Swarm Data Mining for the Fine Structure of Thermals

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Abstract

Accurate thermal models can help to optimize the design of sailplanes. Theoretical models should be based on empirical evidence. However, very few measurements on the structure of thermals are published. This paper uses data mining techniques on data collected by swarms. In this case the swarm consists of the world's best pilots in the world's best gliders competing in a world championship at Uvalde, Texas in 2012. It is pointed out how the data collected by this swarm in the form of ICG files (i.e. GPS recordings) may be processed in order to yield the vertical speed of the air in thermals. This resulted in about 100 hours of data on thermals. From this data a model (GTB) of the fine structure of thermals could be derived consisting of three components: a Gaussian representing the buoyancy, a vortex modeling entrainment and a border vortex caused by the difference in speed between the air inside the thermal and the surrounding air.

1 Introduction

The Diana-2 sailplane was the winner's sailplane in the 15 meter class of the 2012 World Gliding Championship in Uvalde, Texas. The design of this sailplane has been optimized using a model structure of thermals [1]. However, as Kubrynski, the author of [1], puts it: "a very limited amount of measured data is available in the literature, making this problem even more difficult". This means, only a handful of measurements (see chapter 8) form the empirical basis of present day's thermal models. On the other hand, the flight data of most cross country flights and all of the competition flights are measured and logged every second using GPS devices. These in-flight measurements are well documented in the form of ICG-files [2]. This paper reports the methods and results of mining this data with the aim to derive a thermal model with a much broader empirical basis, i.e. about 100 hours of measured flights spent spiraling in thermals.

2 Flight Data and Soaring Conditions

In order to obtain comparable flight data the flights in Ventus-2, ASW-27 and ASW-29 (15m) sailplanes in the 15 meter class of the Uvalde competition were used. Only flights were used which finished with a rank of not more than 25 (of 37 participants) on each day. To the best of my knowledge all pilots used the maximal takeoff weight of their gliders resulting in the same i.e. the maximal wing load of the sailplanes. This assured that all pilots performed on a very high level of competence and that all sailplanes were best prepared. All flights took place during August 5th to 18th 2012. Start and finish airport was Garner Airfield with an elevation of 287m (942ft). Elevation of the terrain ranged from 200-700m (700-2300ft). Only flight data after the start of the race was used. The observed thermals were all between 5.2 and 6.9 hours after sunrise in Uvalde. The thermals are localized in an area between 98.5 to 100.6 degrees West and 27.5 to 30.5 North. Weather conditions in the semi-arid climate zone of south western Texas during that period were rather constant with temperatures around 40° C, 0-3 octas of Cu with a base of 2300-3000m. Wind direction was mostly in the interval of 250-350 degrees. Wind speed was Gauss-distributed with a mean of 19 km/h +-9 km/h s.dev. The average integrated net climb speed of all thermals followed a Gaussian (N(m,s)) with a mean of 3.0 m/sec and s.dev. of 0.81 m/sec. The GPS altitudes of the first and last fixes in the Helix (= centered spiraling and climbing in circles) were N(1800,360) [m] (first) and. N(2200,370) [m] (last). Height gain in these Helices ranged from 200 to 1600m. The thermal data collected at Uvalde in this way is called U-thermals in this paper.

3 Data Mining Methods

Climbs in the IGC files of the flights were identified as successive periods of flight where at least 250m of altitude was gained and the engine noise level indicated normal gliding flight. In these climbs periods a circling flight was identified by least three full circles in the same direction (see Fig. 1)

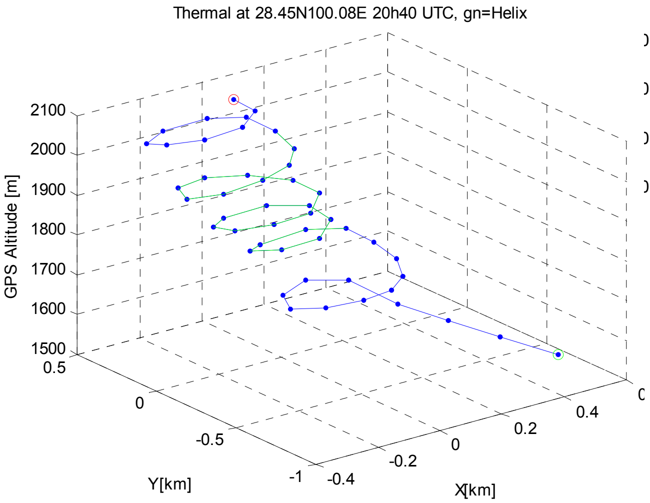

Figure 1: Flight data of a climb and a Helix (green), xy coordinates in Gauss-Kruger centered at the median of the climb fixes.

Wind direction and speed during the spiraling was estimated from the average movement of the centers of these full circles. This wind data was subtracted from the Gauss-Kruger xy-coordinates of the GPS fixes. All fixes were normalized to a one second interval using local spline interpolation for xy and GPS altitude h if necessary.

Periods where the flight path was not turning with at least a rate of 3°/sec (centering) were excluded. This led to a total helix-shaped flight periods of 56.9 h (Ventus-2), 22.1 h (ASW-27) and 24.8 h (ASW-29). The centers of the helix were estimated using the method of Kasa [3] using successive points indicating a direction change of at least 360°. The distance from this center is used as the momentary radius of the turn. This is denoted as Radius [m] in the following figures. Figure 2 shows the coordinates of the wind corrected Helix as part of the spiraling flight shown in Figure 1. For all Helix data of the same sailplane type the turn Radius [m] as well as the successive altitude differences DH [m] were calculated for each successive seconds. The altitude gain was compensated for total-energy.

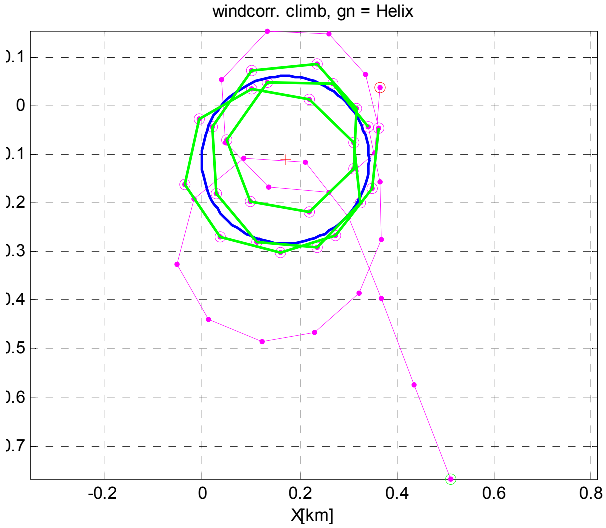

Figure 2: Top view of the fixes of Figure 1 corrected by wind drift. Helix data in green.

Within a range of 50-290 m, the DH values were binned with respect to the radius in bins of 1m. For Radius > 290m, a bin width of 5m was used. This assured at least 50 data points in each bin. Averaging over these bins resulted in the data of Figure 3.

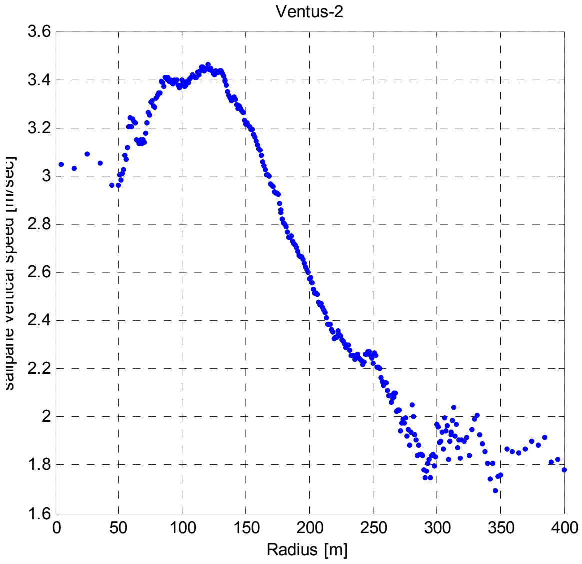

Figure 3: Total vertical speed (= vertical air + sailplane sink) for the Ventus-2 averaged over the Radius bins.

From the radius of the turn and the momentary speed of the sailplane the angle of banking could be calculated. Using the L/D of the particular aircraft an estimation of the sink in the turn was calculated. Adding this velocity to the sailplane's vertical speeds led to an estimation of the momentary encountered total vertical speed, see Figure 4.

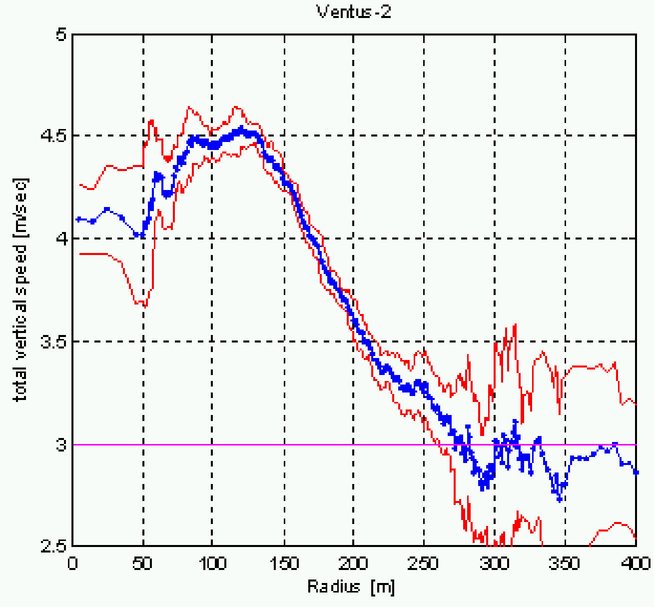

Figure 4: Achieved vertical speed averaged on 47.6 h of Helix flights of Ventus-2. Red lines: bin variances

4 From Swarm Data to Vertical Speeds of Thermals

All the data considered above was not collected with the aim to measure the vertical velocity of air in a thermal. This is a typical example of so called “Swarm Data”. A swarm of world class pilots in high performance sailplanes were sent out to find the best thermals, center as efficiently as possible and try to find the best possible climbing method. The aim of each member of this swarm is to win the competition. This only can be done by making the best (= most efficient) use of every meteorological situation. So we do not possess planned measures. However, we can assume that the pilots fly as best as they can. This means the measures of Figure 4 can be considered to squeeze the maximum out of the meteorology and the maximum out of the sailplane. This gives a hint on how to rescale the data. The vertical speed “far away from the center of the thermal” i.e. for a turn radius>300m can be associated with a vertical air speed of zero (see the horizontal magenta line in Figure 4).The rescaled data is presented in Figure 5. From 300m inward to 130 m (vertical magenta lines in Figure 5), the vertical speed increases from 0 up to a maximum of 4.4 m/sec. A linear model of the data in the range 130-230m results in a gradient of 3.6 m/s per 100m (green line in Figure 5).

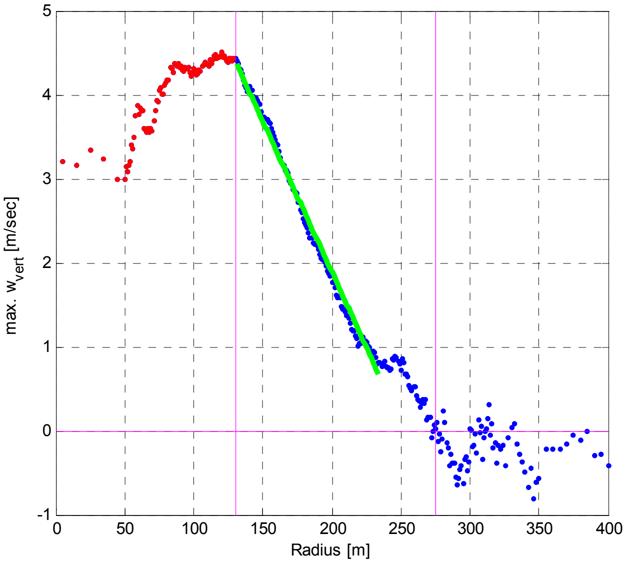

Figure 5: Points are the vertical speed achieved by the swarm of Ventus-2 sailplanes, red: drop of performance due to turning polar, green: linear gradient.

For a turn radius of less than 130m the vertical speed is constant respectively decreasing (red dots in Figure 5). The rule of the swarm data gathering is to collect as much lift as possible. Within these distances from center of the thermal the limiting factor, however, is the performance limit of the aircraft. In order to obtain a smaller turn radius, pilots must turn at a steeper angle. This increases the sink speed, as the well known turn polar shows. This means at these small radii the increased sink due to the bank angle in the turn compensates or exceeds the additional lift of the thermal.

5 The Core of the Thermals

Since all three sailplane types (Ventus-2, ASW-27, ASW-29) showed the same drop in performance, another type of measurements is necessary to explore the core of thermals. For this the same type of swarm data gathering and analysis was done on paragliders. Paragliders operate in a speed range of 30 to 55 km/h and are able to fly very small turning circles. Martin Serner, a high performance German paraglider pilot provided us with the data of 15 long distance cross country flights. The flights covered distances of more than 200km in the lower regions (Lowland) of Germany close to an axis of the towns of Cottbus and Lüsse. This resulted in a total of 4.5 h of helix flight of paragliders. Figure 6 shows the results of swarm data analysis in this case. It can be seen that paragliders reach their performance limits at a turn radius of about 30 m. This figure demonstrates that the vertical speed of the air will increase as we get close to the core of the thermal.

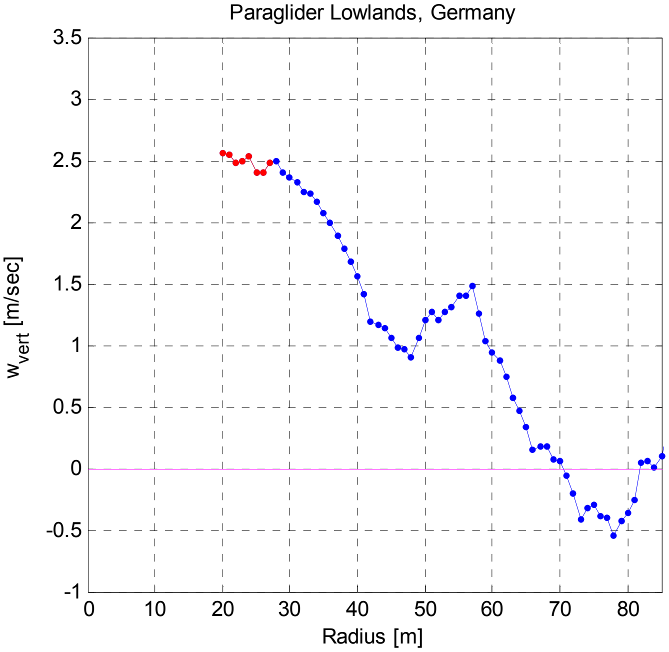

Figure 6: Vertical speeds vs. turning radius achieved by paragliders in Germany, red: drop of performance due to turning polar

In order to extrapolate the data to the core of the thermal we now know, that the vertical speed profile is increasing. The fine structure of Figure 6 in the range 30-40 m indicates, however, that this extrapolation is not linear.

6 Model for the fine structure of thermals

Under zero wind conditions we can assume, that the distribution of vertical speed in a thermal (profile) is symmetric. As the central limit theorem suggests, a plausible hypothesis of the shape of the profile is a Gaussian. This hypothesis was also used in the thermal models of Carmichael in 1954 [8]. A Gaussian was adapted using least square optimization to the symmetric values in the range 130-290 m, see Figure 7. Also a linear model was fitted to this data. The strength of the core at Radius=0 predicted by the Gaussian is 10.15 [m/sec] (s.dev = 150) and 8 [m/sec] for the linear model. A comparison to the paraglider data of Figure 6 suggests that these estimates are unrealistically high. In [5] Gerhard Waibel also insisted that most thermals have a flat top and are rather "hat shaped" than pointed. Therefore a model is sought that delivers a center strength of not significantly more than 6 m/s for U-thermals.

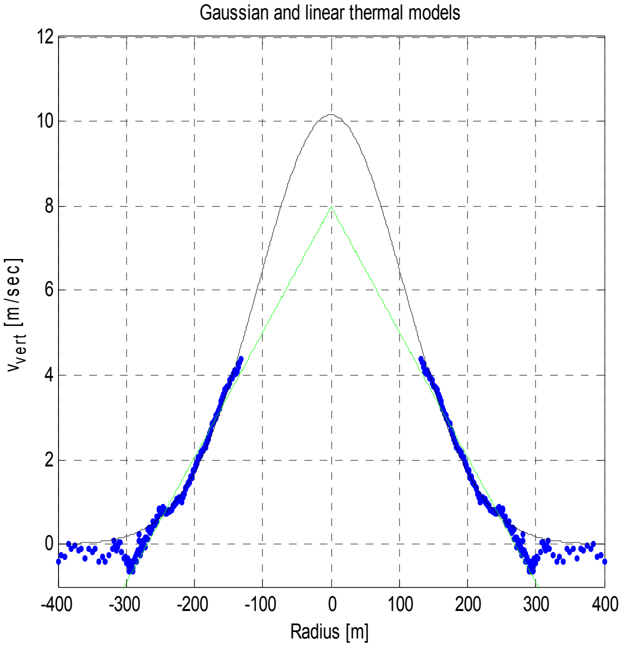

Figure 7: Linear (green) and Gaussian Model (black) for the core of thermals at Uvalde

We propose here an additive model of the thermal profile consisting of three components G, T and B **(GTB model)**:

Component G: consists of a profile of vertical velocities that are Gauss distributed. This models the buoyancy caused by the lower density of the air inside the thermal.

Component T: consist of a vortex modeling the exchange of air between the thermal and the surrounding air (entrainment). For the radial symmetric case this is a vortex torus centered at the middle of the thermal. At Radius=0 the vortex has its maximal downspeed. As a formula, this airflow describes a sinusoid which is symmetrical to the y-axis

Component B: this component models the effects encountered at the boundary layer of the thermal. We assume a small band in which a vortex is rolling. In the horizontal case this would be equivalent to a Kelvin-Helmholtz instability. Also in this small band a constant air flow is confined which models the global air mass flow vertically along the thermal. In the model this is a sinusoid of one period confined between Border_{min} and Border_{max} plus a constant vertical speed w_{border}

Side conditions for the model are :

- at Radius=0 the G and T component sum up to the maximum thermal strength.
- for Radius > R_{max} the vertical speed is zero
- the border vortex B ends at R_{max}
- the model should fit the U-thermals

In the case that the border conditions are less important the model reduces to the components G and T (GT model).

Using least square techniques the components were fitted to the U-thermals. The results were as follows:

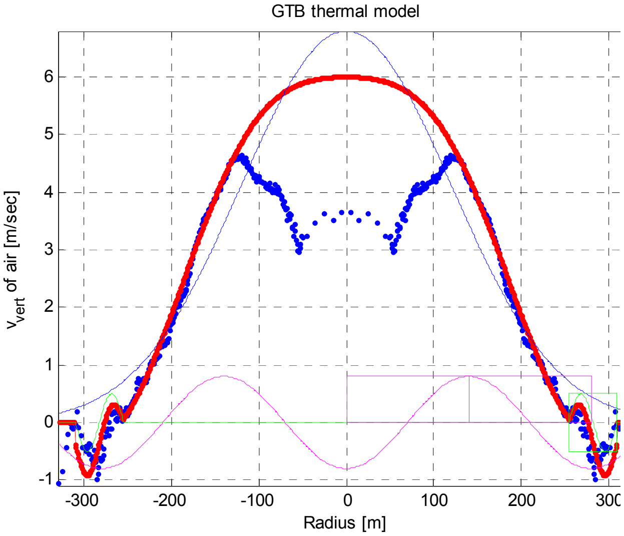

Figure 8: GTB thermal model of the fine structure of the thermal. Blue dots are the data points.

Maximum thermal strength at Radius=0: 6.0 m/sec, R_{max}= 310 m.

Component G: the s.dev. of the Gaussian is 170 m

Component T: A sinusoid with wavelength 140 m. The amplitude of the vertical speed of the entrainment vortex torus is 0.8 m/sec. Entrainment vortex and max buoyancy speed sum up to thermal core velocity of 6 m/sec.

Component B: this vortex has a phase length of 55m and amplitude of 0.5 m/sec. Contained within the boundaries of vortex is a downstream of air at a speed of 0 m/sec.

This border vortex is responsible for the increased sink at a radius of 300m in Figure 4 and the increased lift at about 260m.

The GTB model and its components are shown in Figure 8. Complete formulas and parameters of the models are given in the Appendix. The fine blue line in Figure 8 is the buoyancy Gaussian. The entrainment vortex is drawn in magenta. In the green box the border vortex can be seen.

7 GTB model applied to paragliding data

It may be the case that the GTB model is over adapted to U-thermals due to the nature of the swarm data collection in the semi-arid weather condition of south-west Texas. On the other extreme of the speed of uplift strengths and speeds is the data from the paragliders in Germany's Lowlands. The optimal GTB model for this case gives a maximum thermal strength of 3.2 m/sec, R_{max} = 87m, the s.dev. of the buoyancy Gaussian is 40m, the entrainment vortex has an amplitude of 0.4 m sec, and a period of 31m. The border vortex has a period of 40m with an amplitude of 1.0 m/sec. The border airflow was upward with 0.1m/sec.

Compared to the U-thermals the maximum strength in the core of these thermals is about half and the diameter is about one third. This may well be explained by the metorological conditions in northern Germany vs south-western Texas.

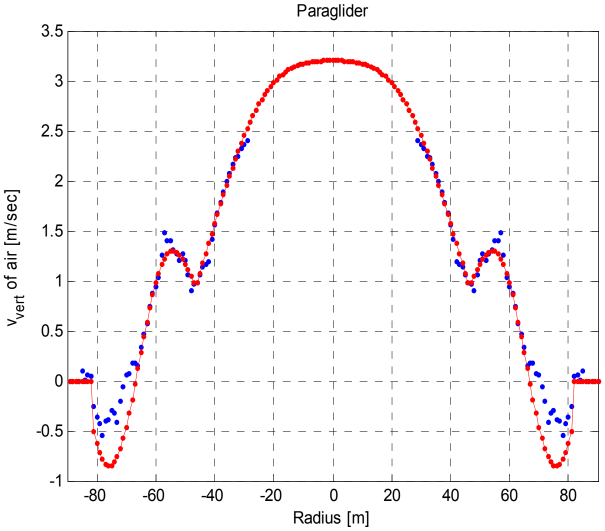

Figure 9: Symmetric data of thermal profiles from paragliding (blue) with adapted GTB model (red)

The period of the border vortex is also as big as in the U-thermals. However, the amplitude of the border vortex is twice as much as in the U-thermals. This may point to a bias in measuring this vortex using gliders or other airplanes.

Due to their mass and speeds (>500 kg, > 100 km/h) it could be, that this vortex is passed trough too quickly by planes and is not captured to it's full extend.

8 Comparison with published measurements of thermals

As pointed out above, very few actual measurements of thermals are published. One of the first is taken from the works of Konovalov [4] cited after Waibel [5]. In Figure 10 the single core type (Type a) is compared with the GTB model of the U-thermals (red line).

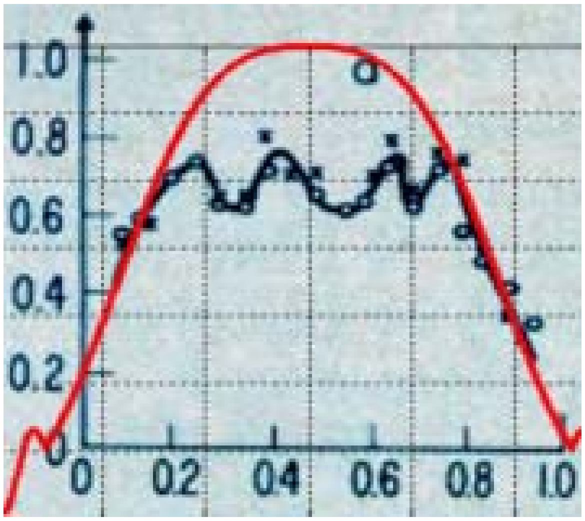

Figure 10 Konovalov's multiple core type thermal (a) reproduced from [5] and the GTB model.

Gerhard Waibel also pointed out an Idaflieg publication [6] with the measurement of a thermal. The model used there was a Fourier analysis (sum of sinusoids), see the black respectively brown lines in Figure 11.

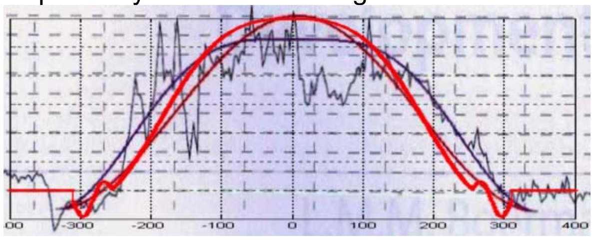

Figure 11: Inflight measurement of a thermal reproduced from [6], Abb 7. Black lines are the measurement data, other lines are interpolations proposed in [6].

In [7] flight data was collected by a specially instrumented Blanik glider flying over Rogers Dry Lake to the north of the Edwards Air Force Base in the Mojave desert in California in September 2006.The GTB model (red line) is compared with these data in Figure 12.

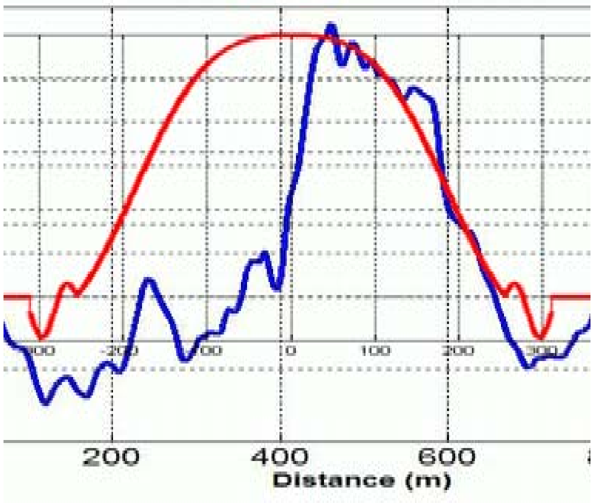

Figure 12 overlay of Blanik glider flying over Rogers Dry Lake, [7] Fig. 5 (blue), with the GTB model (red)

In the measuring flight of C. Lindeman [12] two thermals could be identified see Figure 13. The GTB models of the U-thermals were scaled such that the x-axis, i.e. the width of the thermals, are the same.

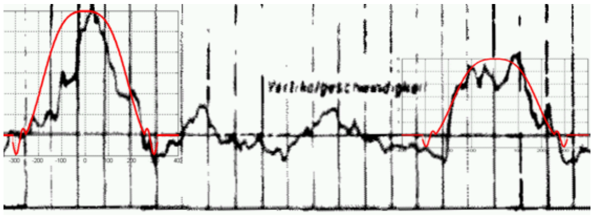

Figure 13 Inflight Measurements of C. Lindemann cited in [12], GTB (red) in the same horizontal scale.

Figures 10 to 13 show a fairly good coincidence with the presented model. This is astounding since the origin of the measurements is Russia, Germany and California. This may point that the average Uvalde–thermal as presented here (U-thermal) may represent some universal characteristics.

9 Comparison to published thermal models

One of the first models of the thermal comes from the Carmichael's publication in 1954 [8]. Compared to our model, these model distributions are too narrow. However, Carmichael derives the central Gaussian from the idea of a thermal jet stream.

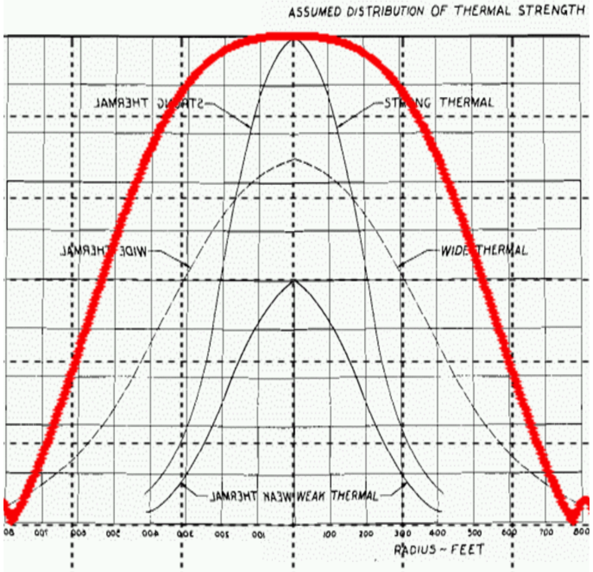

Figure 14 GTB in comparison to Carmichael's models [8] mirrored at y-axis. GTB is rescaled to fit Carmichael's units.

According to Kubrynski [1], Horstmann's [9] models are probably the most realistic approach. They include four standard thermal profiles: combination of strong (2) and weak(1) and wide (B) and narrow (A) thermals, see Figure 15.

Kubrynski [1] specified three thermal families A:narrow, B:wide, C:middle thermal.

Compared to both models the GTB model is much wider. In the GTB model the zone of rising air has a total diameter of about 500m.

It is remarkable that GTM-model of the U-thermals seem to have the same gradient as the A-types. of the Hartman profiles. This may have contributed to the successful optimization of the Diana-2 sailplane described in [1].

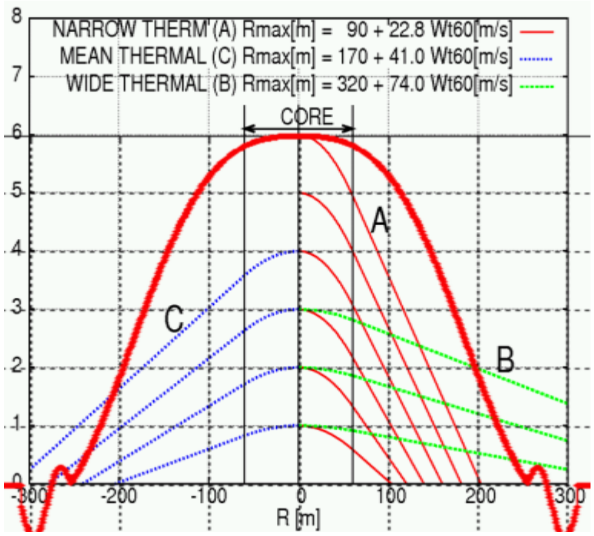

Figure 15 GTB model compared to the adaptation in [1] of Horstmann's [9] profiles

10 "Hat" type thermals

Gerhard Waibel triggered the research presented in this paper. In his talk at the OSTIV Congress in Uvalde 2012 he pointed out, that our thermal models do not fit to the experience of the pilots. He claimed, that the thermals are more of a "hat" type i.e. do have a flat or even impressed core. Gerhard also cited H. W. Grosse and G. Stich, who favor these hat-types of thermals. In the data of Childress [7] two measured thermals appear which fit to this hat-type. These can be modeled with GTB using a strong entrainment component T. Figure 16 and 17 show an possible fitting to such hat-thermal data from the flight measurements of [7] figures 3 (p12) and 7 (p14).

The GTB model explains the flat top respectively the dip in the core of a thermal as the interference of the entrainment vortex with the buoyancy Gaussian.

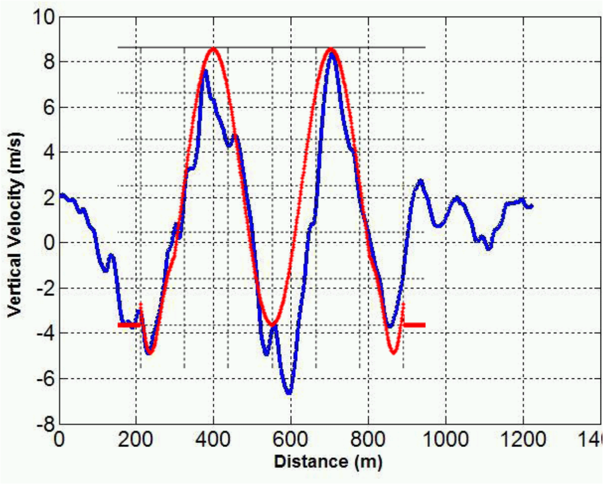

Figure 16 Hat-type of thermal from [7] figure 3 (blue) overlay in red a GTB model with strong T component

Gerhard Waibel also insisted that at the border of the thermal there seems to be sink. His hand drawings of a thermal profile resemble the typical “Mexican Hat” function. The sink at the border is explained in GTB by a border vortex caused by friction. This vortex can be observed in the U-thermals as well as in the paraglider data in Germany and also in the actual flight measurement data of Frey (Figure 11), Lindemann (Figure 13) and Childress (Figures 12, 16, and 17),

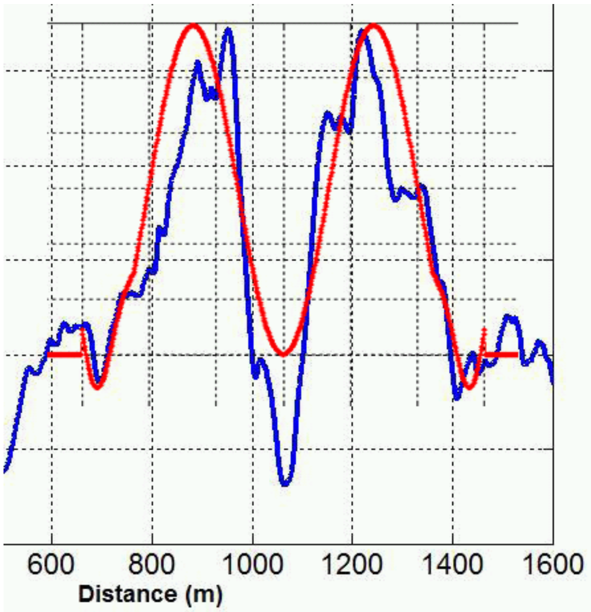

Figure 17 Hat-type of thermal from [7] figure 7(blue); overlay in red a GTB model with strong T component

11Discussion

There is extremely few measurement data published on thermals. So the data base for models of thermals is rather poor. However, the design of better gliders in the future calls for models that fit the nature and not what theoreticians think. The amount of data which, in principle, can be obtained by the swarm data mining method on logged IGC files of flights is enormous. While the published flight data sums up to only some minutes in thermals, the GTB model is derived using about 100 hours of flights flown spiraling in thermals.

However, the data analysis presented here relies heavily on the extraction and preprocessing of the data. As we all know from spectacular events, such as the explosion of the Ariane 5 at first launch, software implementation may be erroneous. So, the best practice would be, that an independent research group repeats the presented data analysis.

The comparison of GTB with the reported data is astoundingly consistent (see Figures 10 to 13). In comparison to the model profiles (Figures 14 and 15) the U-thermal data suggests a diameter of the thermal of approximately 600 m including the border vortex. The paraglider data, however, fits to a thermal diameter of ca. 200m. This may be explained by difference in meteorological conditions.

The weather conditions at Uvalde may not represent a typical flying day. In [10] and [11] the author has investigated the strengths of more than 10.000 thermals in August in Bavaria, Germany. The distribution of the thermal strengths follows extremely precisely a squared Gaussian (see Figure 16).

From this analysis it can be conjectured, that the meteorological conditions which produced U-thermals where the pilots found an average integrated lift of 3m/sec occur in less than 10% of all cases (green line in Figure 18). However, most national and international competitions are timed and placed such, that these very good conditions are likely to occur.

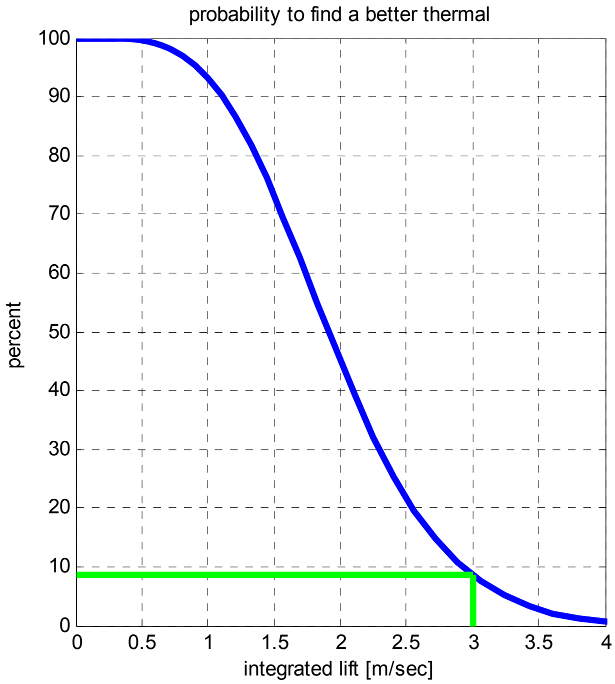

Figure 18: Probability to find a thermal of a given strength or better, first published in [10] and [11]

To explain the general vertical speed within the thermal using a Gaussian stems from the theory of jets, see [13] for a review. Carmichael writes "The shape is pure conjecture but at least the qualitative experience of pilots do not refute the theoretical guidance given by the turbulent free jet." [8], page 9.

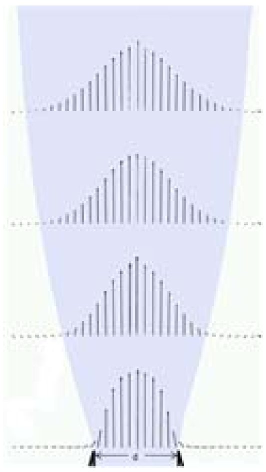

Figure 19: Speed profiles in a vertical jet with divergence, adapted from [14], Abb. 3.2

Figure 19 shows the distribution profiles of speeds in a vertical jet with a slight divergence [14]. In [10] it is observed that the square root of thermal strength is very precisely Gauss-distributed. As deeper reason behind this fact, it can be assumed that the radius of the superheated, respectively super-humid, thermal air bubble in the ground layer is Gauss distributed.

The G-Component of the GTB model can be thought of as the "average" of the speed profiles shown in Figure 19. However, U-thermal data can hardly be explained by one Gaussian alone. A single Gaussian would have a too pointed top in the middle and too heavy tails. Refer to the blue line in Figure 7. The addition of an entrainment vortex compensated for this.

Many pilots report an increased sink rate just "before the thermal begins" The paraglider pilot Martin Serner described even a feeling as "being drawn into the thermal". A border vortex could explain these effects.

One can imagine a series of vertical vortices, similar to the horizontal Helmholtz-Kelvin types, running on the outer border of the thermal (see Figure 20).

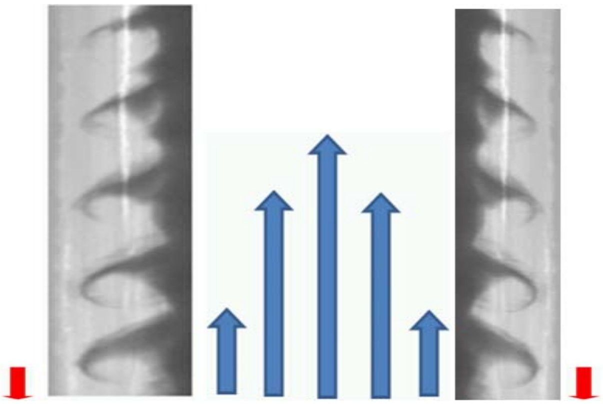

Figure 20 Visualization of the border vortex (B-component), derived from a photograph of Helmholtz-Kelvin wave clouds taken by B. Eppinger close to Spitzbergen (reproduced with permission).

The usage of vortices for a model of thermals is not new. Müller and Kottmeier, for example, compare in [12] thermals with pipes (constant uplift) and bubbles, i.e. toroid vortices. The GTB approach may allow quantifying the mixture ratio of both phenomena.

12 Types of Thermals?

From Carmichael 1954 [8] over Konovalov [4] who reported on data collected around 1960, to Horstmann 1976 [9] the authors suggest that there are different types of thermals: wide vs. narrow, weak vs. strong, pointed top vs. flat top and specific mixtures of this type. See, for example, Figure 15 for such types. For the U-thermals, the average integrated net lift, which the pilots found, follows a Gaussian N(3.0,0.81) m/sec (data not shown). Figure 21 shows the distribution of the average Helix Radius on all 1211 measured Verntus-2 Helices. These fit well to a Gaussian distribution with a slight overrepresentation at about 146 m. At that radius the pilots experienced the best lift. Most spiraling was done in a range of 130 to 200 m.

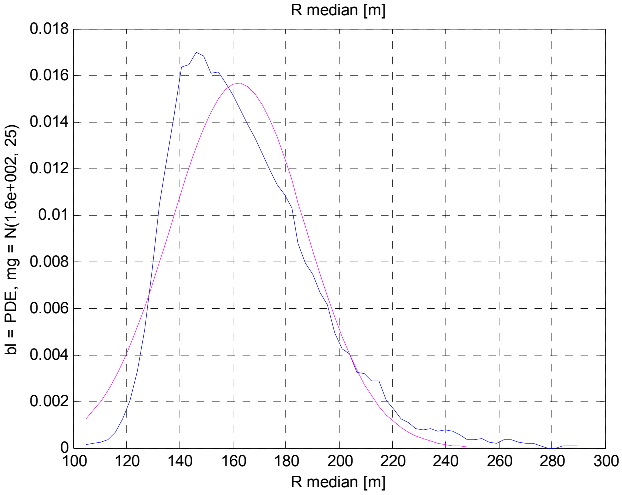

Figure 21: Distribution of the Median of all measured Vantus-2 Helix data, magenta Gaussian N(160, 24) m

In summary, the U-thermal data does not suggest any clusters of thermals. This can be attributed to the very homogeneous climatic and orographic conditions of the flights. Furthermore all measurements are all taken in a very short period of only 2 hours of the best flying time during each day. Furthermore the swarm pilots are conditioned to spiral in only the strongest thermals that can be found each day in order to win the world championship. So the U-thermal measurements are highly selective for only the class of strongest thermals. Other thermals types, in particular in other meteorological conditions, different orographic situations (mountains vs. flat country) and different time of the day (onset of thermal, “Umkehrthermik”) may well exist.

In order to capture these thermal types, data from many cross country flights in different weather conditions and landscapes would be necessary. However, nowadays the policy to obtain our own flight data back from the OLC database (www.onlinecontest.org*)* for scientific purposes is unfortunately too restrictive to allow such research. A submission of our flight data to an open source database such as, for example, www.skylines-project.org/about will help to overcome this problem.

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13 Conclusion

This paper sheds some light on the fine structure of thermal. Up to the present only a handful of inflight measurements of thermal data has been published. This analysis is based on averaging over ca. 100 hours of such flying time where the pilots have already centered the thermal and are climbing in circles (Helix). An enormous amount of flight data is there and – except from the OLC data base - freely available. A first approach was presented here on how to use this data for thermal models that describe what is going on in nature.

The result is a mixture model of buoyancy, entrainment and a vortex on the border. The same model could be applied to strong thermals with 6 m/sec core lift and 600m width in south west Texas down to paraglider flights in Germany’s flatlands with 3 m/sec core lift and 200m width.

As soon as more data is available and the data processing is standardized and error proofed we can expect, that theoretical thermal models can be verified using a solid empirical basis stemming from our “swarm data”.

The vertical fine structure of (U-) thermals remains for further research

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Appendix

GTB Model:
gives the vertical speed of air as a function of the distance r from the center of a thermal.
It is an additive model consisting of a Gaussian G (buoyancy), a large vortex T (mixture/friction) and a border vortex B (Helmholz/Kelvin).

w	[m/sec]	vertical speed of air
r	[m]	distance from center of thermal
M_G	[m/sec]	mean strength of central thermal Gaussian
S_G	[m/sec]	standard deviation of central thermal Gaussian
w_T	[m/sec]	amplitude of (entrainment) vortex
p_T	[m/sec]	width of (entrainment) vortex (period)
w_B	[m/sec]	amplitude of border vortex
p_B	[m/sec]	width of border vortex (period)
w_{0B}	[m/sec]	constant vertical speed of border vortex
r_{max}	[m]	limit radius of thermal: if r> r_{max}, it follows that w = 0

$$\begin{aligned} \mathbf{w = GTB(r, M_G, S_G, w_T, p_T, w_B, p_B, w_{0B}, r_{max}) =} \\ \mathbf{(G(r, M_G, S_G)+} \\ \mathbf{T(r, w_T, d_T) \quad +} \\ \mathbf{B(r, w_B, p_B, w_{0B}, r_{max})) * 1(r \leq r_{max}))} \end{aligned}$$

where:

Component G (central Gaussian):
$G(r, M_G, S_G) = N(r, M_G, S_G)$
where N(r,m,s) denotes a Gaussian with mean m and standard deviation s

Component T(torus):
$T(r, A, p) = -A*\cos(\pi/p*r)$

Component B (border vortex):
$B(r, A, p, w_0, r_{max}) = - A*\sin((2\pi)/(r-rb) + w_0$
with $rb = r_{max} - p/2$

and

1(E) = 1 if expression E is true, 0 otherwise.