Optimal Monetary and Macroprudential Policy in a Currency Union

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Coordination: Bernd Hayo • Philipps-University Marburg
School of Business and Economics • Universitätsstraße 24, D-35032 Marburg
Tel: +49-6421-2823091, Fax: +49-6421-2823088, e-mail: hayo@wiwi.uni-marburg.de
Abstract

The financial crisis proved strikingly that stabilizing the price level is a necessary but not a sufficient condition to ensure macroeconomic stability. The obvious candidate for addressing systemic risk is macroprudential policy. In this paper we study the optimal monetary and macroprudential policy mix in a currency union in the case of different kinds of aggregate and idiosyncratic shocks. The monetary and macroprudential instruments are modelled as independent tools. With a union-wide macroprudential tool, full absorption on the aggregate level is possible, but welfare losses due to fluctuations in relative variables prevail. With country-specific macroprudential tools, full absorption of shocks is always possible. But it is only optimal as long as there is no inefficient labor allocation. Comparing different policy regimes, we get the following ranking in terms of welfare: discretion outperforms strict inflation targeting which outperforms a (euro-area based) Taylor Rule.

JEL-Classification: E32, E44, E58
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†Department of Economics, University of Kassel, Nora-Platiel-Str. 4, D-34127 Kassel, Germany; Tel.: + 49 (0) 561-804-3085; Fax: + 49 (0) 561-804-3083; E-mail: palek@uni-kassel.de.

‡Department of Economics, University of Kassel, Nora-Platiel-Str. 4, D-34127 Kassel, Germany; Tel.: + 49 (0) 561-804-3887; Fax: + 49 (0) 561-804-3083; E-mail: schwanebeck@uni-kassel.de.
1 Introduction

The famous Tinbergen principle states that a given number of targets must be met by at least an equal number of independent instruments (Tinbergen, 1952). Before the recent financial crisis the "divine coincidence" aphorism prevailed the macroeconomic thinking of most central bankers and researchers. It states that a strict inflation targeting policy will even in the presence of certain market imperfections keep output close to its potential. The crisis proved strikingly that stabilizing the price level is a necessary but not a sufficient condition to ensure macroeconomic stability. Many economists as Blanchard et al. (2014) and Woodford (2014) stressed that central bank policy has to move from a one target, one instrument approach to a many targets, many instruments approach. Accordingly, the focus of central banks should be expanded to include not only inflation and output but also financial stability. Several studies have investigated the question whether central bank should include some kind of financial stability measure in an augmented Taylor Rule\footnote{See Käfer (2014) for a literature review on augmented Taylor Rules with financial stability objectives.} However, this inevitably creates new trade-offs between targets for one given instrument - the nominal interest rate. In the sense of Jan Tinbergen policymakers have to expand the instrument set as well. Monetary policy does not need to be the only game in town. Even if monetary policy is able to address financial stability risks to some extent, other measures must be developed as they might be more effective. The main challenge nowadays is the appropriate assignment of instruments to targets. The obvious candidate for addressing systemic risk is macroprudential policy.

The research on macroprudential policy is still at a very early stage, similar to that of monetary policy in the 1940s since at that time it was also not clear how objectives and instruments should be assigned. Financial stability as an objective is a very vague and elusive term and cannot easily be defined or measured. It is therefore necessary to find a proxy or indicator for financial stability especially when trying to quantify the welfare costs of the distortions caused by system risks. In the literature asset prices (e.g. Bernanke and Gertler, 1999; Cecchetti et al., 2002), credit aggregates (e.g. Agénor et al., 2013; Christiano et al., 2010), credit spreads or leverage (e.g. Cúrdia and Woodford, 2009; Carlstrom et al., 2010; De Paoli and Paustian, 2013; Ueda and Valencia, 2014; Smets, 2014) have evolved as tangible measures.

In this paper we study welfare-based monetary and macroprudential policy in a two-country currency union model. In the context of the euro-area, the interaction of monetary and macroprudential policy is of particular interest. With the introduction of the Single Supervisory Mechanism (SSM) the ECB already regulates all significant credit institutions of its member countries besides conducting the common monetary...
Along the standard New Keynesian distortions coming from price rigidity and monopolistic competition, our model is characterized by financial frictions. We do not explicitly model these distortions in an interbank market setup (as in Gertler and Karadi, 2011 or Dellas et al., 2014) but choose the cost channel approach of Ravenna and Walsh (2006) as a shortcut. In Ravenna and Walsh (2006), the interest rate enters the marginal costs as firms have to borrow in order to finance the wage bill. We adapt their framework by assuming that firms must back the borrowing costs with collateral. In order to simplify the analysis, households do not face such a constraint. The Lagrange multiplier associated with the firms’ credit constraint, i.e. the shadow price of borrowing, can be interpreted as a spread between the risk-free interest rate and the borrowing rate. Hence, the quadratic loss functions features along the standard target variables, (aggregate) inflation and output, also (country-specific) credit market distortions. Moreover, in a currency union, a terms of trade gap and the national inflation rates emerge in the welfare criterion of the central bank complicating the optimal policy design compared to the closed economy framework of Carlstrom et al. (2010) and De Paoli and Paustian (2013). The advantage of macroprudential policy over an interest-rate policy is the possibility to design it almost infinitely granular. The nominal interest rate is too blunt of a tool to be cost-effective. Macropudential tools instead can be targeted to specific markets, geographical areas or loan-types. Hence, we show how the introduction of (country-specific) macroprudential policy alters the conduct of the optimal central bank policy in response to aggregate and idiosyncratic shocks.

Clearly one has to distinguish macroprudential from microprudential policy. The latter is concerned with a partial equilibrium (e.g. a certain financial institution) while macroprudential policy looks at the general equilibrium (e.g. spillover effect to other institutions and markets). The policy implications of a negative financial shock for example would led the microprudential authority to raise capital requirements in order to increase the accumulation of net worth, while the macroprudential policymaker would lower capital requirements in order to avoid repercussions on the real economy.

Another hot topic concerns the design (discretion vs. rules) and the frequency of macroprudential policy adjustments. Is it possible to implement macroprudential tools in a way that allows them to be changed dynamically, say on a quarterly basis, or are these tools adjusted infrequently? A large part of the literature agrees that macroprudential policy already has an active role in dynamically stabilizing the business cycle but the frequency depends on the type of the instrument. These tools include for exam-

\footnote{Formally, the SSM is an independent unit within the ECB organizational’s structures.}

\footnote{Introducing a housing sector in the sense of Iacoviello (2005) would be one of the ways to model credit constrained households.}
ple countercyclical capital buffers introduced by Basel III\footnote{In the Basel III framework the choice of adjusting the capital buffer is decided discretionary by national authorities. Increasing the capital buffer has to be pre-announced by 12 months; decreasing the buffer takes immediate effect. Both decisions are made public.} capital requirements by the central bank or caps on loan-to-value (LTV-) ratios which are both adjusted periodically (Angelini et al., 2014; Bank of England, 2009; Committee on the Global Financial System, 2010; Lim et al., 2011). Since financial stability is multidimensional and each financial cycle has its own specific properties, it will be very hard to find a Taylor-like rule that fits for all cases. The uncertainties around macroprudential assessment will therefore make a certain degree of discretion indispensable. Lim et al. (2011) show that some macroprudential instruments (especially caps) need to be adjusted discretionary. Other researchers are more skeptical about time-varying discretionary tools. Cecchetti and Kohler (2012) find that the transmission mechanisms of monetary policy and time-varying capital requirements are nearly identical. Cecchetti (2015) therefore argues that both instruments are nearly perfect substitutes and policymakers should shy away from discretionary regulatory policy. In particular, besides the traditional problems attached to discretion (time lags), Cecchetti (2015) emphasizes that the welfare implications of varying capital requirements are unclear. In contrast to these concerns, there is empirical evidence supporting the importance of macroprudential policy alongside monetary policy. Since the effectiveness of the latter depends on the capitalization of banks (Gambacorta, 2008) both policies could be complements for addressing inflation. Regarding systemic risk, De Nicolò et al. (2010) find that monetary policy has ambiguous effects on the risk taking behavior of banks (also depending on capitalization) which implies a trade-off between the targets price stability and financial stability. Hence, macroprudential policy can complement monetary policy. Borio and Zhu (2012) obtain similar results.

The independence of instruments - as suggested by the Tinbergen principle - implies no perfect substitutability. In fact both monetary and macroprudential policy could have similar transmission channels (through lending rates) by affecting the demand and supply for credits\footnote{See, among others, Aiyar et al. (2014) and Akram (2014) for empirical evidence.} But this does not mean that these instruments have to be perfect substitutes. Even when capital requirements may have similar effects as monetary policy, there is a rich set of macroprudential tool in practice\footnote{See, for instance, Bank of England (2009), Hahm et al. (2012) and Lim et al. (2011) for a list of possible and already implemented macroprudential tools.} Some of these instruments may be nearly perfect substitutes while others such as LTV- or debt-to-income caps are far away from perfect substitution. Our analysis focuses on the latter. Hence, irrespective of the type of the shock, monetary and macroprudential policies are independent. This result originates in our modelling approach of the monetary and macroprudential
transmission mechanisms which are clearly not identical. The former influences house-
holds’ and firms’ behavior (due to the cost channel) while the latter only affects the 
borrowing costs of firms. We can also provide an answer regarding the welfare effects 
of macroprudential policy as we specifically derive a microfounded objective function.

The literature on the mix of monetary and macroprudential policy has been boom-
ing in the recent years and we will provide only a selected review.  

Several studies consider simple rules as the policy framework: Kannan et al. (2012) study monetary 
and macroprudential rules in a New Keynesian model with a housing market. The 
macroprudential instrument is introduced by assuming that the central bank is able to 
affect the spread between the lending rate and the deposit rate (e.g. due to capital re-
quirements). Using macroprudential policy improves welfare in the case of a financial or 
housing demand shock, but not under productivity shocks. Quint and Rabanal (2014) 
consider a similar model but for a two-country currency union. The authors confirm the 
results of Kannan et al. (2012). Angelini et al. (2014) study the interaction between 
capital requirements and monetary policy. The availability of both policy instruments 
yields significant welfare gains, especially in the case of financial shocks. Other studies 
consider optimal policies: Cecchetti and Kohler (2012) find that capital requirements 
and interest rates are substitutes to a certain degree since the transmission mechanism 
of both policies is similar. A coordinated approach improves welfare as both instru-
ments serve to enhance macroeconomic stability. Collard et al. (2014) study locally 
Ramsey-optimal interactions between monetary and macroprudential policy where the 
latter sets bank capital requirements. In their framework, interest-rate policy does not 
affect the risk-taking behavior of banks. For shocks that do not influence risk-taking 
monetary policy should move while prudential policy should be inactive. For shocks 
affecting the risk-taking behavior, prudential policy should stabilize the shock while 
monetary policy should mitigate the negative externalities of the prudential policy. To 
our knowledge only Carlstrom et al. (2010) and Cúrdia and Woodford (2009) use a 
second-order approximation of household’s utility function in order to derive a micro-
founded objective function, which includes a financial stability measure as an objective. 
However, both study only optimal monetary policy. De Paoli and Paustian (2013) in-
clude a macroprudential instrument into the framework of Carlstrom et al. (2010).

Our model is an extension of De Paoli and Paustian (2013) and includes a two-
country currency union. De Paoli and Paustian (2013) study the strategic interaction 
of a monetary and macroprudential authority while we assume that all policy is con-
ducted under the roof of the central bank. Our focus lies on the optimal policy mix in 
the case of different kind of aggregate and idiosyncratic shocks. We show that macro-
prudential policy is able to fully absorb fluctuations of the economies on the union

\footnote{A comprehensive list of relevant literature can be found in Loisel (2014).}
level for a large set of different scenarios which improves welfare significantly. The additional welfare gain from introducing country-specific macroprudential tools is small even though variations of relative target variables can be reduced. This result may seem counterintuitive but stems from the fact that our microfoundation suggests that the relative variables have a small weight in the welfare objective. Our final analysis compares different policy regimes. For all kinds of shocks we get the following ranking in terms of welfare: discretion outperforms strict inflation targeting which outperforms a (euro-area based) Taylor Rule.

The organization of the paper is as follows. In Section 2 we outline our model; the building blocks are households, entrepreneurs and firms. Section 3 frames the joint policy problem of the monetary and macroprudential authority. In Section 4, we present and discuss the inflation and output dynamics of various shocks. In Section 5 we perform a welfare analysis and compare discretion to simple rules. Section 6 concludes.

2 Model

Our model is a two-country version of a monetary union consisting of households, entrepreneurs and firms. In each country, households supply two types of labor inputs to entrepreneurs. In particular, one of these production factors is credit-constrained which gives rise to an agency problem between households and entrepreneurs as in Carlstrom et al. (2010). Entrepreneurs must back the borrowing costs of the constrained input with securities. This leads to a manifestation of a credit distortion that depends on the tightness of the credit constraint. Entrepreneurs combine both labor inputs in a constant-returns-to-scale production function to produce intermediate goods which are sold to sticky-price firms. Firms are monopolistically competitive and use a linear production function to produce a final good which is traded without any barriers. In the spirit of De Paoli and Paustian (2013), we introduce a (country-specific) regulatory policy instrument which affects the borrowing costs of entrepreneurs. In contrast to these authors, we do not take game theoretical considerations arising from multiple policymakers into account. Instead, we restrict our analysis to the case of full optimization, i.e. the joint optimal monetary and macroprudential policy problem is solved by a single authority - the central bank.

2.1 Households

The total population is ordered on a continuum of measure one. The population on the segment $[0, n)$ belongs to (H)ome, while the population on $[n, 1]$ belongs to (F)oreign. The representative infinitely-lived household $j$ will seek to maximize the following utility
function
\[ E_{0} \sum_{t=0}^{\infty} \beta^{t} \left[ \frac{(C^{j})^{1-\sigma}}{1-\sigma} - B_{1} \frac{(L^{j})^{1+\theta}}{1+\theta} - B_{2} \frac{(u^{j})^{1+\theta}}{1+\theta} \right], \]

where \( C^{j} \) denotes consumption of the final good, \( L^{j} \) and \( u^{j} \) denote the constrained and unconstrained labor inputs, respectively. \( \beta \in [0, 1] \) is the discount factor, \( \sigma \) is the inverse of the intertemporal elasticity of substitution, and \( \theta \) is the inverse Frisch elasticity of labor supply. More precisely, the private composite consumption index is defined as
\[ C^{j}_{t} \equiv \left[ \frac{(C^{j}_{H,t})^{n}(C^{j}_{F,t})^{1-n}}{n^{(1-n)^{1-n}}} \right], \]
where \( C^{j}_{H,t} \) and \( C^{j}_{F,t} \) are the Home and Foreign private consumption indices given by
\[ C^{j}_{H,t} \equiv \left[ \frac{1}{n} \int_{0}^{n} C^{j}_{i}(h)^{-\frac{1}{n}} dh \right]^{-\frac{1}{n}}, \]
\[ C^{j}_{F,t} \equiv \left[ \frac{1}{1-n} \int_{n}^{1} C^{j}_{i}(f)^{-\frac{1}{n}} df \right]^{-\frac{1}{n}}. \]
The parameter \( \varepsilon > 1 \) denotes the elasticity of substitution between any two varieties. There are no trade barriers, so the law of one price holds for each brand. And since preferences are assumed to be identical in the entire union, the consumer price index of the final good \( P_{t} \) is identical across countries: \( P_{t} = P_{t}^{H} = P_{t}^{F} \). The consumer price index is given by
\[ P_{i} = (P_{H,i})^{n} (P_{F,i})^{1-n}, \]
where \( P_{i,t} \) is the producer price index in region \( i = \{H,F\} \). It is useful to define the terms of trade as the relative price of the Foreign bundle of goods in terms of the Home bundle, i.e. \( S_{t} \equiv P_{F,t}/P_{H,t} \). By aggregating consumption over all households (\( C^{W}_{t} \equiv \int_{0}^{1} C^{j}_{i} dj \) and using the appropriate production indices, aggregate demands can be written as
\[ Y^{H}_{t} = S^{1-n}_{t} C^{W}_{t}, \quad Y^{F}_{t} = S^{-n}_{t} C^{W}_{t}. \]

Because of liquidity constraints in the factor market, households engage in the asset market before the goods market opens. Both households and entrepreneurs can purchase shares at price \( Q^{j}_{i} \) which pay out dividends \( D^{j}_{i} \). \( e^{j}_{i} \) denotes the fraction of shares bought by entrepreneurs. At the beginning of period \( t \), the representative agent in country \( i \) has cash holdings \( M^{i}_{t-1} \), receives nominal wage income \( P_{i,t} (w^{j}_{i} L^{i}_{t} + r^{j}_{i} u^{j}_{i}) \) and nominal returns on her shares \( P_{i,t} (Q^{j}_{i} + D^{j}_{i})(1 - e^{j}_{i-1}) \) which can be used as a mean of payment. This cash payoff is used for depositing funds \( A^{i}_{t} \) at financial intermediaries and buying new shares. Hence, the agent faces the following cash-in-advance constraint in the goods market
\[ P_{t} C^{W}_{t} \leq M^{i}_{t-1} + (1 + \Omega^{j}_{w}) P_{i,t} w^{j}_{i} L^{i}_{t} + (1 + \Omega^{j}_{r}) P_{i,t} r^{j}_{i} u^{j}_{i} + P_{i,t} (Q^{j}_{i} + D^{j}_{i})(1 - e^{j}_{i-1}) - A^{i}_{t} - T^{i}_{t} \]
where \( T^{i}_{t} \) is a lump-sum tax and \( \Omega^{j}_{w} \) and \( \Omega^{j}_{r} \) are steady-state employment subsidies. We follow Rotemberg and Woodford (1997) and large parts of the literature by assuming
that steady-state subsidies are used to offset all distortions.

At the end of the period, the agent gets deposits plus interest \((R_t A^i_t)\) back, where \(R_t\) is the gross nominal interest rate set by the central bank. Thus, we obtain the following equation for the cash holdings \(M^i_t\), cash that is carried over to the next period:

\[
M^i_t = M^i_{t-1} + (1 + \Omega^i_{w}) P_{i,t} w^i_t L^i_t + (1 + \Omega^i_r) P_{i,t} r^i_t u^i_t + P_{i,t} (Q^i_t + D^i_t)(1 - e^i_{t-1}) - A^i_t - P_{i,t} Q^i_t(1 - e^i_t) - P_t C^W_t - T^i_t + R_t A^i_{t-1}.
\]

Assuming a positive interest rate, (3) will always bind in equilibrium. Hence, the budget constraint is given by

\[
P_t C^W_t = (1 + \Omega^i_{w}) P_{i,t} w^i_t L^i_t + (1 + \Omega^i_r) P_{i,t} r^i_t u^i_t + P_{i,t} (Q^i_t + D^i_t)(1 - e^i_{t-1}) - A^i_t - P_{i,t} Q^i_t(1 - e^i_t) - T^i_t + R_{t-1} A^i_{t-1}.
\]

The representative household in country \(i\) maximizes utility (1), subject to the cash-in-advance constraint (3) and the budget constraint (5). By rearranging the resulting first-order conditions, we get

\[
B_1 \frac{(L^i_t)^\rho}{(C^W_t)^\sigma} = (1 + \Omega^i_{w}) w^i_t P_{i,t} \frac{P_{i,t}}{P_t}
\]

\text{and}

\[
B_2 \frac{(u^i_t)^\rho}{(C^W_t)^\sigma} = (1 + \Omega^i_r) r^i_t P_{i,t} \frac{P_{i,t}}{P_t}
\]

\[
1 = \beta R_t E_t \left[ \left( \frac{C^W_t}{C^W_{t+1}} \right)^\sigma \left( \frac{P_t}{P_{t+1}} \right) \right]
\]

\[
1 = \beta E_t \left[ \left( \frac{C^W_t}{C^W_{t+1}} \right)^\sigma \left( \frac{Q^i_{t+1} + D^i_{t+1}}{Q^i_t} \right) \left( \frac{P_t}{P_{t+1}} \right) \left( \frac{P_{i,t+1}}{P_{i,t}} \right) \right].
\]

### 2.2 Entrepreneurs

There is a continuum of long-lived entrepreneurs with linear consumption preferences of measure \(n\) in country \(H\) and of measure \(1-n\) in country \(F\). Entrepreneurs use both labor inputs to produce an intermediate good according to the Cobb-Douglas production function \(x^i_t = (L^i_t)^\alpha (u^i_t)^{1-\alpha}\). The parameter \(\alpha\) determines the fraction of the credit-constrained labor input in the production process and can be interpreted as agency costs. If \(\alpha = 0\), the model will collapse to a standard two-country version of a currency union as in Benigno (2004). If \(\alpha = 1\), there will be agency costs but no distortions in terms of allocation of resources between factors of production. Entrepreneurs supply
only the respective domestic market; intermediate goods are not traded internationally. Profits in each country are given by

\[ \text{profits}_i = mc_i x_i - \tau_i R_i w_i L_i - r_i u_i, \quad (10) \]

where real marginal costs in country \( i \) are defined as \( mc_i = MC_i / P_{it} \). In order to pay the wage bill for the constrained labor input, entrepreneurs must borrow from intermediaries at the lending rate \( R_l \). This feature is known as the cost channel. Most importantly, we introduce a (country-specific) macroprudential policy instrument \( \tau_i \) which influences the borrowing costs of the entrepreneurs. It captures every kind of macroprudential instrument that affects the cost of borrowing, e.g. the quality of net worth the central bank accepts as collateral. This kind of approach to model macroprudential policy is well established in the literature (see, for instance, Kannan et al., 2012; De Paoli and Paustian, 2013, Quint and Rabanal, 2014, or Unsal, 2013).

There is a friction in borrowing for the wage bill \( \tau_i R_i w_i L_i \) as entrepreneurs face the following collateral constraint

\[ \tau_i R_i w_i L_i \leq (nw_i)^b (mc_i x_i - r_i u_i)^{1-b}, \quad (11) \]

where \( nw_i \) is a country-specific exogenous net worth shock which is assumed to follow an AR(1) process. The constraint always binds in the steady state and in its neighborhood. The intuition behind the borrowing constraint (11) is a holdup problem known from Hart and Moore (1994). Since there is no final good without the entrepreneur's own human capital input, entrepreneurs might "hold up" production after the L-input has been added to the production process. By holding up, entrepreneurs gain bargaining power in order to renegotiate wages ex-post. As L-suppliers anticipate this behavior they demand securities in the form of net worth or operating profits before producing. The parameter \( b \) determines the fraction of the net worth which may serve as collateral.

Entrepreneurs maximize profits (10) subject to the borrowing constraint (11). The optimization conditions are

\[ \alpha mc_i x_i = \tau_i R_i w_i L_i (1 + b \phi_i) \quad (12) \]

\[ (1 - \alpha) mc_i x_i = r_i u_i, \quad (13) \]

where \( \phi_i \) denotes the Lagrange multiplier on the borrowing constraint. Equation (12) pins down the loan size and shows that similar to the cost channel, \( b \phi_i \) looks like an interest rate on a loan financing the wage bill. Moreover, it captures any wedges between the lending rate \( R_i \) and the risk-free interest rate \( R_t \). We can therefore approximate
$R^i_t$ with $R_t$ and interpret $b \phi^i_t$ as a credit spread. $^8$ Substituting both FOC (12) and (13) into the borrowing constraint (11), we obtain

\[ \left( \frac{\alpha m c^i_t x^i_t}{nw^i_t} \right)^b = (1 + b \phi^i_t). \]  

Equation (14) shows that the credit spread is an increasing function of leverage which is defined as the ratio of loan size to net worth. Fluctuations in the leverage therefore cause fluctuations in the credit spread and induce credit distortions.

In the neighborhood of the steady state, the rate on return on internal funds exceed the discount factor on consumption, $\beta$. Since their preferences are linear, entrepreneurs will always prefer to accumulate net worth over consumption. In order to avoid entrepreneurs accumulating so much net worth that they do not need to borrow funds, we assume that each period a fraction $1 - \gamma$ of entrepreneurs dies unexpectedly. Their funds are redistributed in a lump-sum way to households and an equal number of new entrepreneurs enter the market. The entrepreneur’s budget constraint is then given by

\[ e^i_t Q^i_t = \gamma \left[ e^i_{t-1} (Q^i_t + D^i_t) \cdot c^i_{nw,t} + \text{profits}^i_t \right], \]

which can be rewritten as

\[ e^i_t Q^i_t = \gamma \alpha m c^i_t x^i_t \left[ \frac{b \phi^i_t}{1 + b \phi^i_t} + \left( \frac{1}{1 + b \phi^i_t} \right)^{1/b} \right]. \]  

### 2.3 Firms

Monopolistically competitive firms of measure $n$ in country $H$ and of measure $1 - n$ in country $F$ produce final goods $y^i_t$. For the sake of simplicity, we assume that each firm simply labels the intermediate good without any costs with its own specific brand according to the technology $y^i_t = x^i_t$. Price setting follows Calvo (1983) scheme of price adjustment where each firm producing in country $i$ may reset its price with probability $1 - \varphi^i$ in any given period. Assuming that the steady state is characterized by zero inflation in both countries, the evolution of the producer inflation rate in region $i$ is given by the marginal cost based (log-linearized) Phillips curve:

\[ \pi^i_t = \beta E_t \pi^i_{t+1} + \lambda^i (\hat{m}c^i_t + \epsilon^i_{x,t}). \]  

\[ \text{Equation (16)} \]

\[ \text{Even though $\phi^i_t$ is not truly a risk premium but formally a shadow price of the borrowing constraint, it can be shown that the linearized model is isomorphic to a costly-state verification (CSV) framework (see Carlstrom et al., 2010, for a detailed derivation). This isomorphism enables us to find relevant values for the structural parameters in the calibration of the model.} \]
where the composite parameter $\lambda^i$ is given by

$$\lambda^i \equiv \frac{(1-\phi^i)(1-\beta\phi^i)}{\phi^i}$$

(see, e.g., Gali, 2008) and $\epsilon_{\pi,t}^i$ is a country-specific exogenous markup shock which is assumed to follow an AR(1) process. The firm’s profits are paid out as dividends to shareholders according to $D_t^i = y_t^i(1 - mc_t^i)$.

### 2.4 Equilibrium Dynamics

The appendix presents a detailed derivation of the log-linearized equations. In this section, we present important dynamics for the Home country and provide a clear economic interpretation. Then we present a reduced form of the model. The Home firm’s real marginal costs are given by

$$\dot{mc}_t^H = \theta\dot{y}_t^H + \sigma\dot{c}_t^W + (1 - n)\dot{S}_t + \alpha(\dot{R}_t + \dot{\gamma}_t^H + b\phi_t^H).$$

(17)

An exogenous increase in Home income, world consumption or a terms of trade improvement directly increase labor demand which in turn induces a rise in the real wage for both production factors. The profit maximizing entrepreneur passes the higher costs through by increasing the relative price of the intermediate good - marginal costs of the firm rise. The term $\dot{R}_t + \dot{\gamma}_t^H + b\phi_t^H$ mirrors the cost channel, macroprudential policy and the borrowing constraint. An increase in any of these three variables tightens the borrowing constraint which result in an increase in marginal costs. Hence, monetary and macroprudential instruments serve as perfect substitutes regarding their impact via the supply side of the economy (as long as the cost channel is present). In general equilibrium this is not true anymore since only monetary policy affects aggregate demand.

Next, we need a dynamic equation in $\hat{\epsilon}_t^H$ which determines the accumulation of net worth:

$$\beta\hat{s}_t^H = \hat{s}_t^H + (1 - \beta)\hat{e}_{t}^H + (1 + \beta\Lambda^H)\hat{\phi}_t^H + \epsilon_{\text{nw},t}^H.$$  

(18)

where $\Lambda^H \equiv \frac{F_H}{F^H}(1+b\phi^H), F^H \equiv \frac{b\phi^H}{1+b\phi^H} + \left(\frac{1}{1+b\phi^H}\right)^{1/b}$ and $F^H_H = \frac{b\phi^H}{(1+b\phi^H)^2} - \left(\frac{1}{1+b\phi^H}\right)^{1/(1+b)}$. An increase in marginal costs relaxes the entrepreneur’s borrowing constraint as operating profits rise. Therefore, the fraction of shares owned by entrepreneurs decrease and less net worth is accumulated. A rise in the Lagrange multiplier, $\hat{\phi}_t^H$, has two opposing effects. First, it tightens directly the borrowing constraint and more net worth is accumulated. Second, it increases marginal costs which lowers the accumulation of net worth. For the benchmark specification the direct effect outweighs the indirect effect.

The last relationship we need is a dynamic equation for the evolution of the Lagrange

\[ A \ ^\wedge \ ^\wedge \ ^\wedge \ ^\wedge n \ symbol is used to denote the percentage deviation of a variable from its steady state value.

11
multiplier, \( \tilde{\phi}_t^H \), which we interpret as (part of) the credit spread (see (26)-(27)).

Before we proceed, some simplifying notation is useful: An aggregate (union) variable \( x^x_t \) is defined as the weighted average of the national variables, \( x^x_t \equiv nx^H_t + (1-n)x^F_t \), while a relative variable \( x^R_t \) is defined as \( x^R_t \equiv x^H_t - x^F_t \). We can summarize the system by the following reduced equations:

\[
\begin{align*}
\hat{y}_t^H &= (1-n)\hat{S}_t + E_t\hat{y}_{t+1}^W - \sigma^{-1}(\hat{R}_t - E_t\pi_{t+1}^W) + \epsilon^H_{d,t} \\
\hat{y}_t^F &= -n\hat{S}_t + E_t\hat{y}_{t+1}^W - \sigma^{-1}(\hat{R}_t - E_t\pi_{t+1}^W) + \epsilon^F_{d,t} \\
\pi_t^H &= \beta E_t\pi_{t+1}^H + \lambda^H \left[ (\sigma + \theta)\hat{y}_t^H + (1-n)(1-\sigma)\hat{S}_t + \alpha(\hat{R}_t + \tau_t^H) \right] + [1 + \beta \Lambda^H - \alpha b(1-\beta)]\tilde{\phi}_t^H + \epsilon^H_{n,w,t} \\
\pi_t^F &= \beta E_t\pi_{t+1}^F + \lambda^F \left[ (\sigma + \theta)\hat{y}_t^F - n(1-\sigma)\hat{S}_t + \alpha(\hat{R}_t + \tau_t^F) \right] + [1 + \beta \Lambda^F - \alpha b(1-\beta)]\tilde{\phi}_t^F + \epsilon^F_{n,w,t} \\
(1 + \Lambda^H)\tilde{\phi}_t^H &= E_t \begin{cases} 
(\theta + 1)\Delta\hat{y}_{t+1}^H + \alpha(\Delta\tau_{t+1}^H + \Delta\hat{R}_{t+1}) \\
-(1-\alpha b)\Delta\tilde{\phi}_{t+1}^H - \epsilon_{n,w,t+1}^H 
\end{cases} \\
(1 + \Lambda^F)\tilde{\phi}_t^F &= E_t \begin{cases} 
(\theta + 1)\Delta\hat{y}_{t+1}^F + \alpha(\Delta\tau_{t+1}^F + \Delta\hat{R}_{t+1}) \\
-(1-\alpha b)\Delta\tilde{\phi}_{t+1}^F - \epsilon_{n,w,t+1}^F 
\end{cases} .
\end{align*}
\]

Equations (19) - (20) are the Home and Foreign IS curves which are obtained by combining the Euler equation (8) with the aggregate demand functions (2) plus adding country-specific demand shocks. Inserting real marginal costs into (16) delivers the Home (21) and Foreign Phillips curves (22). In addition to the common features of the Phillips curve, the term \( \hat{R}_t + \tau_t + b\tilde{\phi}_t \) is a substantial component. A rise in any of these variables increases marginal costs and thereby inflation. Equation (23) states that the current period terms of trade is a function of its past value, thus it is a state variable. Substituting Home real marginal costs (17) into equation (18) yields the Home dynamic net worth equation (24). Similarly we obtain the Foreign dynamic net worth equation (25). Finally, equations (26)-(27) describe the Home and Foreign forward-looking conditions for the evolution of the credit spreads (see Appendix A). Expectations about the
future play an important role for the actual credit spread. A rise in production leads
to tighter credit conditions since credit demand increases, whereas increases in the
instruments set by monetary and macroprudential policy lead to higher borrowing costs
and therefore higher credit spreads. The model is closed by a description of monetary
and macroprudential policy.

3 Framing the Policy Problem

In this section we describe the nature of optimal discretionary policy and optimal com-
mitment policy by the monetary and macroprudential authority. In contrast to De Paoli
and Paustian (2013), who study strategic interaction between both policymaker, we re-
strict our analysis to the case of full optimization, i.e. the central bank is responsible
for both types of policies and chooses jointly the union-wide nominal interest rate \( \hat{R}_t \)
and the (national) macroprudential tool \( \hat{\tau}_t \) to maximize the utility of the representative
household given by (1). This case of a centralized single policymaker corresponds to
that of full coordination of monetary and supervisory authorities.

3.1 Welfare Objective

We obtain the objective function of the single policymaker from a second-order Taylor
expansion of (1) around the deterministic steady state (see Appendix B for details):

\[
- E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \Psi_t \right\} + t.i.p. + o \left( \| \xi \|_3^3 \right),
\]  

(28)

where \( t.i.p. \) stands for terms independent of policy and \( o \left( \| \xi \|_3^3 \right) \) represents terms of
order three and higher. The per-period quadratic deadweight loss function \( \Psi_t \) is given by

\[
\Psi_t = (\sigma + \theta)(\hat{y}_t^W)^2 + n\frac{\varepsilon}{\lambda^H}(\pi_t^H)^2 + (1-n)\frac{\varepsilon}{\lambda^F}(\pi_t^F)^2 + n(1-n)(1+\theta)(\hat{S}_t)^2
\]

\[
+ n \frac{\alpha(1-\alpha)}{1+\theta}(\hat{R}_t + \hat{\tau}_t^H + b_\phi_t^H)^2 + (1-n) \frac{\alpha(1-\alpha)}{1+\theta}(\hat{R}_t + \hat{\tau}_t^F + b_\phi_t^F)^2.
\]

(29)

The advantage using a second-order Taylor expansion of the utility function is that we
obtain a microfounded objective function where the weights of the respective variables
are all functions of deep model parameters. The variables in the upper line of the loss
function (29) are the standard target variables and weights for a two-country currency
union (see for example Benigno, 2004, or Beetsma and Jensen, 2005). The variables
in the bottom line are new to the baseline New Keynesian model (see De Paoli and Paustian, 2013, for a closed-economy version of our model). The credit spread, $b_{it}$, is part of the loss function because the tightness of the borrowing constraint reflects the credit market tightness. Movements in the credit spread induce inefficient factor allocations. The central bank tries to stabilize the business cycle by choosing the nominal interest rate and the (country-specific) macroprudential tool, $\hat{R}_t$ and $\hat{\tau}_t$.

However, the use of these instruments is not for free and causes a welfare loss per se. Varying the interest rate will cause an inefficient distribution between both labor inputs. Macroprudential policy has a similar cost-push effect as the borrowing costs are directly affected. Hence, the sum of $\hat{R}_t$, $\hat{\tau}_t^H$ and $b\hat{\phi}_t$ and not only the individual targets must be considered when evaluating the dynamics of the system. Setting $\alpha = 0$ or $\alpha = 1$, the objective reduces to the standard two-country currency union loss function. If all output is produced by the unconstrained labor input ($\alpha = 0$), the effective interest rate drops out of the target criterion since the borrowing constraint is not active. If all output is produced by the constrained labor input ($\alpha = 1$), the term $\hat{R}_t + \hat{\tau}_t^i + b\hat{\phi}_t^i$ drops out because there is no inefficient allocation between both labor inputs. For $\alpha \neq 0$ and $b = 0$, we have the case of a currency union without credit spreads but with cost channel.

If the duration of price contracts is identical across countries, $\lambda^H = \lambda^F = \lambda$, the per-period loss function can be rewritten in area and relative terms as

$\Psi_t = (\sigma + \theta)(\hat{\gamma}_t^W)^2 + \frac{\varepsilon}{\lambda} \left[ (\pi_t^W)^2 + n(1-n)(\pi_t^R)^2 \right] + n(1-n)(1+\theta)(\hat{S}_t)^2$

$+ \frac{\alpha(1-\alpha)}{1+\theta} \left[ (\hat{R}_t + \hat{\tau}_t^W + b\hat{\phi}_t^W)^2 + n(1-n)(\hat{\tau}_t^R + b\hat{\phi}_t^R)^2 \right].$ \hspace{1cm} (30)

Regarding the case that there is only a union-wide macroprudential tool, $\hat{\tau}_t^R$ drops out of the objective function. Then policymakers have only two aggregate tools ($\hat{R}_t$ and $\hat{\tau}_t^W$) with whom they cannot affect differentials and the objective further simplifies to

$\Psi_t = (\sigma + \theta)(\hat{\gamma}_t^W)^2 + \frac{\varepsilon}{\lambda} \left[ (\pi_t^W)^2 \right] + \frac{\alpha(1-\alpha)}{1+\theta} \left[ (\hat{R}_t + \hat{\tau}_t^W + b\hat{\phi}_t^W)^2 \right]$ which is equivalent to the loss function in De Paoli and Paustian (2013). Fluctuations in relative variables still create loss but the central bank ignores them.

\hspace{1cm} \cite{De Paoli and Paustian, 2013} The costly use of instruments is already known in the literature regarding optimal fiscal policy where varying government spendings creates a welfare loss (see e.g. Beetsma and Jensen, 2005, or Gali and Monacelli, 2008). Note that in most of the literature regarding the cost channel (e.g. Ravenna and Walsh, 2006), there are no direct welfare losses when varying the interest rate. This is because these models include only one unconstrained labor input.
3.2 Calibration

Let us outline the parametrization for the quantitative policy analysis. The model is calibrated to a quarterly frequency. Key parameters are chosen in a manner that matches the average features of countries belonging to the EMU and are taken from Benigno (2004): The discount factor $\beta$ is set equal to 0.99, so that the steady-state real interest rate is 4% p.a. By calibrating the elasticity of substitution between goods $\varepsilon$ to a value of 7.66, we assume that the steady-state mark-up of prices over marginal costs is around 15% which is a reasonable value for the European economies. The price rigidity is assumed to be equal in both countries. Therefore the Calvo parameter $\theta^i$ is set equal to a standard value of 0.75 which implies an average duration of price contracts of four quarters. We divide the monetary union into two equal-sized groups; thus, $n = 0.5$. $\alpha$ is the share of constrained labor and set equal to 0.5 following Carlstrom et al. (2010). These authors also demonstrate the formal linkage between this model and a costly-state-verification model in order to parametrize $b$ and $\phi$. We follow them by assuming monitoring costs of 0.15 and a bankruptcy rate of 1% as these are on average the same in the euro area as in the U.S. According to data from the Bank of America Merrill Lynch (2015), the average spread between below investment grade corporate debt issued in the euro area and a spot Treasury curve from 1997 to 2015 implies an aggregate risk premium of 680 annual basis points. Hence, the CSV-conversion implies that $b$ is equal to 0.4 and $\phi$ is equal to 0.11. Following Woodford (2003) the inverse of the Frisch elasticity of labor supply ($\phi$) and the inverse of the intertemporal elasticity of substitution ($\sigma$) are set equal to 0.47 and 0.16 respectively. Moreover, we adopt a degree of persistence in the shocks of 0.9.

4 Dynamics

The objective of this section is to analyze the dynamic response of the relevant endogenous variables to different kind of shocks, i.e. demand shocks, net worth shocks and markup shocks. We distinguish between aggregate and idiosyncratic shocks and focus on the latter. In order to avoid (too) many case differentiations, the presentation focuses on optimal discretionary policy. Let us describe our assumptions on the sequence of events before turning our focus on the dynamics. First, the economy is in the deterministic steady state. Then, period $t$ shocks are revealed. Given the realizations of the shocks, the policy authority decides on the optimal response of the nominal interest rate and the (national) macroprudential tool. Next, wage setters decide on the wage, entrepreneurs decide on the relative price of the intermediate good and take up a loan to finance the wage bill. Employment is pinned down, and production of the intermediate good takes place. After selling the products to the final goods firms, entrepreneurs
repay the loan. Firms decide on the profit maximizing price of the final good and label the product without any costs.

4.1 Aggregate Shocks

In this subsection we briefly discuss the optimal policy mix in the case of different kind of aggregate shocks. Since both countries are symmetrical, the analysis is isomorphic to the case of a closed economy as considered by De Paoli and Paustian (2013). We can replicate the results of these authors except for the demand shock, which is not considered in their study.

Proposition 1 Optimal monetary and macroprudential policy fully absorbs aggregate demand and net worth shocks.

Proof. The instruments are set such that $\tilde{R}_t = \sigma \hat{\varepsilon}^W_t$ and $\tilde{\tau}^W_t = -b \hat{\phi}^W_t - \tilde{R}_t$ so that $\tilde{y}^W_t = \pi^W_t = 0$ which insures efficiency $\Psi_t = 0$ (see (30)) in every period. ■

We start by considering a positive aggregate demand (preference) shock, i.e. $\varepsilon^W_{d,0} = H^d_{0} = F^d_{0} = 1$. Without a dynamic macroprudential tool ($b_i^t = 0$) optimal policy consists only of an increase in the interest rate. Because of the cost channel, monetary policy cannot perfectly absorb the shock. The rise of the interest rate pushes inflation up via the supply side of the economy. Moreover there is an inefficient allocation between labor inputs which creates a welfare loss. Allowing macroprudential policy to act dynamically changes the picture. Now the interest rate is varied in order to perfectly absorb fluctuations in inflation and output ($\tilde{R}_t = \sigma \hat{\varepsilon}^W_t$). Distortions caused by variations in the interest rate are perfectly offset by the macroprudential instrument ($\tilde{\tau}^W_t = -\tilde{R}$). Hence, in accordance to the Tinbergen principle, both policies are not perfect substitutes, i.e. they are independent. Since there are no relative distortions between Home and Foreign, it is obvious that there is no advantage of having country-specific macroprudential tools.

Following a negative aggregate net worth shock ($\epsilon^W_{nw,0} = \epsilon^H_{nw,0} = \epsilon^F_{nw,0} = -1$) the credit constraint (11) is tightened, hence the credit spread becomes positive, marginal costs and thus inflation increase (see (17)). Without macroprudential policy, tightening monetary policy reduces the positive inflation gap but the output gap becomes negative. The first best outcome is only feasible when the macroprudential tool is available as a policy instrument: Loosening macroprudential policy perfectly offsets fluctuations in the credit spread ($\tilde{\tau}^W_t = -b \hat{\phi}^W_t$). As a result, the interest rate is kept constant ($\tilde{R}_t = 0$).

Note that $\tilde{\tau}^i_t = 0$ also implies the case of a macroprudential regulation that is in place but cannot be dynamically used to smoothen the business cycle.
Proposition 2 Consider an aggregate markup shock. a) For $\alpha = 1$, optimal monetary and macroprudential policy fully eliminates economic distortions. b) For $\alpha \neq 1$, full stabilization of aggregate output and inflation is possible but not optimal.

Proof. a) The instruments are set such that $\hat{R}_t = 0$ and $\hat{\pi}_t^W = -b\hat{\phi}_t - \epsilon^W$ so that $\hat{y}_t^W = \pi_t^W = 0$ which insures efficiency $\Psi_t = 0$ (see (30)) in every period since there is no inefficient labor allocation. b) Due to the fact that there is an inefficient labor allocation, varying instruments is costly per se and causes additional losses according to (30).

A positive markup shock ($\epsilon^W_{\pi,0} = \epsilon^H_{\pi,0} = \epsilon^F_{\pi,0} = 1$) drives a wedge between the inflation and output target and cannot be offset if there is no macroprudential tool. The central bank mitigates the inflationary cost-push effect by increasing the interest rate but there will not be full accommodation. In presence of the macroprudential tool, the markup shock can be fully absorbed if $\alpha = 1$, i.e. all output is produced by the constrained labor input only. In this case, the macroprudential tool is a perfect supply-side instrument since it does not produce any inefficient labor input allocations. Hence, monetary policy remains inactive ($\hat{R}_t = 0$). This is not true anymore if $\alpha \neq 1$ as now the use of the macroprudential tool is costly per se. $\hat{y}_t^W = \pi_t^W = 0$ is possible but not optimal. Following the shock, there will be a positive inflation gap and a negative output gap. In this case, it is optimal to increase the nominal interest rate (in order to reduce inflation) and to decrease the macroprudential tool (in order to relax the borrowing constraint which reduces marginal costs and inflation). The interest hike, the inflation and the output gap are significantly lower compared to the case without macroprudential policy.

4.2 Idiosyncratic Shocks

Let our focus now turn to the more interesting case of idiosyncratic shocks. In the following we will consider a demand shock, a net worth shock and a markup shock in the Home country. In all cases we are interested in the change in the inflation and output dynamics when the instrument set of the central bank is enhanced by adding macroprudential policy (tools). Even though both countries are symmetrical, there will be relative distortions following a shock in only one of the countries. It is well known that monetary policy is not able to affect differences across countries in a monetary union as the nominal interest rate is a union-wide instrument only. When country-specific macroprudential policy is available, the central bank has a (relative) instrument in order to mitigate relative fluctuations across countries.
4.2.1 Demand Shock

**Proposition 3** Consider an idiosyncratic demand shock. a) In the case that there is only a union-wide macroprudential tool, optimal monetary and macroprudential policy fully absorbs aggregate fluctuations, but a welfare loss due to variations in relative variables remains. b) In the case that there are country-specific macroprudential tools and $\alpha = 1$, optimal monetary and macroprudential policy fully eliminates fluctuations in aggregate and relative target variables. c) In the case that there are country-specific macroprudential tools and $\alpha \neq 1$, full stabilization of relative output and inflation is possible but not optimal.

**Proof.** a) The optimal policy mix implies that $\hat{R}_t = \sigma n \epsilon^H_{d,t} = -\hat{\tau}^W_t$ so that aggregate distortions are eliminated, i.e. $\hat{y}^W_t = \pi^W_t = 0$. Since the policymaker ignores differentials there is still a welfare loss arising from fluctuations in $\pi^R_t$, $\hat{S}_t$, and $\hat{\phi}^R_t$. b) The instruments are set such that $\hat{R}_t = \sigma n \epsilon^H_{d,t} = -\hat{\tau}^W_t$, and $\hat{\tau}^R_t = -b \hat{\phi}^R_t - b \hat{\phi}^R_t - \theta \epsilon^H_{d,t}$ so that $\hat{y}^W_t = \pi^W_t = \pi^R_t = \hat{S}_t = 0$ which insures efficiency $\Psi_t = 0$ (see (30)) in every period. Due to the fact that there is no inefficient labor allocation ($\alpha = 1$), varying instruments is not costly per se. Hence, $\hat{\tau}^R_t$ can be used to eliminate fluctuations in relative marginal costs $\hat{m} \tilde{c}^R_t = \theta \hat{\phi}^R_t + (1 - n) \hat{S}_t + \hat{\tau}^R_t + \hat{\phi}^R_t$ (see (17)) to stabilize $\pi^R_t = \beta E_t \pi^R_{t+1} + \lambda \hat{m} \tilde{c}^R_t$ and $\hat{S}_t$ (23) which ultimately results in the first best outcome. c) The inefficient labor allocation implies that varying instruments is costly per se and causes additional losses according to (30). ■

Figure (1) displays the impulse responses to a positive Home demand shock for the benchmark specification. Without a macroprudential tool (blue line), the shock cannot be absorbed due to the cost channel and the collateral constraint which lead to an inefficient labor allocation. Due to these distortions, monetary policy becomes more aggressive than in a world without these frictions (see Michaelis and Palek, 2014) where the central bank could stabilize the economy by setting the nominal interest rate according to $\hat{R}_t = \sigma n \epsilon^H_{d,t}$. In presence of macroprudential policy (red and green line), all aggregate fluctuations are eliminated as monetary policy absorbs the shock ($\hat{R}_t = \sigma n \epsilon^H_{d,t}$) while macroprudential policy offsets the distortions caused by using the nominal interest rate ($\hat{\tau}^W_t = -\hat{R}_t$). Nevertheless, there is still a welfare loss due to fluctuations in relative inflation, the terms of trade and relative credit spread.

In the case that there are country-specific macroprudential tools, optimal monetary and macroprudential policy fully absorb idiosyncratic demand shocks if $\alpha = 1$. In this case varying instruments is not costly per se anymore. Country-specific macroprudential policy is used to offset all fluctuations in relative variables. More precisely, $\hat{\tau}^R_t$ stabilizes relative marginal costs $\hat{m} \tilde{c}^R_t = \theta \hat{\phi}^R_t + (1 - n) \hat{S}_t + \hat{\tau}^R_t + \hat{\phi}^R_t$ according to $\hat{\tau}^R_t = -b \hat{\phi}^R_t - \theta \hat{y}^R_t$.
Figure 1: Impulse responses to a Home demand shock. No macroprudential tool (blue line), union-wide macroprudential tool (red line), national macroprudential tools (green line)
which implies that relative inflation and the terms of trade do not move \((\pi_t^R = \tilde{S}_t = 0)\). Fluctuations in \(\tilde{y}_t^R = c_{d,t}^R\) remain but cause no welfare loss.

If \(\alpha \neq 1\), reaching the first-best outcome is not possible anymore as it implies an inefficient labor allocation. So it is not optimal to fully stabilize relative marginal costs since the use of \(\tilde{\tau}_t^R\) is costly. Therefore there are fluctuations in relative inflation, the terms of trade and the relative output gap. Having country-specific macroprudential tools improves welfare compared to one union macroprudential tool since the gaps in relative inflation, the relative credit spread and the terms of trade become smaller (see green line).

4.2.2 Net Worth Shock

**Proposition 4** Consider an idiosyncratic net worth shock. a) In the case that there is only a union-wide macroprudential tool, optimal macroprudential policy fully absorbs aggregate fluctuations, but a welfare loss due to fluctuations in relative variables remains. b) In the case that there are country-specific macroprudential tools, optimal macroprudential policy fully eliminates economic distortions.

**Proof.** a) Macropruitudinal policy is superior to monetary policy, \(\tilde{R}_t = 0\), and only \(\tilde{\tau}_t^W = -b\tilde{\phi}_t^W\) is used to eliminate aggregate distortions, \(\tilde{y}_t^W = \pi_t^W = 0\). Since the policymakers ignore differentials there is still a welfare loss arising from fluctuations in \(\pi_t^R, \tilde{S}_t, \) and \(b\tilde{\phi}_t^R\). b) Macropruitudinal policy \(\tilde{\tau}_t^W = -b\tilde{\phi}_t^W\) remains superior to monetary policy \(\tilde{R}_t = 0\) which implies \(\tilde{y}_t^W = \pi_t^W = 0\). Furthermore, the macroprudential tools are set according to \(\tilde{\tau}_t^R = -b\tilde{\phi}_t^R\) which implies that \(\pi_t^R = \tilde{S}_t = 0\) (see (21),(22),(23)) and insures efficiency \(\Psi_t = 0\) (see (30)) in every period. 

Following a negative Home net worth shock (see Figure (2)), the union and relative credit spread rises and therefore union and relative inflation increases. Without macroprudential policy (blue line), the central bank acts exactly as in the case of an aggregate net worth shock by increasing the interest rate in order to mitigate the effects on the union target variables. But contrary to the aggregate shock, there are additional welfare losses due to fluctuations in the relative variables which cannot be affected by monetary policy. With a union-wide macroprudential tool at hand (red line) the shock can be fully absorbed at the union but not on the relative level. The central bank sets \(\tilde{\tau}_t^W = -b\tilde{\phi}_t^W\) and \(\tilde{R}_t = 0\) so that \(\tilde{y}_t^W = \pi_t^W = 0\). Welfare losses arise due to fluctuations in \(\pi_t^R, \tilde{S}_t, \) and \(b\tilde{\phi}_t^R\). Country-specific macroprudential tools (green line) are able to eliminate these distortions by decreasing the relative macroprudential instrument such that \(\tilde{\tau}_t^R = -b\tilde{\phi}_t^R\). Hence, there are no welfare losses irrespective of the value of \(\alpha\).
Figure 2: Impulse responses to a Home net worth shock. No macroprudential tool (blue line), union-wide macroprudential tool (red line), national macroprudential tools (green line)
4.2.3 Markup Shock

**Proposition 5** Consider an idiosyncratic markup shock. a) In the case that there is only a union-wide macroprudential tool and for $\alpha = 1$, optimal monetary and macroprudential policy fully absorbs aggregate fluctuations, but a welfare loss due to fluctuations in relative variables remains. b) In the case that there is only a union-wide macroprudential tool and for $\alpha \neq 1$, full stabilization of aggregate output and inflation is possible but not optimal. Hence, losses arise from variations in aggregate and relative variables. c) In the case that there are country-specific macroprudential tools and for $\alpha = 1$, optimal monetary and macroprudential policy fully eliminates economic distortions. d) In the case that there are country-specific macroprudential tools and for $\alpha \neq 1$, full stabilization of aggregate and relative output and inflation is possible but not optimal.

**Proof.** a) Macroprudential policy is superior to monetary policy, hence $R_t = 0$. The macroprudential tool is set according to $\hat{\tau}_t^W = -b\hat{\phi}_t^W - n\epsilon H_{\pi,t}^w$ which eliminates aggregate distortions, $\hat{y}_t^W = \pi_t^W = 0$. Since the policymaker ignores differentials there is still a welfare loss arising from fluctuations in $\pi_t^R, \hat{S}_t, \text{ and } b\hat{\phi}_t$. b) The inefficient labor allocation implies that varying instruments causes additional losses according to (30). c) The optimal policy mix implies $\hat{\tau}_t^W = -b\hat{\phi}_t^W - n\epsilon H_{\pi,t}^w$ and $R_t = 0$ in order to stabilize the economies on the union level, $\hat{y}_t^W = \pi_t^W = 0$. Furthermore, the country-specific tools are set such that $\hat{\tau}_t^R = -b\hat{\phi}_t^R - \epsilon^H_{\pi,t}$ which eliminates relative distortions, $\pi_t^R = \hat{S}_t = 0$ (see (21), (22), (23)), and insures efficiency $\Psi_t = 0$ (see (30)) in every period since there are no sectoral distortions. d) Due to the fact that there is an inefficient labor allocation, varying instruments is costly per se and causes additional losses according to (30).

A positive Home markup shock raises union and relative inflation. Figure (3) displays the impulse responses for the benchmark specification. The positive inflation differential lets the Home terms of trade deteriorate and the relative output gap becomes negative due to the decline in relative demand. In absence of macroprudential policy (blue line), the optimal interest rate hike only diminishes union-wide fluctuations. Without sectoral distortions ($\alpha = 1$), the nominal interest rate is kept constant ($R_t = 0$) as the union-wide macroprudential tool ($\hat{\tau}_t^W = -b\hat{\phi}_t^W - n\epsilon H_{\pi,t}^w$) is able to stabilize the economies on the aggregate level ($\hat{y}_t^W = \pi_t^W = 0$). Variations in all relative target variables remain as they cannot be addresses with aggregate instruments. Again, country-specific macroprudential tools are able to eliminate these distortions by setting $\hat{\tau}_t^R = -b\hat{\phi}_t^R - \epsilon^H_{\pi,t}$ so that $\pi_t^R = \hat{S}_t = 0$ which implies no welfare losses.

When there are different labor inputs ($\alpha \neq 1$), using policy instruments is costly per se so that the first-best outcome is not feasible anymore. Hence, a policy mix is required. In the case of a union-wide macroprudential tool (red line), it is optimal to
Figure 3: Impulse responses to a Home markup shock. No macroprudential tool (blue line), union-wide macroprudential tool (red line), national macroprudential tools (green line).
increase the nominal interest rate (in order to reduce union inflation) and to decrease the macroprudential tool (in order to relax the borrowing constraint which reduces aggregate marginal costs and union inflation). The availability of a dynamic macroprudential policy tool makes the optimal interest hike, the union inflation and the union output gap significantly lower compared to the case where only monetary policy serves as shock stabilizer.

With country-specific macroprudential tools (green line), the level of the union-wide instruments remains the same but the composition of $b^W_t$ changes such that $b^R_t$ decreases. Lowering the relative macroprudential tool reduces relative marginal costs and thus the inflation differential by relaxing the relative borrowing constraint.

5 Welfare Analysis

The objective of this section is twofold. First, we will show the size of welfare gain of having macroprudential policy at disposition. Second, we compare optimal policy under discretion with simple rules.

Table (1) displays the welfare losses of various types of shocks when optimal policy is conducted under discretion. For each shock type, Table (1) shows the losses for a different kind of macroprudential instrument set, expressed as a fraction of steady-state consumption that must be given up to equate welfare in the stochastic economy to that in a deterministic steady state.

Throughout all types of shocks there are significant welfare improvements by introducing a union-wide macroprudential tool. These gains range from 0.04 (idosyncratic demand shock) to 22.86 percent of steady-state consumption (aggregate markup shock).\footnote{Note that markup shocks create large welfare losses due to high deviations of the inflation gap since microfounded welfare functions attach a weight to inflation that can be over ten or twenty times higher than the one attached to the output term (see Woodford, 2003, Ch.6). For many macroeconomists this sounds counterintuitive. Either the intuition is wrong or the model does not capture important cost} When macroprudential policy is available the additional gain from having
country-specific macroprudential is relatively small (except for the idiosyncratic demand shock in the baseline specification). This result is robust to a large set of alternative parametrizations.

So far we have discussed the optimal policy mix under discretion. However, almost all of the research on macroprudential policy assumes that the design of the policy follows simple rules. Implementing some kind of elaborate banking sector or housing market into the standard general equilibrium model comes at the cost that it is not easily possible to derive a microfounded objective function of the policymaker. Therefore we now highlight the welfare differences between the optimal discretionary policy and various kinds of simple rules. Since Lim et al. (2011) show that some macroprudential instruments need to be adjusted discretionary, we assume that there is no credible technology to influence expectations systematically. Thus, we do not consider optimal policy under commitment.\[13\]

Regarding monetary policy we differentiate between a Taylor Rule and strict inflation targeting (SIT). In the first case, the nominal interest rate evolves according to: 

\[ \hat{R}_t = 0.8 \cdot \hat{R}_{t-1} + 1.5 \cdot \pi_t^w + 0.2 \cdot \hat{y}_t^w \]

as in Quint and Rabanal (2014) who estimate a Taylor Rule for the euro area. In the second case, the interest rate is set such that \( \pi_t^w = 0 \) for all \( t \). Our analysis in Section 4 has shown that macroprudential policy often seeks to eliminate distortions coming from fluctuations in the credit spread and the cost channel on both the union and the relative level. Hence, the macroprudential policy rules are set according to: 

\[ \hat{\pi}_t^w = - (\hat{b}_t \hat{\phi}_t^w + \hat{R}_t) \] and \[ \hat{\pi}_t^R = - \hat{b}_t \hat{\phi}_t^R. \]

Figure (4) shows the impulse responses to an aggregate markup shock under different policy regimes. Under discretion, the optimal policy mix consists of an increase in the nominal interest rate (in order to reduce inflation) and a decrease in the macroprudential tool (in order to relax the borrowing constraint which reduces marginal costs and inflation) as already discussed in Section 4.1. SIT eliminates all fluctuations in union inflation at cost of the largest output gap and credit spread by a small interest rate response but a large macroprudential policy reaction. The macroprudential tool is increased in order to counteract the large drop in the credit spread. Hence, in contrast to discretionary policy, macroprudential and monetary policy become (imperfect) substitutes as both policies tighten the borrowing constraint. The Taylor Rule exhibits the strongest interest rate response and a rise in the macroprudential tools as well. However, this leads to the highest inflation as the cost-push effects of the policy mix are relatively large.

Table (2) depicts the welfare losses under different policy regimes for various kinds drivers of the output gap. For a pragmatic view - conduct a robustness check by varying the weights - see Wren-Lewis (2011) and Kirsanova et al. (2013).

\[13\] We refer to De Paoli and Paustian (2013) who consider both discretion and commitment.
Figure 4: Impulse responses to an aggregate markup shock under different policy regimes

<table>
<thead>
<tr>
<th>policy regime</th>
<th>demand shock</th>
<th>net worth shock</th>
<th>markup shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>aggregate</td>
<td>idiosyncratic</td>
<td>aggregate</td>
</tr>
<tr>
<td>Taylor</td>
<td>0.1282</td>
<td>0.1929</td>
<td>0</td>
</tr>
<tr>
<td>SIT</td>
<td>0</td>
<td>0.1609</td>
<td>0</td>
</tr>
<tr>
<td>discretion</td>
<td>0</td>
<td>0.0762</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: Welfare losses under different policy regimes
of shocks. Several comments are in order.

First, there is a clear-cut welfare ranking: discretion outperforms SIT and SIT outperforms the Taylor Rule. Since our welfare measure puts a relatively high weight on inflation, SIT comes relatively close to the optimal policy response. In contrast, the Taylor Rule allows for the highest variations in inflation and performs therefore the worst. Second, stabilizing union inflation is optimal in the case of an aggregate demand shock. Hence optimal discretionary policy and SIT coincide and lead to the first-best outcome. In response to an idiosyncratic demand shock, both discretion and SIT perfectly offset fluctuations in union target variables. However variations in relative target variables remain and these are better stabilized by the optimal discretionary policy. Third, the net worth shock can be perfectly absorbed by all of the policy regimes. The optimal response to offset the net worth shocks implies that the interest rate is held constant \((\hat{R}_t = 0)\) and that the macroprudential tools are set according to \(\hat{\tau}_t^W = -b\hat{\phi}_t^W\) and \(\hat{\tau}_t^R = -b\hat{\phi}_t^R\) (as stated in Proposition 1 and 4) which coincides with the macroprudential rules.

The main results are summarized in the following:

**Proposition 6** a) There are significant welfare improvements by introducing a union-wide macroprudential tool. The additional welfare gain from varying the relative macroprudential tool is small. b) Regarding policy regimes we get the following ranking in terms of welfare: discretion outperforms SIT and SIT outperforms the Taylor Rule.

A final question that may arise is whether an augmented Taylor Rule is not superior to the case of introducing macroprudential policy. We have investigated this issue for several augmented Taylor Rules and found that in almost all possible scenarios a simple (monetary) rule without macroprudential policy performs worse than the policy regimes we have considered in our analysis.

### 6 Conclusions

This paper investigates the optimal monetary and macroprudential policy mix in a currency union in presence of aggregate and idiosyncratic demand, net worth and markup shocks. We incorporate a credit friction into the standard New Keynesian model by assuming that firms with the need for external finance have to back their borrowing with collateral. Given our assumptions on nominal rigidities and financial frictions trade-offs arise between stabilizing inflation, output, the credit spread as well as the terms of trade. As the interest rate is a too blunt of an instrument there is a rationale for the use of macroprudential policy as a stabilization tool. In particular, the presence of idiosyncratic shocks calls for actions in country-specific macroprudential policy.
In our analysis, the monetary and macroprudential instruments are modelled as independent tools. By introducing macroprudential policy, fluctuations on the union level can be fully absorbed for a large set of different scenarios. Welfare losses due to variations on the relative level remain even when country-specific macroprudential tools are available as long as there is an inefficient allocation between labor inputs. Introducing a union-wide macroprudential tool therefore improves welfare significantly. The additional welfare gain from varying the relative macroprudential tool is small though. The setup of our model allows us to study welfare-based (optimal) monetary and macroprudential policy in a currency union which is the main difference to other studies who assume a simple rules policy design. Therefore, we compare optimal discretionary policy with macroprudential and monetary simple rules. Evaluating the performance of these policy regimes with a microfounded welfare criterion yields the following results: discretion outperforms SIT and SIT outperforms the Taylor Rule.

Our analysis can be extended in several directions: Introducing an elaborated interbank market (as in Gertler and Karadi, 2011 or Dellas et al., 2014, for example) or a housing sector (as in Iacoviello, 2005), or modelling the strategic interaction between the central bank and regulatory authorities in the sense of De Paoli and Paustian (2013) are important issues for future research.

Appendix

Appendix A: Log-linearizing the Model

Following De Paoli and Paustian (2013), we focus on the dynamics induced by distortions (monopolistic competition, cost channel and collateral constraint). Hence, we want to derive the equilibrium conditions of the model in terms of log deviations from an efficient steady state. Additionally, we assume that the collateral constraint always binds in the steady state and in its neighborhood. For Home, log-linearizing the household’s first order conditions (6)-(9) and aggregate demands (2) results in

\[
\begin{align*}
\sigma \dot{c}_t^H + \theta \tilde{L}_t^H &= \tilde{w}_t^H - (1 - n) \tilde{S}_t, \\
\sigma \dot{c}_t^W + \theta \tilde{u}_t^H &= \tilde{r}_t^H - (1 - n) \tilde{S}_t, \\
\sigma (E_t \tilde{c}_{t+1}^W - \tilde{c}_t^W) &= \tilde{R}_t - E_t \tilde{\pi}_{t+1}^W, \\
\sigma (E_t \tilde{c}_{t+1}^H - \tilde{c}_t^H) &= \beta E_{t+1} \tilde{q}_t^H - \tilde{q}_t^H + (1 - \beta) E_t \tilde{D}_{t+1}^H + E_t \tilde{\pi}_{t+1}^H - E_t \tilde{\pi}_{t+1}^W, \\
\tilde{y}_t^H &= (1 - n) \tilde{S}_t + \tilde{c}_t^W. 
\end{align*}
\]

By combining (A.3) and (A.5) plus adding country-specific demand shocks, we obtain the Home IS curve (19) and, similarly, the Foreign IS curve (20).
For entrepreneurs in country $i$, we write the production function, the collateral constraint (11), the first-order conditions (12)-(13), and the budget constraint (15) in terms of log deviations

\[
\begin{align*}
\tilde{y}_i^t &= \alpha \tilde{L}_i^t + (1 - \alpha) \tilde{u}_i^t, \\
\tilde{r}_i^t + \tilde{R}_i + \tilde{\omega}_i^t + \tilde{L}_i^t &= b(\tilde{c}_{i-1}^t + \beta \tilde{q}_i^t + (1 - \beta) \tilde{D}_i^t + \epsilon_{nw,t}^i) + (1 - b)(\tilde{y}_i^t + \tilde{mc}_i^t), \quad \text{(A.6)} \\
\tilde{y}_i^t + \tilde{mc}_i^t &= \tilde{r}_i^t + \tilde{R}_i + \tilde{\omega}_i^t + \tilde{D}_i^t, \quad \text{(A.7)} \\
\tilde{c}_i^t + \tilde{q}_i^t &= \tilde{y}_i^t + \tilde{mc}_i^t + \Lambda^i \tilde{\phi}_t, \quad \text{(A.8)} \\
\tilde{y}_i^t + \tilde{mc}_i^t &= \tilde{r}_i^t + \tilde{\omega}_i^t, \quad \text{(A.9)} \end{align*}
\]

where $\Lambda^i \equiv F^i_t (1 + b \phi^i)^i$, $F^i_t \equiv \frac{b \phi^i}{1+b \phi^i} + \left( \frac{1}{1+b \phi^i} \right)^{1/b}$, $F^i_\phi = \frac{b \phi^i}{(1+b \phi^i)^2} - \left( \frac{1}{1+b \phi^i} \right)^{1/(1+b)}$ and

\[
\tilde{\phi}_t = \frac{(\phi_t^i - \phi^i)}{(1+b \phi^i)}. \]

Using (A.1)-(A.2), (A.6), and (A.8)-(A.9), we arrive at an expression for the Home firm’s real marginal costs (17). Similarly, we obtain

\[
\tilde{mc}_c F = \theta \tilde{y}_t^F + \sigma \tilde{W}_t + n \tilde{S}_t + \alpha(\tilde{R}_t + \tilde{\omega}_t + \tilde{r}_t^F + b \tilde{\phi}_t), \quad \text{(A.11)}
\]

for Foreign.

For firms in Home, inserting (17) and (A.5) into (16) results in the Phillips curve (21). The same applies to (22). Next, we write dividends in terms of log deviations

\[
\tilde{D}_i^t = \tilde{y}_i^t - (\varepsilon - 1) \tilde{mc}_i^t. \quad \text{(A.12)}
\]

We can obtain a backward-looking condition for the evolution of net worth (18) by combining (A.7), (A.10), and (A.12). Inserting firm’s real marginal costs (17) or (A.11) yields the dynamic net worth equations (24) and (25).

Using (A.4), (A.10) in $t$ and $t + 1$, (18), and (A.12) scrolled forward, we get

\[
\sigma(E_t \tilde{c}_t^W - \tilde{c}_t^W) = E_t \Delta \tilde{y}_{t+1}^i + E_t \Delta \tilde{mc}_{t+1}^i - E_t \tilde{\phi}_{t+1}^i - \Lambda^i \tilde{\phi}_t^i + E_t \tilde{\pi}_{t+1}^i - E_t \tilde{\pi}_{t+1}^W - E_t \epsilon_{nw,t+1}^i. \quad \text{(A.13)}
\]

Inserting firm’s real marginal costs (17) or (A.11) yields the forward-looking conditions for the evolution of the credit spreads (26) and (27).

Finally, (23) can be obtained by log-linearizing the definition of the terms of trade.
Appendix B: Union’s Welfare Loss

The central bank’s loss function is given by

\[ \psi_t = U(C^W_t) - nV(L^H_t) - (1 - n)V(L^F_t) - nV(u^H_t) - (1 - n)V(u^F_t). \]  

(B.1)

Subtracting the corresponding steady-state values gives

\[ \psi_t - \bar{\psi} = U(C^W_t) - U(C) - n[V(L^H_t) - V(\bar{L}^H)] - (1 - n)[V(L^F_t) - V(\bar{L}^F)] \\
- n[V(u^H_t) - V(\bar{\pi}^H)] - (1 - n)[V(u^F_t) - V(\bar{\pi}^F)]. \]  

(B.2)

We take a second-order approximation of the consumption part in the utility function (1), \( U(C^W_t) \) around its steady-state value \( \bar{C} \)

\[ U(C^W_t) - U(C) = \bar{C}^{1-\sigma} \left[ c^W_t + \frac{1 - \sigma}{2} (c^W_t)^2 \right] + o \left( \| \xi \| ^3 \right). \]  

(B.3)

For country \( i \), taking a quadratic approximation of both labor supply terms yields

\[ V(L^i_t) - V(L^i) = B_1(L^i)^{1+\theta} \left[ \bar{L}^i_t + \frac{1 + \theta}{2} (\bar{L}^i_t)^2 \right] + o \left( \| \xi \| ^3 \right), \]  

(B.4)

\[ V(u^i_t) - V(\bar{\pi}^i) = B_2(\bar{\pi}^i)^{1+\theta} \left[ \bar{u}^i_t + \frac{1 + \theta}{2} (\bar{u}^i_t)^2 \right] + o \left( \| \xi \| ^3 \right). \]  

(B.5)

We insert (B.3), (B.4) and (B.5) into (B.2)

\[ \psi_t - \bar{\psi} = \bar{C}^{1-\sigma} \left[ c^W_t + \frac{1 - \sigma}{2} (c^W_t)^2 \right] - nB_1(\bar{L}^H)^{1+\theta} \left[ \bar{L}^H_t + \frac{1 + \theta}{2} (\bar{L}^H_t)^2 \right] \\
- (1 - n)B_1(\bar{L}^F)^{1+\theta} \left[ \bar{L}^F_t + \frac{1 + \theta}{2} (\bar{L}^F_t)^2 \right] - nB_2(\bar{\pi}^H)^{1+\theta} \left[ \bar{u}^H_t + \frac{1 + \theta}{2} (\bar{u}^H_t)^2 \right] \\
- (1 - n)B_2(\bar{\pi}^F)^{1+\theta} \left[ \bar{u}^F_t + \frac{1 + \theta}{2} (\bar{u}^F_t)^2 \right] + o \left( \| \xi \| ^3 \right). \]  

(B.6)

Assuming perfect risk-sharing (\( \bar{C} = \bar{C}^H = \bar{C}^F \)) and that the steady-state employment subsidies \( \Omega^H_i \) and \( \Omega^F_i \) are used to offset the distortions by monopolistic competition, the cost channel and the collateral constraint (see De Paoli and Paustian, 2013), we can obtain the following relations

\[ B_1(\bar{L}^i)^{1+\theta} = \alpha(\bar{C})^{1-\sigma}, \]  

(B.7)

\[ B_2(\bar{\pi}^i)^{1+\theta} = (1 - \alpha)(\bar{C})^{1-\sigma}. \]  

(B.8)
Combining the production function with the total demand function for \( h \) yields

\[
(L_t)^\alpha (u_t)^{1-\alpha} = S_t^{1-n} C_t^W \int_0^n \left( \frac{P_t(h)}{P_H;H_t} \right)^{-\varepsilon} dh
= Y_t^H \int_0^n \left( \frac{P_t(h)}{P_H;H_t} \right)^{-\varepsilon} dh.
\] (B.9)

It can be shown (see Gali 2008) that

\[
\alpha \hat{L}_t^H + (1 - \alpha) \hat{u}_t^H = \hat{y}_t^H + \ln \int_0^n \left( \frac{P_t(h)}{P_H;H_t} \right)^{-\varepsilon} dh + o(\| \xi \|^3)
= \hat{y}_t^H + \frac{\varepsilon}{2} \text{var}_h \tilde{P}(h) + o(\| \xi \|^3).
\] (B.10)

Similarly, we can obtain the following relation for Foreign

\[
\alpha \hat{L}_t^F + (1 - \alpha) \hat{u}_t^F = \hat{y}_t^F + \frac{\varepsilon}{2} \text{var}_f \tilde{P}(f) + o(\| \xi \|^3).
\] (B.11)

Inserting (B.7), (B.8), (B.10), (B.11), (A.5) and \( \hat{y}_t^F = -nS_t + c_t^W \) into (B.6) yields

\[
\frac{\psi_t - \bar{\psi}}{U_C C} = \frac{1 - \sigma}{2} (c_t^W)^2 - n \frac{\varepsilon}{2} \text{var}_h \tilde{P}(h) - (1 - n) \frac{\varepsilon}{2} \text{var}_f \tilde{P}(f)
- n \frac{1 + \theta}{2} [\alpha (\hat{L}_t^H)^2 + (1 - \alpha) (\hat{u}_t^H)^2]
- (1 - n) \frac{1 + \theta}{2} [\alpha (\hat{L}_t^F)^2 + (1 - \alpha) (\hat{u}_t^F)^2] + o(\| \xi \|^3),
\] (B.12)

which can be rearranged

\[
\frac{\psi_t - \bar{\psi}}{U_C C} = \frac{1 - \sigma}{2} (c_t^W)^2 - n \frac{\varepsilon}{2} \text{var}_h \tilde{P}(h) - (1 - n) \frac{\varepsilon}{2} \text{var}_f \tilde{P}(f)
- n \frac{1 + \theta}{2} [\alpha (1 - \alpha)(\hat{u}_t^H - \hat{L}_t^H)^2 + (\hat{y}_t^H)^2]
- (1 - n) \frac{1 + \theta}{2} [\alpha (1 - \alpha)(\hat{u}_t^F - \hat{L}_t^F)^2 + (\hat{y}_t^F)^2] + o(\| \xi \|^3).
\] (B.13)

Now, we combine (6), (7), (12), and (13) to get an expression for labor choice

\[
\frac{1 - \alpha}{\alpha} \tau_t^i R_t(1 + b \phi^i) = \frac{(1 + \Omega^w_i) B_2(u_t^i)^{1+\theta}}{(1 + \Omega^w_i) B_1(L_t^i)^{1+\theta}}.
\] (B.14)
In terms of log deviations from an efficient steady state

\[ \tilde{R}_t + \tilde{\tau}_t + \tilde{b}\phi_t = (1 + \theta)(\tilde{u}_t - \tilde{L}_t), \]  

where \( \tilde{\phi}_t = \frac{\phi^*_t - \phi^*_0}{1 + b\phi^*_0} \).

Next, we take a second-order approximation of aggregate demands

\[ \begin{align*}
\tilde{y}_t^H + \frac{1}{2}(\tilde{y}_t^H)^2 &= (1 - n)S_t + \tilde{c}_t^W + \frac{1}{2}(1 - n)^2S_t^2 + \frac{1}{2}(\tilde{c}_t^W)^2 \\
&+ (1 - n)S_t\tilde{c}_t^W + O(\|\xi\|^3) \\
\tilde{y}_t^F + \frac{1}{2}(\tilde{y}_t^F)^2 &= -nS_t + \tilde{c}_t^W + \frac{1}{2}n^2S_t^2 + \frac{1}{2}(\tilde{c}_t^W)^2 \\
&- nS_t\tilde{c}_t^W + O(\|\xi\|^3). \end{align*} \]  

By inserting (B.15) and (B.16) we can simplify (B.13) as

\[ \frac{\psi_t - \bar{\psi}}{U_{CC}} = \frac{-\sigma + \theta}{2}(\tilde{y}_t^H)^2 - n\frac{\varepsilon}{2}var_h\tilde{P}_t(h) - (1 - n)\frac{\varepsilon}{2}var_f\tilde{P}_t(f) \\
- \frac{1}{2}n(1 - n)(1 + \theta)(\tilde{S}_t)^2 - \frac{1}{2}n\frac{\alpha(1 - \alpha)}{1 + \theta}(\tilde{R}_t + \tilde{\tau}_t + \tilde{b}\phi_t)^2 \\
- \frac{1}{2}(1 - n)\frac{\alpha(1 - \alpha)}{1 + \theta}(\tilde{R}_t + \tilde{\tau}_t + \tilde{b}\phi_t)^2 + O(\|\xi\|^3). \]  

Finally, it can be shown (see Woodford, 2003, chap. 6) that

\[ \sum_{t=0}^{\infty} \beta^t var_i p_t(i) = \frac{\theta^i}{(1 - \theta^i)(1 - \beta\theta^i)} \sum_{t=0}^{\infty} \beta^t(\pi_t^i)^2. \]  

Using this expression, the union’s welfare function can be written as

\[ W = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\psi_t - \bar{\psi}}{U_{CC}} \right\} = -E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \psi_t \right\} + O(\|\xi\|^3), \]  

where

\[ \psi_t = (\sigma + \theta)(\tilde{y}_t^H)^2 + n\frac{\varepsilon}{\lambda^H}(\pi_t^H)^2 + (1 - n)\frac{\varepsilon}{\lambda^F}(\pi_t^F)^2 + n(1 - n)(1 + \theta)(\tilde{S}_t)^2 \\
+ n\frac{\alpha(1 - \alpha)}{1 + \theta}(\tilde{R}_t + \tilde{\tau}_t + \tilde{b}\phi_t)^2 + (1 - n)\frac{\alpha(1 - \alpha)}{1 + \theta}(\tilde{R}_t + \tilde{\tau}_t + \tilde{b}\phi_t)^2. \]  

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References


