The structure of a group is one of the most basic ones in mathematics. It appears in other fields, for instance through the notion of symmetry. Its generality allows one to classify geometric objects (platonic solids, tilings etc.): the natural starting point of such a classification is ‘passing to the symmetry group’.

Conversely, when a family of groups is poorly understood, the other way can be followed: that is, designing a ‘custom made’ family of spaces for which the given groups are sets of symmetries. The main idea of the talk will be to go both ways (i.e. from geometry to groups and vice versa) and, finally, to motivate the introduction of a family of spaces called buildings. In first approximation, the latter spaces are ‘higher-dimensional trees’. Another motivation for buildings is to provide some synthetic proofs of simplicity (a fundamental property in group theory, from the very beginning), and even to produce new simple groups.