coorbit spaces and in view of possible applications it would be desirable to know how these new spaces behave under trace operations.

The aim of this short note is to provide a first approach on how to derive traces for shearlet coorbit spaces on smooth domains. First trace results for hyperplanes have been established in [2,3]. Therefore, our hope was that these results carry over to boundaries of smooth domains. However, it turns out that the corresponding problem on domains is somewhat delicate to solve since a lot of standard tools are not available in the framework of shearlet coorbit spaces.

One natural idea would be to use the fact that shearlet coorbit spaces are can be embedded into sums of homogeneous Besov spaces $\dot{B}^s_{p,q}(\mathbb{R}^d)$. However, these spaces lack a lot of properties in comparison with their inhomogeneous counterparts, e.g. smooth functions are not multipliers in these spaces and they turn out not to be invariant with respect to diffeomorphisms, which are necessary properties in order to reduce the problem of determining traces on domains to the corresponding problem on hyperplanes.

To overcome these difficulties, our trace results are established for certain subspaces of shearlet coorbit spaces within the framework of L_p-spaces. We make use of the embeddings into sums of homogeneous Besov spaces and the fact that

$$\dot{B}^s_{p,q}(\mathbb{R}^d) \cap L_p(\mathbb{R}^d) = B^s_{p,q}(\mathbb{R}^d), \qquad s > 0,$$

$1 \le p \le \infty$, and $1 \le q \le \infty$. Concerning the homogeneous spaces we refer to [10, Ch. 5]. This enables us to benefit from trace results for inhomogeneous Besov spaces $B^s_{p,q}$ on domains, which are well-known. Our main results are stated in Theorem 3.1.

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2 Shearlet Coorbit Spaces

In this section, we recall basic facts from shearlet coorbit theory [3, 5] which are necessary to understand our new trace results. For $a \in \mathbb{R}^* := \mathbb{R} \setminus \{0\}$ and $s \in \mathbb{R}^{d-1}$, let

$$A_a := \begin{pmatrix} a & 0_{d-1}^{\mathrm{T}} \\ 0_{d-1} & \operatorname{sgn}(a)|a|^{\frac{1}{d}} I_{d-1} \end{pmatrix} \quad \text{and} \quad S_s := \begin{pmatrix} 1 & s^{\mathrm{T}} \\ 0_{d-1} & I_{d-1} \end{pmatrix}$$

be the *parabolic scaling matrix* and the *shear matrix*, respectively, where $\operatorname{sgn}(a)$ denotes the sign of a. The *shearlet group* $\mathbb{S}$ is defined to be the set $\mathbb{R}^* \times \mathbb{R}^{d-1} \times \mathbb{R}^d$ endowed with the group operation

$$(a, s, t)\,(a', s', t') = (aa', s + |a|^{1-1/d}s', t + S_s A_a t').$$

The left-invariant Haar measure of $\mathbb{S}$ is given by $\mu_{\mathbb{S}} = |a|^{-d-1}\,da\,ds\,dt$. The mapping $\pi : \mathbb{S} \to \mathcal{U}(L_2(\mathbb{R}^d))$ defined by $\pi(a, s, t)\,\psi(x) := |\det A_a|^{-\frac{1}{2}}\psi(A_a^{-1}S_s^{-1}(x - t))$ is a unitary representation of $\mathbb{S}$, see [4, 5]. It is also *square integrable*, i.e., it is irreducible and there exists a nontrivial *shearlet* $\psi \in L_2(\mathbb{R}^d)$ fulfilling the *admissibility condition*

$$\int_{\mathbb{S}} |\langle f, \pi(a, s, t)\psi\rangle|^2\, d\mu_{\mathbb{S}}(a, s, t) < \infty.$$

For a shearlet ψ the transform $\mathcal{SH}_\psi : L_2(\mathbb{R}^d) \to L_2(\mathbb{S})$ defined by

$$\mathcal{SH}_\psi(f)(a, s, t) := \langle f, \pi(a, s, t)\psi\rangle,$$

is called *continuous shearlet transform.*

Let w be a real-valued, continuous and submultiplicative weight on $\mathbb{S}$ which fulfills in addition all the coorbit-theory conditions as stated in [9, Section 2.2]. For $1 \le p \le \infty$, consider $L_{p,w}(\mathbb{S}) := \{F \text{ measurable} : Fw \in L_p(\mathbb{S})\}$ with the norm $\|F\|_{L_{p,w}} := \left(\int_{\mathbb{S}} |F(g)w(g)|^p\, d\mu_{\mathbb{S}}(g)\right)^{1/p}$. For a vector ψ contained in

$$\mathcal{A}_w := \{\psi \in L_2(\mathbb{R}^d) : \mathcal{SH}_\psi(\psi) = \langle \psi, \pi(\cdot)\psi\rangle \in L_{1,w}(\mathbb{S})\},$$

we introduce the space

$$\mathcal{H}_{1,w} := \{f \in L_2(\mathbb{R}^d) : \mathcal{SH}_\psi(f) = \langle f, \pi(\cdot)\psi\rangle \in L_{1,w}(\mathbb{S})\},$$

with norm $\|f\|_{\mathcal{H}_{1,w}} := \|\mathcal{SH}_\psi(f)\|_{L_{1,w}(\mathbb{S})}$ and its anti-dual $\mathcal{H}_{1,w}^\sim$, the space of all continuous conjugate-linear functionals on $\mathcal{H}_{1,w}$. The spaces $\mathcal{H}_{1,w}$ and $\mathcal{H}_{1,w}^\sim$ are π-invariant Banach spaces with continuous embedding $\mathcal{H}_{1,w} \hookrightarrow L_2(\mathbb{R}^d) \hookrightarrow \mathcal{H}_{1,w}^\sim$. Then the following sesquilinear form on $\mathcal{H}_{1,w}^\sim \times \mathcal{H}_{1,w}$ is well-defined:

$$\mathcal{SH}_\psi(f)(a, s, t) := \langle f, \pi(a, s, t)\psi\rangle_{\mathcal{H}_{1,w}^\sim \times \mathcal{H}_{1,w}}.$$

We are interested in the special weights

$$m(a, s, t) = m(a) := |a|^{-r}, \quad r \ge 0$$

and use the abbreviation $L_{p,r}(\mathbb{S}) := L_{p,m}(\mathbb{S})$. Then the following Banach spaces are called *shearlet coorbit spaces*

$$\mathcal{SC}_{p,r}(\mathbb{R}^d) := \{f \in \mathcal{H}_{1,w}^\sim : \mathcal{SH}_\psi(f) \in L_{p,r}(\mathbb{S})\}, \quad \|f\|_{\mathcal{SC}_{p,r}} := \|\mathcal{SH}_\psi(f)\|_{L_{p,r}(\mathbb{S})}.$$

Note that the definition of $\mathcal{SC}_{p,r}(\mathbb{R}^d)$ is independent of the analyzing vector ψ, see [6, Theorem 4.2].

A (countable) family $X = \{g_i := (a_i, s_i, t_i) : i \in \mathcal{I}\}$ in $\mathbb{S}$ is said to be *U-dense* if $\bigcup_{i\in\mathcal{I}} g_i U = \mathbb{S}$, and *separated* if for some compact neighborhood Q of $e = (1, 0, 0) \in \mathbb{R}_* \times \mathbb{R}^{d-1} \times \mathbb{R}^d$ we have $g_i Q \cap g_j Q = \emptyset, i \neq j$, and *relatively separated* if X is a finite union of separated sets. Based on U-dense and relatively separated sets we can state the existence of atomic decompositions of shearlet coorbit spaces [4, 6].

Theorem 2.1 (Atomic decompositions) *Let $1 \leq p \leq \infty$ and $\psi \in \mathcal{B}_w$, $\psi \neq 0$, where $\mathcal{B}_w$ denotes the so-called* ***b****etter subset $\mathcal{B}_w$ of $\mathcal{A}_w$ defined in [3]. Then there exists a (sufficiently small) neighborhood U of e so that for any U-dense and relatively separated set $X = \{(a_i, s_i, t_i) : i \in \mathcal{I}\}$ the set $\{\pi(g_i)\psi)\}$ provides an atomic decomposition for $\mathcal{SC}_{p,r}(\mathbb{R}^d)$: If $f \in \mathcal{SC}_{p,r}(\mathbb{R}^d)$, then*

$$f = \sum_{i\in\mathcal{I}} c_i(f)\pi(a_i, s_i, t_i)\psi,$$

where the sequence of coefficients depends linearly on f and satisfies

$$\|(c_i(f))_{i\in\mathcal{I}}\|_{\ell_{p,r}} \lesssim \|f\|_{\mathcal{SC}_{p,r}}$$

with $\ell_{p,r}$ being defined by $\ell_{p,r} := \{c = (c_i)_{i\in\mathcal{I}} : \|c\|_{\ell_{p,r}} := \|c\,|a|^{-r}\|_{\ell_p} < \infty\}$, where $a = (a_i)_{i\in\mathcal{I}}$. Conversely, if $(c_i)_{i\in\mathcal{I}} \in \ell_{p,r}$, then $f = \sum_{i\in\mathcal{I}} c_i \pi(g_i)\psi$ is in $\mathcal{SC}_{p,r}$ and

$$\|f\|_{\mathcal{SC}_{p,r}} \lesssim \|(c_i)_{i\in\mathcal{I}}\|_{\ell_{p,r}}.$$

It was shown in [3] that for a neighborhood

$$U \supset [\alpha^{\frac{1}{d}-1}, \alpha^{\frac{1}{d}}) \times [-\frac{\beta}{2}, \frac{\beta}{2})^{d-1} \times [-\frac{\tau}{2}, \frac{\tau}{2})^d, \quad \alpha > 1,\ \beta, \tau > 0$$

of the identity $(1, 0, 0) \in \mathbb{R}_* \times \mathbb{R}^{d-1} \times \mathbb{R}^d$, the set

$$X := \left\{(\varepsilon\alpha^{-j}, \beta\alpha^{-j(1-\frac{1}{d})}k, S_{\beta\alpha^{-j(1-\frac{1}{d})}k} A_{\alpha^{-j}}\tau l) : j \in \mathbb{Z}, k \in \mathbb{Z}^{d-1}, l \in \mathbb{Z}^d,\ \varepsilon \in \{-1, 1\}\right\}$$

is U-dense and relatively separated. Without loss of generality, we can restrict our attention to to the case $a > 0$ such that $\varepsilon = +1$. For $a := \alpha^{-j}$, $s := \beta\alpha^{-j(1-\frac{1}{d})}k$ and $t := S_{\beta\alpha^{-j(1-\frac{1}{d})}k} A_{\alpha^{-j}}\tau l$ we use the abbreviation $\psi_{j,k,l} := \pi(a, s, t)\psi$.

By Theorem 2.1, every function $f \in \mathcal{SC}_{p,r}(\mathbb{R}^d)$ can be written as

$$f(x) = \sum_{j\in\mathbb{Z}} \sum_{k\in\mathbb{Z}^{d-1}} \sum_{l\in\mathbb{Z}^d} c(j, k, l)\psi_{j,k,l}(x). \tag{2.1}$$

To derive our embedding theorems, we introduce the following cone-like subspaces of $\mathcal{SC}_{p,r}(\mathbb{R}^d)$: For fixed $\psi \in B_w$, we denote by $\mathcal{SCC}_{p,r}$ the closed subspace of $\mathcal{SC}_{p,r}(\mathbb{R}^d)$ consisting of those functions which are representable as in (2.1) but with integers $|k_i| \leq \alpha^{j(1-\frac{1}{d})}$, $i = 1, \ldots, d-1$:

$$f(x) = \sum_{j\in\mathbb{Z}} \sum_{|k|\leq\alpha^{j(1-\frac{1}{d})}} \sum_{l\in\mathbb{Z}^d} c(j, k, l)\psi_{j,k,l}(x).$$

We will further need the decomposition $f = f_1 + f_2$ given by

$$f_1(x) := \sum_{j\geq 0} \sum_{|k|\leq\alpha^{j(1-\frac{1}{d})}} \sum_{l\in\mathbb{Z}^d} c(j, k, l)\psi_{j,k,l}(x) \tag{2.2}$$

$$f_2(x) := \sum_{j<0} \sum_{l\in\mathbb{Z}^d} c(j, 0, l)\psi_{j,k,l}(x) \tag{2.3}$$

and denote the space containing functions of the form (2.2) by $\mathcal{SCC}^{(1)}_{p,r}(\mathbb{R}^d)$ and those of the form (2.3) by $\mathcal{SCC}^{(2)}_{p,r}(\mathbb{R}^d)$.
The following embedding results of the described subspaces of shearlet coorbit spaces in $\mathbb{R}^2$ and $\mathbb{R}^3$ into (sums of) homogeneous Besov spaces may be found in [3, Thm. 4.1] and [2, Thm. 5.1], respectively.

Theorem 2.2 (Embeddings into Besov spaces)

(i) The embedding $\mathcal{SCC}_{p,r}(\mathbb{R}^2) \subset \dot{B}^{\sigma_1}_{p,p}(\mathbb{R}^2) + \dot{B}^{\sigma_2}_{p,p}(\mathbb{R}^2)$, holds true, where

$$\sigma_1 + \lfloor\sigma_1\rfloor = 2r - \frac{9}{2} + \frac{4}{p} \quad \text{and} \quad \sigma_2 - \frac{\lfloor\sigma_2\rfloor}{2} = r + \frac{3}{2p} + \frac{1}{4}. \tag{2.4}$$

(ii) The embedding $\mathcal{SCC}_{p,r}(\mathbb{R}^3) \subset \dot{B}^{\sigma_1}_{p,p}(\mathbb{R}^3) + \dot{B}^{\sigma_2}_{p,p}(\mathbb{R}^3)$, holds true, where

$$\sigma_1 + 2\lfloor\sigma_1\rfloor = 3r - \frac{21}{2} + \frac{9}{p} \quad \text{and} \quad \sigma_2 - \frac{2}{3}\lfloor\sigma_2\rfloor = r + \frac{5}{3p} + \frac{7}{6}. \tag{2.5}$$

The sums of homogeneous Besov spaces arises from the splitting (2.2) - (2.3). More precisely it is shown that $\mathcal{SCC}^{(1)}_{p,r}(\mathbb{R}^d) \subset \dot{B}^{\sigma_1}_{p,p}(\mathbb{R}^d)$ and $\mathcal{SCC}^{(2)}_{p,r}(\mathbb{R}^d) \subset \dot{B}^{\sigma_2}_{p,p}(\mathbb{R}^d)$, $d = 2, 3$.

3 Trace Results

So far we have only dealt with functions on the whole Euclidean plane. We define spaces on smooth domains $\Omega \subset \mathbb{R}^d$ via restriction, i.e.,

$$\|f\|_{\dot{B}^{\sigma}_{p,p}(\Omega)} := \inf\{\|g\|_{\dot{B}^{\sigma}_{p,p}(\mathbb{R}^d)} : g|_{\Omega} = f\}.$$

In terms of traces on domains in the context of shearlet coorbit spaces we obtain the following theorem.

Theorem 3.1 (Traces on domains)

(i) Let $\Omega \subset \mathbb{R}^2$ be a smooth domain with boundary Γ, $1 \le p \le \infty$, $r > \frac{7}{4} - \frac{1}{p}$, and σ_1 as in (2.4). Then for the trace operator we have

$$\mathrm{Tr}_{\Gamma}\left(\mathcal{SCC}^{(1)}_{p,r} \cap L_p + \mathcal{SCC}^{(2)}_{p,r} \cap L_p\right)(\Omega) \subset B^{\sigma_1 - \frac{1}{p}}_{p,p}(\Gamma).$$

(ii) Let $\Omega \subset \mathbb{R}^3$ be a smooth domain with boundary Γ, $1 \le p \le \infty$, $r > \frac{17}{6} - \frac{2}{p}$, and σ_1 as in (2.5). Then for the trace operator we have

$$\mathrm{Tr}_{\Gamma}\left(\mathcal{SCC}^{(1)}_{p,r} \cap L_p + \mathcal{SCC}^{(2)}_{p,r} \cap L_p\right)(\Omega) \subset B^{\sigma_1 - \frac{1}{p}}_{p,p}(\Gamma).$$

Proof : Since $\mathcal{SCC}_{p,r}(\mathbb{R}^d) = \mathcal{SCC}^{(1)}_{p,r}(\mathbb{R}^d) + \mathcal{SCC}^{(2)}_{p,r}(\mathbb{R}^d)$, where $\mathcal{SCC}^{(1)}_{p,r}(\mathbb{R}^d) \subset \dot{B}^{\sigma_1}_{p,p}(\mathbb{R}^d)$ and $\mathcal{SCC}^{(2)}_{p,r}(\mathbb{R}^d) \subset \dot{B}^{\sigma_2}_{p,p}(\mathbb{R}^d)$ we see that

$$\begin{aligned}
&\mathcal{SCC}^{(1)}_{p,r}(\mathbb{R}^d) \cap L_p(\mathbb{R}^d) + \mathcal{SCC}^{(2)}_{p,r}(\mathbb{R}^d) \cap L_p(\mathbb{R}^d) \\
&\quad \subset \dot{B}^{\sigma_1}_{p,p}(\mathbb{R}^d) \cap L_p(\mathbb{R}^d) + \dot{B}^{\sigma_2}_{p,p}(\mathbb{R}^2) \cap L_p(\mathbb{R}^d) \\
&\quad = B^{\sigma_1}_{p,p}(\mathbb{R}^d) + B^{\sigma_2}_{p,p}(\mathbb{R}^d) \\
&\quad \hookrightarrow B^{\min(\sigma_1,\sigma_2)}_{p,p}(\mathbb{R}^d),
\end{aligned} \tag{3.1}$$

where we used the fact that $B^{\sigma_i}_{p,p}(\mathbb{R}^d) = \dot{B}^{\sigma_i}_{p,p}(\mathbb{R}^d) \cap L_p(\mathbb{R}^d)$ if $\sigma_i > 0$ and $1 \leq p \leq \infty$, cf. Remark 3 in [10, Sect. 5.2.3]. Since the spaces on domains Ω are defined via restriction, we see from (3.1) that

$$\left(\mathcal{SCC}^{(1)}_{p,r} \cap L_p + \mathcal{SCC}^{(2)}_{p,r} \cap L_p\right)(\Omega) \hookrightarrow B^{\min(\sigma_1,\sigma_2)}_{p,p}(\Omega).$$

Concerning traces this yields

$$\mathrm{Tr}_\Gamma \left(\mathcal{SCC}^{(1)}_{p,r} \cap L_p + \mathcal{SCC}^{(2)}_{p,r} \cap L_p\right)(\Omega) \subset \mathrm{Tr}_\Gamma B^{\min(\sigma_1,\sigma_2)}_{p,p}(\Omega) = B^{\min(\sigma_1,\sigma_2)-\frac{1}{p}}_{p,p}(\Gamma),$$

where the results on traces for Besov spaces may be found in [10, Sect. 3.3] and hold true if $\min(\sigma_1, \sigma_2) > \frac{1}{p}$ is satisfied.

Let us show that $\min(\sigma_1, \sigma_2) = \sigma_1$. In $\mathbb{R}^2$ we use the restriction (2.4) and have

$$\sigma_1 + \lfloor\sigma_1\rfloor = 2r - \frac{9}{2} + \frac{4}{p}, \qquad \text{i.e.,} \qquad \sigma_1 = 2r - \frac{9}{2} + \frac{4}{p} - \lfloor\sigma_1\rfloor.$$

This leads to

$$\sigma_1 \leq 2r - \frac{9}{2} + \frac{4}{p} - (\sigma_1 - 1), \qquad \text{i.e.,} \qquad \sigma_1 \leq r - \frac{7}{4} + \frac{2}{p}. \tag{3.2}$$

The condition for σ_2 gives

$$r + \frac{3}{2p} + \frac{1}{4} = \sigma_2 - \frac{\lfloor\sigma_2\rfloor}{2} \leq \sigma_2 - \frac{\sigma_2 - 1}{2}, \qquad \text{i.e.,} \qquad \frac{\sigma_2}{2} \geq r + \frac{3}{2p} - \frac{1}{4},$$

which yields

$$\sigma_2 \geq 2r + \frac{3}{p} - \frac{1}{2}. \tag{3.3}$$

Combining (3.2) and (3.3) we obtain

$$\sigma_1 \leq r - \frac{7}{4} + \frac{2}{p} \leq 2r + \frac{3}{p} - \frac{1}{2} \leq \sigma_2,$$

thus, $\min(\sigma_1, \sigma_2) = \sigma_1$. Finally, for the traces to make sense we require $\frac{1}{p} < \sigma_1 \leq r - \frac{7}{4} + \frac{2}{p}$, which is satisfied for

$$r > \frac{7}{4} - \frac{1}{p}.$$

In $\mathbb{R}^3$ we use the restriction (2.5) and a similar calculation as above gives $\min(\sigma_1, \sigma_2) = \sigma_1$. By the condition on σ_1 we have

$$\sigma_1 + 2\lfloor\sigma_1\rfloor = 3r - \frac{21}{2} + \frac{9}{p}, \qquad \text{i.e.,} \qquad \sigma_1 = 3r - \frac{21}{2} + \frac{9}{p} - 2\lfloor\sigma_1\rfloor.$$

which gives

$$\sigma_1 \leq 3r - \frac{21}{2} + \frac{9}{p} - 2(\sigma_1 - 1), \qquad \text{i.e.,} \qquad \sigma_1 \leq r - \frac{17}{6} + \frac{3}{p}.$$

Again we require $\frac{1}{p} < \sigma_1 \leq r - \frac{17}{6} + \frac{3}{p}$, which is satisfied if

$$r > \frac{17}{6} - \frac{2}{p}.$$

This completes the proof. □

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