Lecture series within the Graduate Programm
”Lie Theory and Complex Geometry“
on

Information Geometry

Panagiotis Konstantis
Fachbereich für Mathematik und Informatik
Philipps–Universität Marburg

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**Abstract**

Information Geometry is concerned with geometric structures of statistical models. For an introduction we will start to study concrete statistical examples to discover their geometric nature. Here, the spaces of interest are submanifolds in the space of probability measures and the Fisher Information Matrix can be used to define a Riemannian metric. Moreover, a lot of worthwhile statistical information like higher order properties of statistical inference are encoded in a family of affine connections.

Based on these examples we will develop an abstract notion of a statistical manifold, which will be defined as a triple \((M, g, D)\) where \(M\) is a smooth finite dimensional manifold, \(g\) a Riemannian metric and \(D\) a symmetric trilinear map. Now a family of connections arises naturally which coincides with the ones explored in the introductory examples. In that way we uncouple the statistical background and survey just the differential geometric features. From here on we will study the geometrical properties of statistical manifolds in terms of those affine connections.

Finally, the series of lectures will close with some interesting examples and open questions on the field of Information Geometry.

**Preliminary Outline**

1. In the first lecture we will recall the basic definitions in statistics and stochastics. In particular we will discuss the definitions of the Fisher Information matrix, tests, estimators and higher–order asymptotics. Afterwards we introduce the notion of a statistical model which is a smooth parametrized family of measures which are all dominated by a fixed probabilistic measure. Finally we finish the first part with some examples of exponential families.

2. The next step will be to discover the geometric nature of statistical models. At the beginning we study the Fisher Information Matrix as a Riemannian metric on the family of probabilistic measures of a statistical model. Moreover a natural family of connections may be defined, which are also known as \(\alpha\)–connections. Now, the crucial fact is that the difference between those \(\alpha\)–connections and the Levi–Civita connection of the Fisher Information matrix induces a symmetric trilinear \(D\) form on the underlying manifold of the statistical model.

3. Based on the geometric objects found in the preceding sessions we uncouple the statistical background and define a statistical manifold as a triple \((M, g, D)\) where \(M\) is a smooth manifold, \(g\) a Riemannian metric and \(D\) a trilinear symmetric map. For every \(\alpha \in \mathbb{R}\) we define the affine connection

\[
\nabla^\alpha := \nabla + \frac{\alpha}{2} D
\]
where $\nabla$ is the Riemannian connection of $g$ and $D$ is considered as a $(2, 1)$–tensor with respect to $g$. The remaining part of this lecture series is devoted to analyze the geometric properties of affine connections. Some keywords will be: conjugate connections, geodesics and curvatures of affine connections and foliations into affine geodesic submanifolds. This part will be divided probably in two separated sessions.

**Requirements**

Basis courses in stochastics and differential geometry are sufficient for this lecture series.

**References**


